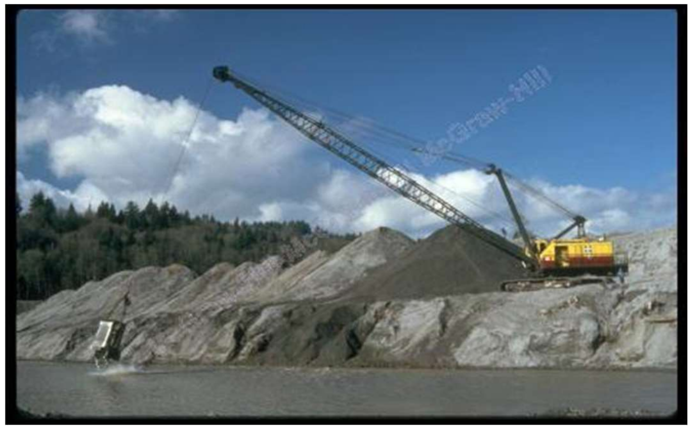
ME211 Statics and Strength of Materials

CHAPTER 4
EQUILIBRIUM OF RIGID BODIES

Contents

Application

Engineers designing this crane will need to determine the forces that act on this body under various conditions.



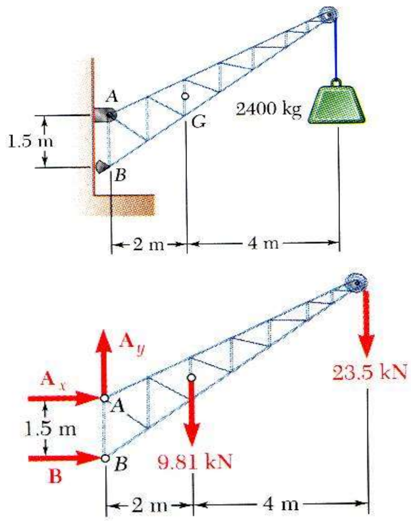
Introduction

- For a rigid body, the condition of static equilibrium means that **the *body under study* does not translate or rotate under** the given loads that act on the body
- The necessary and sufficient conditions for the static equilibrium of a body are that the **forces sum to zero, and the moment about any point sum to zero:**

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

- Equilibrium analysis can be applied to two-dimensional or three-dimensional bodies, but the first step in any analysis is the creation of the ***free body diagram***

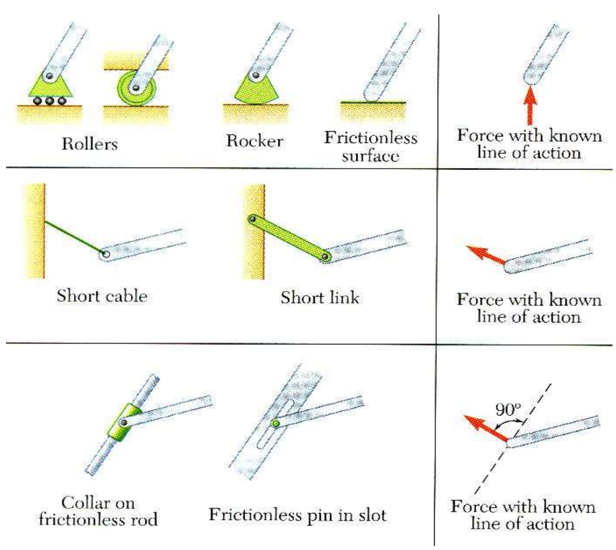
Free-Body Diagram



The first step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free body diagram*.

