

ME211 Statics and Strength of Materials

CHAPTER 5.5 FORCES IN BEAMS AND CABLES

Contents

[Introduction](#)

[Internal Forces in Members](#)

[Sample Problem 7.1](#)

[Various Types of Beam Loading and
Support](#)

[Shear and Bending Moment in a Beam](#)

[Sample Problem 7.2](#)

[Sample Problem 7.3](#)

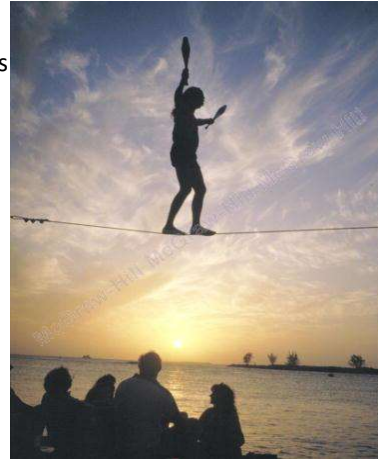
[Relations Among Load, Shear, and
Bending Moment](#)

[Sample Problem 7.4](#)

[Sample Problem 7.6](#)

Application

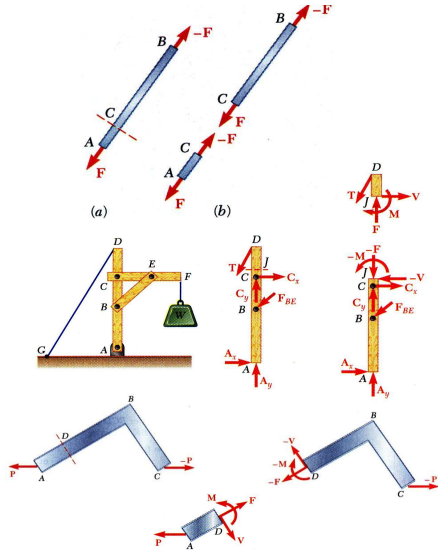
Forces that are *internal* to the structural members beams are the subject of this chapter



Introduction

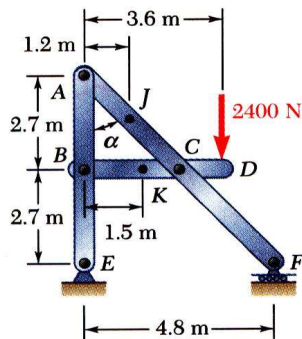
- Preceding chapters dealt with:
 - a) determining **external** forces acting on a structure and
 - b) determining forces which hold together the various members of a structure. Lets call joint forces!**
- The current chapter is concerned with determining the **internal forces** (i.e., tension/compression, shear, and bending) which hold together the various parts of a given member.
- Focus is on:
 - a) *Beams* - usually long, straight, prismatic members designed to support loads applied at various points along the member.

Internal Forces in Members



- Straight two-force member AB is in equilibrium under application of F and $-F$.
- **Internal forces** equivalent to F and $-F$ are required for equilibrium of free-bodies AC and CB .
- Multiforce member $ABCD$ is in equilibrium under application of cable and member contact forces.
- Internal forces equivalent to a force-couple system are necessary for equilibrium of free-bodies JD and $ABCJ$.
- An internal force-couple system is required for equilibrium of two-force members which are not straight.

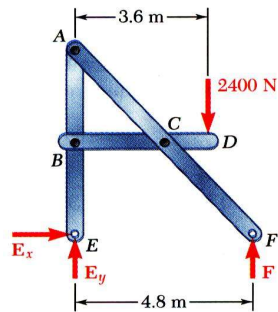
Sample Problem 7.1



Determine the internal forces (a) in member ACF at point J and (b) in member BCD at K .

SOLUTION:

- Compute reactions and forces at connections for each member.
- **Cut member ACF at J .** The internal forces at J are represented by equivalent force-couple system which is determined by considering equilibrium of either part.
- **Cut member BCD at K .** Determine force-couple system equivalent to internal forces at K by applying equilibrium conditions to either part.



SOLUTION:

- Compute reactions and connection forces.

Consider entire frame as a free-body, and apply equilibrium conditions:

$$\sum M_E = 0 :$$

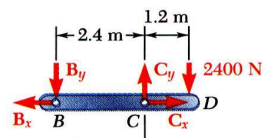
$$-(2400 \text{ N})(3.6 \text{ m}) + F(4.8 \text{ m}) = 0 \quad F = 1800 \text{ N}$$

$$\sum F_y = 0 :$$

$$-2400 \text{ N} + 1800 \text{ N} + E_y = 0 \quad E_y = 600 \text{ N}$$

$$\sum F_x = 0 :$$

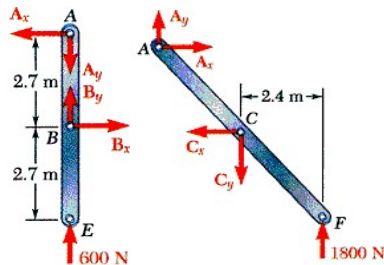
$$E_x = 0$$



Drawing the FBD for member BCD:

- Why are forces at B and C drawn in these directions? Is there a choice on the directions?

- Why are there two force components at each point instead of just a single force?

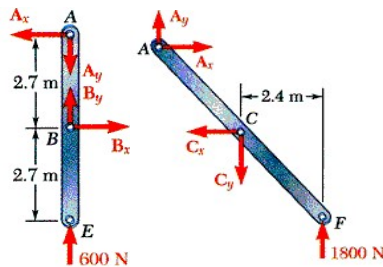
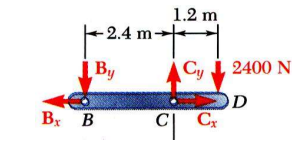


Drawing the FBD for member ABE:

- Why are forces at B in these directions? Is there a choice on the directions?

- Why are there two force components at A instead of just a single force?

Finally, the FBD for member ACF.



Consider member *BCD* as free-body:

$$\sum M_B = 0 :$$

$$-(2400 \text{ N})(3.6 \text{ m}) + C_y(2.4 \text{ m}) = 0 \quad C_y = 3600 \text{ N}$$

$$\sum M_C = 0 :$$

$$-(2400 \text{ N})(1.2 \text{ m}) + B_y(2.4 \text{ m}) = 0 \quad B_y = 1200 \text{ N}$$

$$\sum F_x = 0 : \quad -B_x + C_x = 0$$

Consider member *ABE* as free-body:

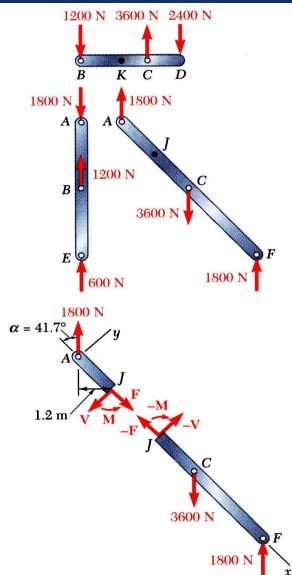
$$\sum M_A = 0 : \quad B_x(2.4 \text{ m}) = 0 \quad B_x = 0$$

$$\sum F_x = 0 : \quad B_x - A_x = 0 \quad A_x = 0$$

$$\sum F_y = 0 : \quad -A_y + B_y + 600 \text{ N} = 0 \quad A_y = 1800 \text{ N}$$

From member *BCD*,

$$\sum F_x = 0 : \quad -B_x + C_x = 0 \quad C_x = 0$$



- Cut member *ACF* at *J*. The internal forces at *J* are represented by equivalent force-couple system.

Consider free-body *AJ*:

$$\sum M_J = 0 :$$

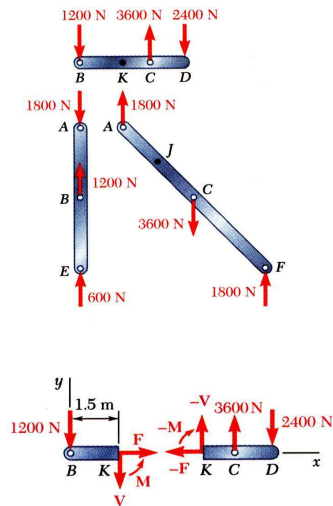
$$-(1800 \text{ N})(1.2 \text{ m}) + M = 0 \quad M = 2160 \text{ N} \cdot \text{m}$$

$$\sum F_x = 0 :$$

$$F - (1800 \text{ N})\cos 41.7^\circ = 0 \quad F = 1344 \text{ N}$$

$$\sum F_y = 0 :$$

$$-V + (1800 \text{ N})\sin 41.7^\circ = 0 \quad V = 1197 \text{ N}$$



- Cut member BCD at K . Determine a force-couple system equivalent to internal forces at K .