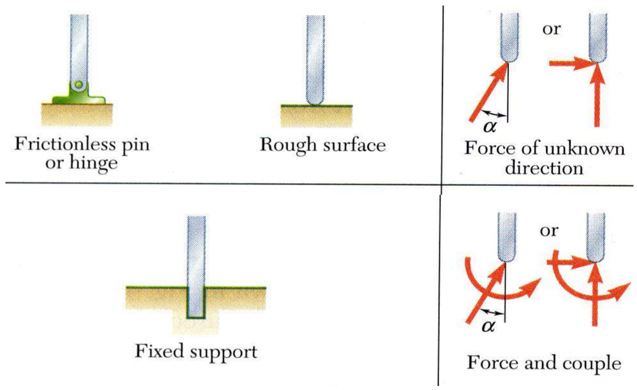
- Select the body to be analyzed and **detach** it from **the ground and all other bodies and/or supports**.
- Indicate point of application, magnitude, and direction of **external forces**, including **the rigid body weight**.
- Indicate **point of application and assumed direction of unknown forces** from reactions of the ground and/or other bodies, such as the supports.
- **Include the dimensions**, which will be needed to compute the moments of the forces.

Reactions at Supports and Connections for a Two-Dimensional Structure



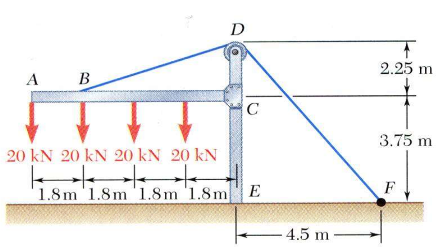
- Reactions equivalent to a force with known line of action.

Reactions at Supports and Connections for a Two-Dimensional Structure



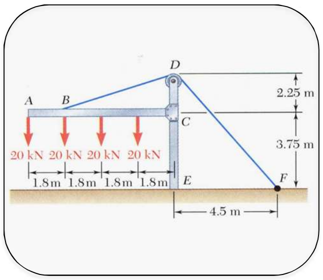
- Reactions equivalent to a force of unknown direction and magnitude.
- Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude

Practice



The frame shown supports part of the roof of a small building. Your goal is to draw the free body diagram (FBD) for the frame.

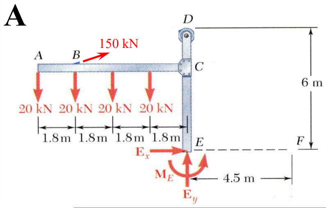
On the following page, you will choose the most correct FBD for this problem.



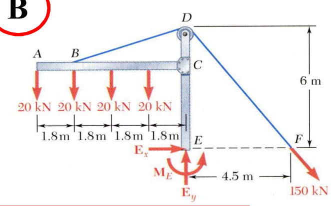
Choose the most correct FBD for the original problem.

Discuss with a neighbor why each choice is correct or incorrect.

A

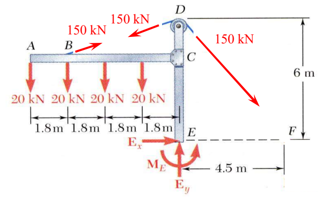


B

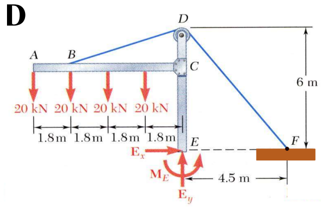


B is the most correct, though C is also correct. A & D are incorrect; why?

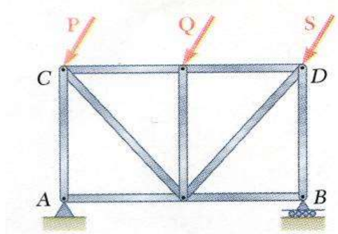
C



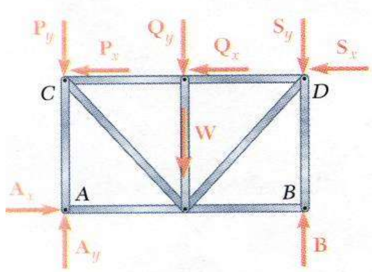
D



Equilibrium of a Rigid Body in Two Dimensions



(a)



(b)

- For known forces and moments that act on a two-dimensional structure, the following are true:

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

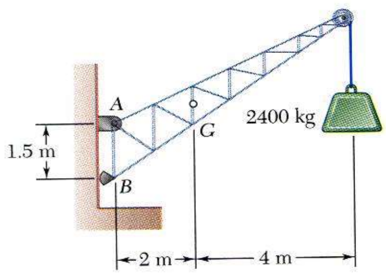
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

where A can be any point in the plane of the body.