Consider free-body BK :

$$\sum M_K = 0 :$$

$$(1200 \text{ N})(1.5 \text{ m}) + M = 0$$

$$M = -1800 \text{ N} \cdot \text{m}$$

$$\sum F_x = 0 :$$

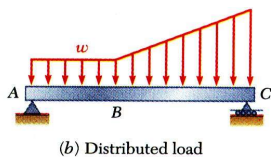
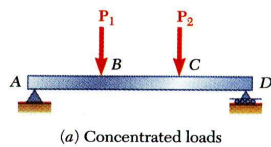
$$F = 0$$

$$\sum F_y = 0 :$$

$$-1200 \text{ N} - V = 0$$

$$V = -1200 \text{ N}$$

Various Types of Beam Loading and Support



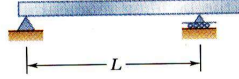
- *Beam* - structural member designed to support loads applied at various points along its length.
- Beam can be subjected to **concentrated** loads or **distributed** loads or combination of both.
- *Beam design* is two-step process:
 - 1) determine shearing forces and bending moments produced by applied loads
 - 2) select cross-section best suited to resist shearing forces and bending moments

Forces in Beams and Cables

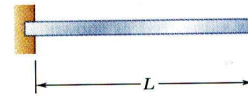
Statically
Determinate
Beams



(a) Simply supported beam

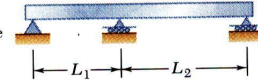


(b) Overhanging beam

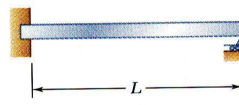


(c) Cantilever beam

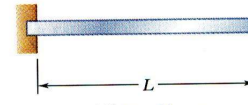
Statically
Indeterminate
Beams



(d) Continuous beam



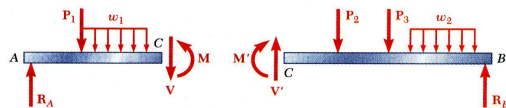
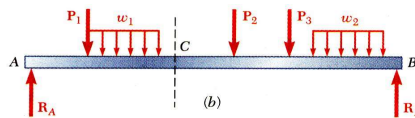
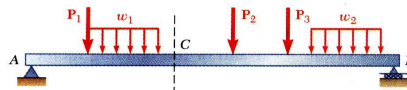
(e) Beam fixed at one end
and simply supported
at the other end



(f) Fixed beam

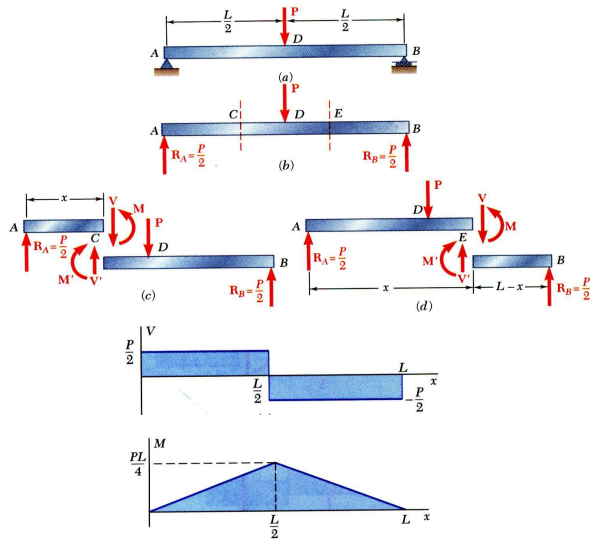
- Beams are classified according to way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.

Shear and Bending Moment in a Beam



- Wish to determine bending moment and shearing force at any point (for example, point C) in a beam subjected to concentrated and distributed loads.
- Determine reactions at supports by treating whole beam as free-body.
- Cut beam at C and draw free-body diagrams for AC and CB. **By definition, positive sense for internal force-couple systems are as shown for each beam section.**
- From equilibrium considerations, determine M and V or M' and V' .

Shear and Bending Moment Diagrams

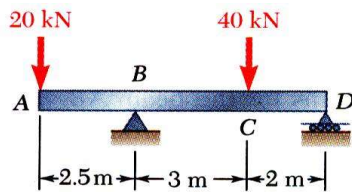


- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at C and consider member AC,

$$V = +P/2 \quad M = +Px/2$$
- Cut beam at E and consider member EB,

$$V = -P/2 \quad M = +P(L-x)/2$$
- For a beam subjected to **concentrated loads**, shear is constant between loading points and moment varies linearly.

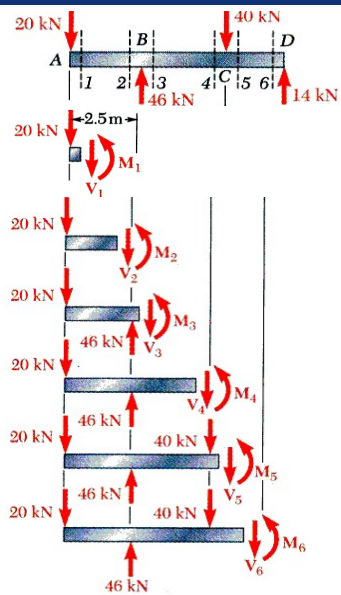
Sample Problem 7.2



Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at B and D.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results.



SOLUTION:

- Taking entire beam as a free-body, calculate reactions at B and D.
- Find equivalent internal force-couple systems at sections on either side of load application points.

$$\sum F_y = 0: \quad -20 \text{ kN} - V_1 = 0$$

$$V_1 = -20 \text{ kN}$$

$$\sum M_2 = 0: \quad (20 \text{ kN})(0 \text{ m}) + M_1 = 0$$

$$M_1 = 0$$

Similarly,

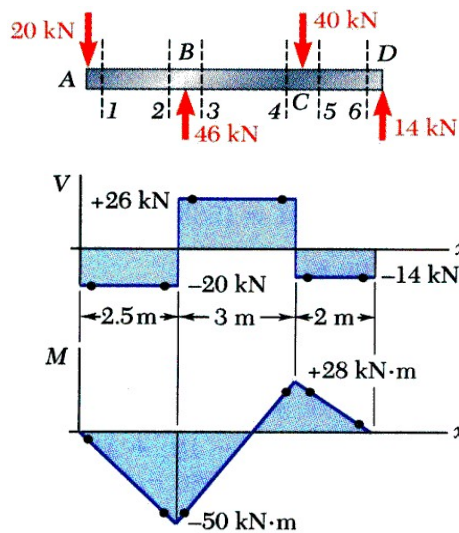
$$V_2 = -20 \text{ kN} \quad M_2 = -50 \text{ kN} \cdot \text{m}$$

$$V_3 = 26 \text{ kN} \quad M_3 = -50 \text{ kN} \cdot \text{m}$$

$$V_4 = 26 \text{ kN} \quad M_4 = +28 \text{ kN} \cdot \text{m}$$

$$V_5 = -14 \text{ kN} \quad M_5 = +28 \text{ kN} \cdot \text{m}$$

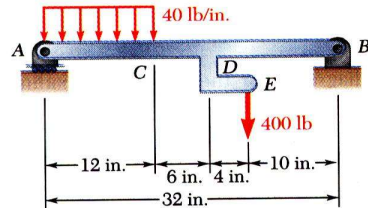
$$V_6 = -14 \text{ kN} \quad M_6 = 0 \text{ kN} \cdot \text{m}$$



- Plot results.

Note that shear is of constant value between concentrated loads and bending moment varies linearly.

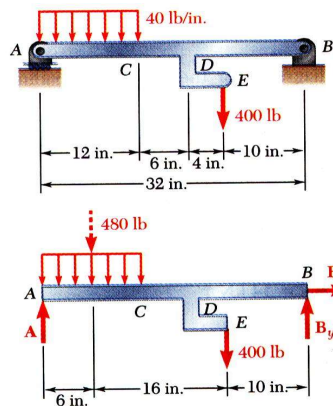
Sample Problem 7.3



Draw the shear and bending moment diagrams for the beam AB. The distributed load of 40 lb/in. extends over 12 in. of the beam, from A to C, and the 400 lb load is applied at E.

SOLUTION:

- Taking entire beam as free-body, calculate reactions at A and B.
- Determine equivalent internal force-couple systems at sections cut within segments AC, CD, and DB.
- Plot results.