- **The 3 equations can be solved for no more than 3 unknowns**.
- The 3 equations cannot be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

Sample Problem 4.1



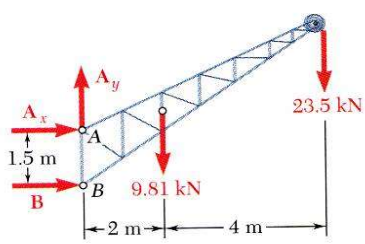
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Sample Problem 4.1



- Create the free-body diagram.

- Determine *B* by solving the equation for the sum of the moments of all forces about *A*.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\text{ kN}(2\text{m}) - 23.5\text{ kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\text{ kN}}$$

- Determine the reactions at *A* by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0: \quad A_x + B = 0$$

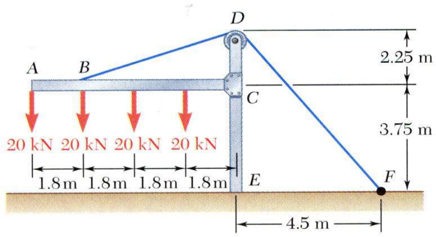
$$\boxed{A_x = -107.1\text{kN}}$$

$$\sum F_y = 0: \quad A_y - 9.81\text{kN} - 23.5\text{kN} = 0$$

$$\boxed{A_y = +33.3\text{ kN}}$$

- Check the values obtained.

Sample Problem 4.4



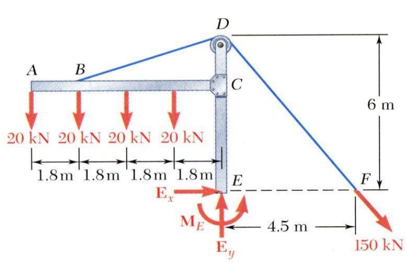
The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.

SOLUTION:

- Create a free-body diagram for the frame and cable.
- Apply the equilibrium equations for the reaction force components and couple at *E*.

Sample Problem 4.4



- The free-body diagram was created in an earlier exercise.
- Apply one of the three equilibrium equations. Try using the condition that the sum of forces in the x-direction must sum to zero.

- Which equation is correct?

A. $\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\text{ kN}) = 0$

$E_x = -90.0\text{ kN}$

B. $\sum F_x = 0: \quad E_x + \cos 36.9^o (150\text{ kN}) = 0$

C. $\sum F_x = 0: \quad E_x + \sin 36.9^o (150\text{ kN}) = 0$

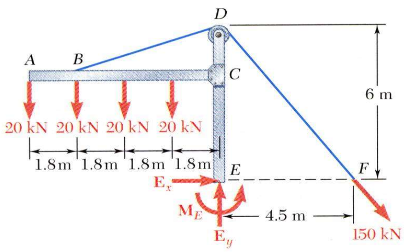
$E_x = -90.0\text{ kN}$

D. $\sum F_x = 0: \quad E_x + \frac{6}{7.5}(150\text{ kN}) = 0$

E. $\sum F_x = 0: \quad E_x - \sin 36.9^o (150\text{ kN}) = 0$

- What does the negative sign signify?

Sample Problem 4.4



- Now apply the condition that the sum of forces in the y-direction must sum to zero.

- Which equation is correct?