SOLUTION:

- Taking entire beam as a free-body, calculate reactions at A and B.

$$\sum M_A = 0 :$$

$$B_y(32 \text{ in.}) - (480 \text{ lb})(6 \text{ in.}) - (400 \text{ lb})(22 \text{ in.}) = 0$$

$$B_y = 365 \text{ lb}$$

$$\sum M_B = 0 :$$

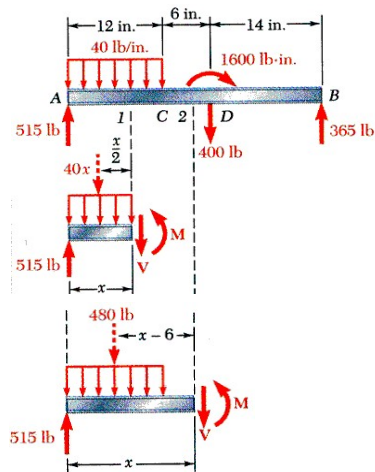
$$(480 \text{ lb})(26 \text{ in.}) + (400 \text{ lb})(10 \text{ in.}) - A(32 \text{ in.}) = 0$$

$$A = 515 \text{ lb}$$

$$\sum F_x = 0 :$$

$$B_x = 0$$

- Note: The 400 lb load at E may be replaced by a 400 lb force and 1600 lb-in. couple at D.



- Evaluate equivalent internal force-couple systems at sections cut within segments AC, CD, and DB.

From A to C:

$$\sum F_y = 0: \quad 515 - 40x - V = 0$$

$$V = 515 - 40x$$

$$\sum M_1 = 0: \quad -515x - 40x\left(\frac{1}{2}x\right) + M = 0$$

$$M = 515x - 20x^2$$

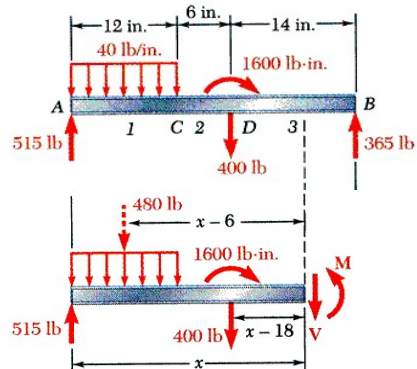
From C to D:

$$\sum F_y = 0: \quad 515 - 480 - V = 0$$

$$V = 35 \text{ lb}$$

$$\sum M_2 = 0: \quad -515x + 480(x - 6) + M = 0$$

$$M = (2880 + 35x) \text{ lb} \cdot \text{in.}$$



- Evaluate equivalent internal force-couple systems at sections cut within segments AC, CD, and DB.

From D to B:

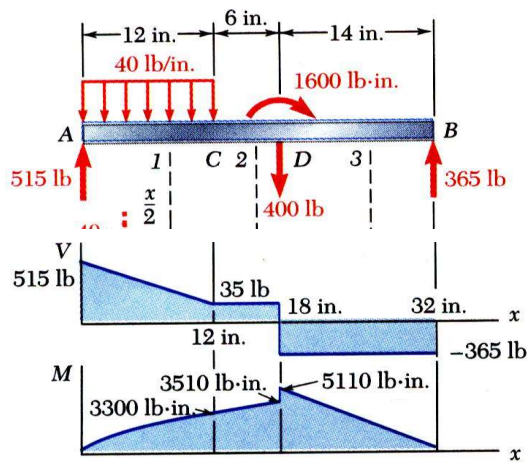
$$\sum F_y = 0: \quad 515 - 480 - 400 - V = 0$$

$$V = -365 \text{ lb}$$

$$\sum M_2 = 0:$$

$$-515x + 480(x - 6) - 1600 + 400(x - 18) + M = 0$$

$$M = (11,680 - 365x) \text{ lb} \cdot \text{in.}$$



- Plot results.

From A to C:

$$V = 515 - 40x$$

$$M = 515x - 20x^2$$

From C to D:

$$V = 35 \text{ lb}$$

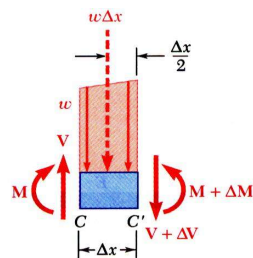
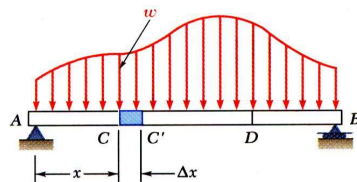
$$M = (2880 + 35x) \text{ lb} \cdot \text{in.}$$