A. $\sum F_y = 0: E_y - 4(20\text{ kN}) - \sin 36.9^o(150\text{ kN}) = 0$

B. $\sum F_y = 0: \quad E_y - 4(20\text{ kN}) + \frac{6}{7.5}(150\text{ kN}) = 0$

C. $\sum F_y = 0: E_y - 4(20\text{ kN}) - \cos 36.9^o(150\text{ kN}) = 0$

$E_y = +200\text{ kN}$

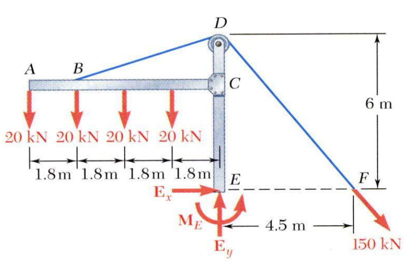
D. $\sum F_y = 0: \quad E_y - 4(20\text{ kN}) - \frac{6}{7.5}(150\text{ kN}) = 0$

$E_y = +200\text{ kN}$

E. $\sum F_y = 0: \quad E_y + 4(20\text{ kN}) - \frac{6}{7.5}(150\text{ kN}) = 0$

- What does the positive sign signify?

Sample Problem 4.4



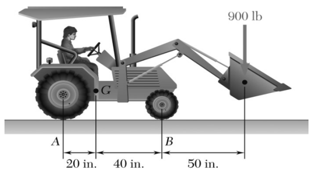
- Finally, apply the condition that the sum of moments about *any point* must equal zero.

- Three good points are D, E, and F. Discuss what advantage each point has over the others, or perhaps why each is equally good.
- Assume that you choose point E to apply the sum-of-moments condition. Write the equation and compare your answer with a neighbor.

$$\sum M_E = 0: \quad +20\text{ kN}(7.2\text{ m}) + 20\text{ kN}(5.4\text{ m}) + 20\text{ kN}(3.6\text{ m}) + 20\text{ kN}(1.8\text{ m}) - \frac{6}{7.5}(150\text{ kN})4.5\text{ m} + M_E = 0$$

$M_E = 180.0\text{ kN}\cdot\text{m}$

Practice



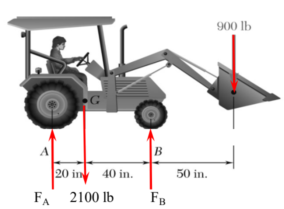
A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two rear wheels and two front wheels

- First, create a *free body diagram*.
- Second, apply the *equilibrium conditions* to generate the three equations, and use these to solve for the desired quantities.

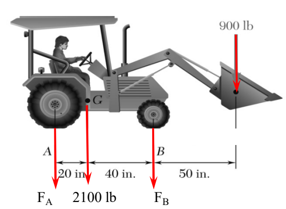
Practice

- Draw the free body diagram of the tractor (on your own first).
- From among the choices, choose the best FBD, and discuss the problem(s) with the other FBDs.

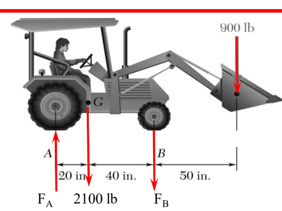
A.



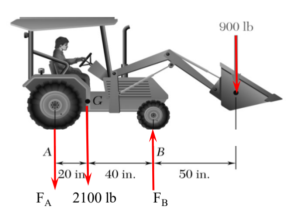
B.



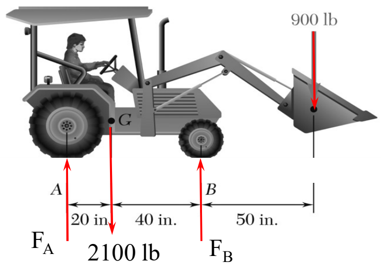
C.



D.



Practice

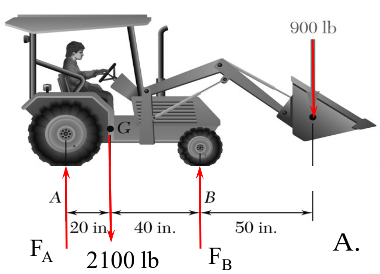


Now let's apply the equilibrium conditions to this FBD.

- Start with the moment equation:

$$\sum M_{pt} = 0$$

Points A or B are equally good because each results in an equation with only one unknown.



Assume we chose to use point B. Choose the correct equation for

$$\sum M_B = 0.$$

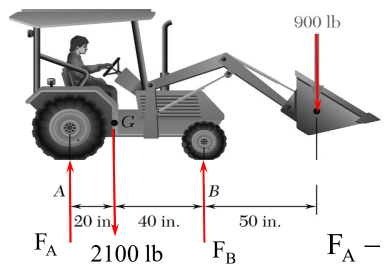
A. $+F_A(60\text{ in.}) - 2100\text{lb}(40\text{ in.}) - 900\text{ lb}(50\text{ in.}) = 0$

B. $+F_A(20\text{ in.}) - 2100\text{lb}(40\text{ in.}) - 900\text{ lb}(50\text{ in.}) = 0$

C. $-F_A(60\text{ in.}) - 2100\text{lb}(40\text{ in.}) + 900\text{ lb}(50\text{ in.}) = 0$

D. $-F_A(60\text{ in.}) + 2100\text{lb}(40\text{ in.}) - 900\text{ lb}(50\text{ in.}) = 0$

F_A=650 lb, so the reaction *at each wheel* is 325 lb



Now apply the final equilibrium condition, $\Sigma F_y = 0$.

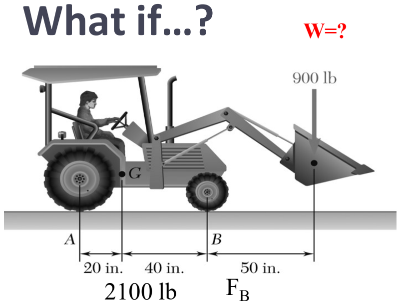
$$F_A - 2100\ \text{lb} + F_B - 900\ \text{lb} = 0$$

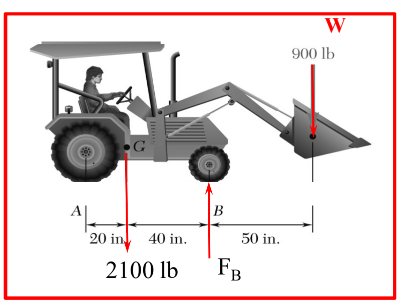
$$\text{or}\ \ +650\ \text{lb} - 2100\ \text{lb} + F_B - 900\ \text{lb} = 0$$

$$\Rightarrow \boxed{F_B = 2350\ \text{lb, or 1175 lb at each front wheel}}$$

Why was the third equilibrium condition, $\Sigma F_x = 0$ not used?

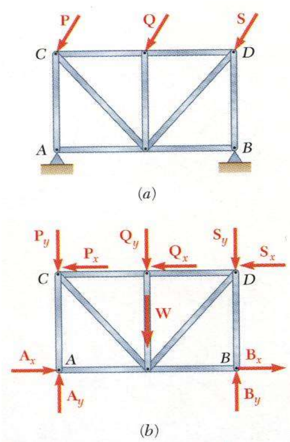
What if…?



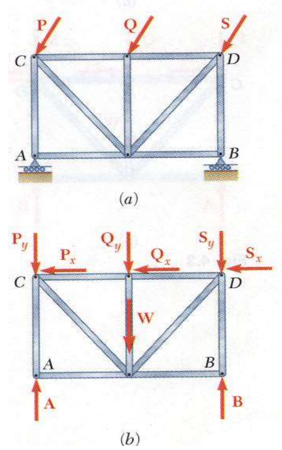


- Now suppose we have a different problem: How much gravel can this tractor carry before it tips over?
- Discuss with a neighbor how you would solve this problem.
- Hint: Think about what the free body diagram would be for this situation…

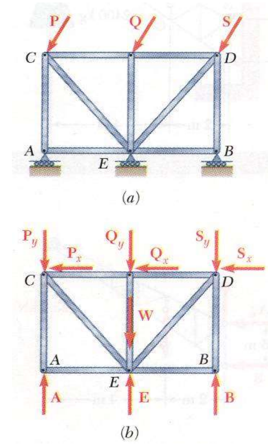
Statically Indeterminate Reactions



- More unknowns than equations



- Fewer unknowns than equations, **partially constrained**



- Equal number unknowns and equations but **improperly constrained**

Equilibrium of a Rigid Body in Three Dimensions

- **Six scalar equations** are required to express the conditions for the equilibrium of a rigid body in the general **three dimensional case**.

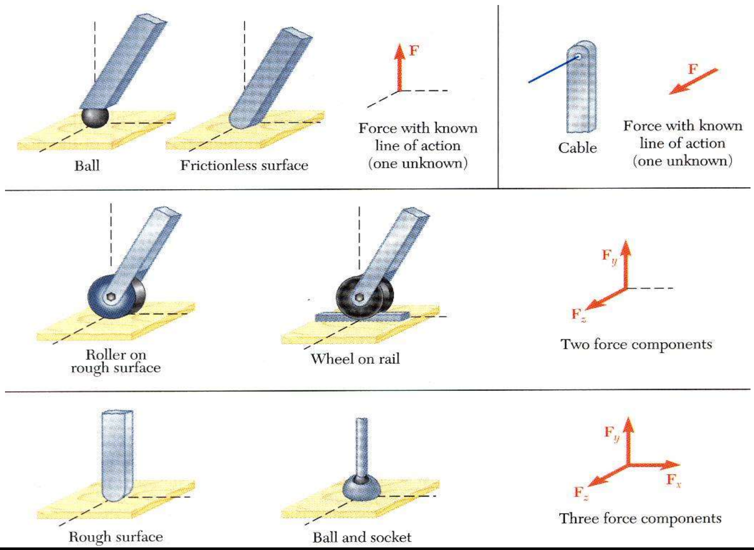
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections or unknown applied forces.

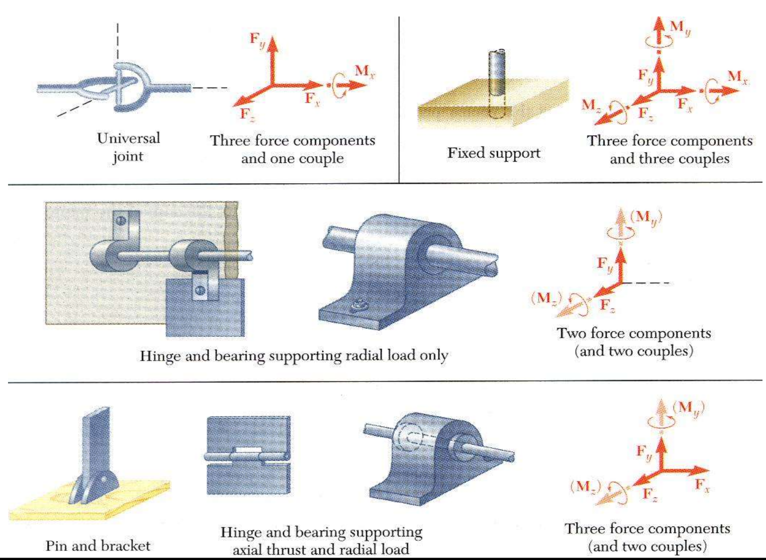
- **The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,**

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F}\right) = 0$$

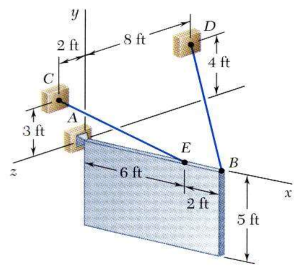
Reactions at Three-Dimensional Structure



Reactions at Three-Dimensional Structure



Sample Problem 4.8



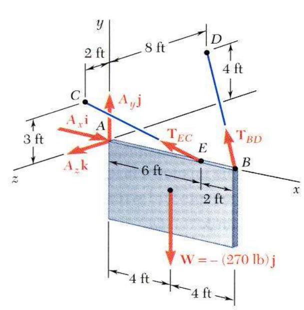
A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at *A* and by two cables.

Determine the tension in each cable and the reaction at *A*.

SOLUTION:

- Create a free-body diagram for the sign.
- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

Sample Problem 4.8



- Create a free-body diagram for the sign.

  Since there are only 5 unknowns, the sign is partially constrained. All forces intersect with the x-axis, so $\Sigma M_X=0$, so this equation is not useful to the solution.

$$\vec{T}_{BD} = T_{BD}\frac{\vec{r}_D - \vec{r}_B}{\left|\vec{r}_D - \vec{r}_B\right|}$$

$$= T_{BD}\frac{-8\vec{i} + 4\vec{j} - 8\vec{k}}{12}$$

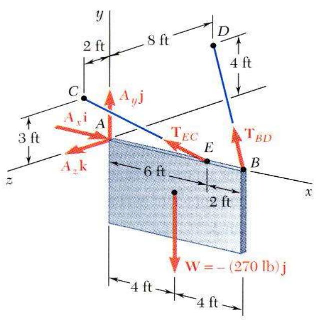
$$= T_{BD}\left(-\tfrac{2}{3}\vec{i} + \tfrac{1}{3}\vec{j} - \tfrac{2}{3}\vec{k}\right)$$

$$\vec{T}_{EC} = T_{EC}\frac{\vec{r}_C - \vec{r}_E}{\left|\vec{r}_C - \vec{r}_E\right|}$$

$$= T_{EC}\frac{-6\vec{i} + 3\vec{j} + 2\vec{k}}{7}$$

$$= T_{EC}\left(-\tfrac{6}{7}\vec{i} + \tfrac{3}{7}\vec{j} + \tfrac{2}{7}\vec{k}\right)$$

Sample Problem 4.8



- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

$$\sum \vec{F} = \vec{A} + \vec{T}_{BD} + \vec{T}_{EC} - (270\text{ lb})\vec{j} = 0$$

$$\vec{i}: \quad A_x - \tfrac{2}{3}T_{BD} - \tfrac{6}{7}T_{EC} = 0$$

$$\vec{j}: \quad A_y + \tfrac{1}{3}T_{BD} + \tfrac{3}{7}T_{EC} - 270\text{ lb} = 0$$

$$\vec{k}: \quad A_z - \tfrac{2}{3}T_{BD} + \tfrac{2}{7}T_{EC} = 0$$

$$\sum \vec{M}_A = \vec{r}_B \times \vec{T}_{BD} + \vec{r}_E \times \vec{T}_{EC} + (4\text{ ft})\vec{i} \times (-270\text{ lb})\vec{j} = 0$$

$$\vec{j}: \quad 5.333\,T_{BD} - 1.714\,T_{EC} = 0$$

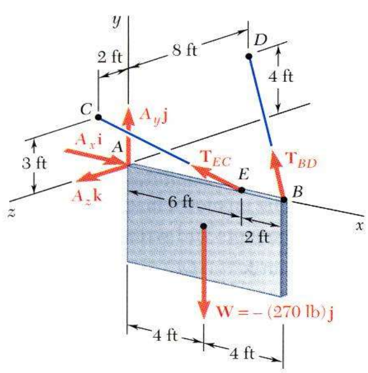
$$\vec{k}: \quad 2.667\,T_{BD} + 2.571\,T_{EC} - 1080\text{ lb} = 0$$

Solve the 5 equations for the 5 unknowns,

$$T_{BD} = 101.3\text{ lb} \qquad T_{EC} = 315\text{ lb}$$

$$\vec{A} = (338\text{ lb})\vec{i} + (101.2\text{ lb})\vec{j} - (22.5\text{ lb})\vec{k}$$

What if…?



Could this sign be in static equilibrium if cable BD were removed?

The sign could not be in static equilibrium because T_{EC} causes a moment about the y-axis (due to the existence of $T_{EC,Z}$) which must be countered by an equal and opposite moment. This can only be provided by a cable tension that has a z-component in the negative-z direction, such as what T_{BD} has.