From D to B:

$$V = -365 \text{ lb}$$

$$M = (11,680 - 365x) \text{ lb} \cdot \text{in.}$$

Relations Among Load, Shear, and Bending Moment



- Relations between load and shear:

$$V - (V + \Delta V) - w\Delta x = 0$$

$$\frac{dV}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta V}{\Delta x} = -w$$

$$V_D - V_C = - \int_{x_C}^{x_D} w dx = -(\text{area under load curve})$$

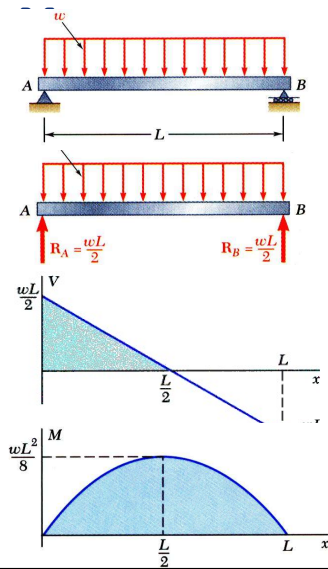
- Relations between shear and bending moment:

$$(M + \Delta M) - M - V\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\frac{dM}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta M}{\Delta x} = \lim_{\Delta x \rightarrow 0} \left(V - \frac{1}{2} w\Delta x \right) = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V dx = (\text{area under shear curve})$$

Relations Among Load, Shear, and Bending



- Reactions at supports, $R_A = R_B = \frac{wL}{2}$

- Shear curve,

$$V - V_A = -\int_0^x w dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

- Moment curve,

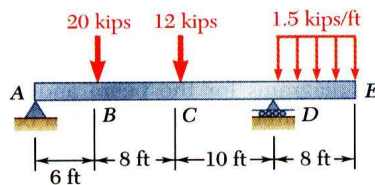
$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right) dx = \frac{w}{2}(Lx - x^2)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0 \right)$$

Sample Problem 7.4

SOLUTION:



Draw the shear and bending-moment diagrams for the beam and loading shown.

- Taking entire beam as a free body, determine reactions at supports.

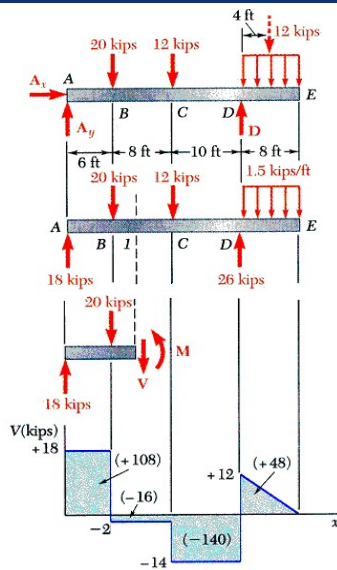
- Between concentrated load application points, and shear is constant. $dV/dx = -w = 0$

- With uniform loading between D and E , the shear variation is linear.

- Between concentrated load application points, The change in moment between load application points is equal to area under shear curve between points.

$$dM/dx = V = \text{constant}$$

- With a linear shear variation between D and E , the bending moment diagram is a parabola.



SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.

$$\sum M_A = 0 :$$

$$D(24 \text{ ft}) - (20 \text{ kips})(6 \text{ ft}) - (12 \text{ kips})(14 \text{ ft}) - (12 \text{ kips})(28 \text{ ft}) = 0$$

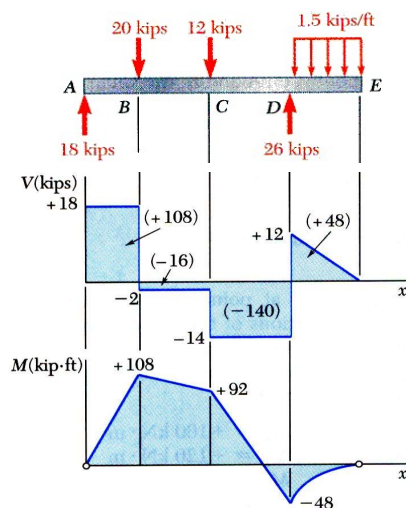
$$D = 26 \text{ kips}$$

$$\sum F_y = 0 :$$

$$A_y - 20 \text{ kips} - 12 \text{ kips} + 26 \text{ kips} - 12 \text{ kips} = 0$$

$$A_y = 18 \text{ kips}$$

- Between concentrated loads, $dV/dx = -w = 0$ and shear is constant and determined by appropriate section cut and solution.
- With uniform loading between D and E, the shear variation is linear.



- Between concentrated load application points, The change in moment between load application points is equal to area under the shear curve between points. $dM/dx = V = \text{constant}$.

$$M_B - M_A = +108 \quad M_B = +108 \text{ kip} \cdot \text{ft}$$

$$M_C - M_B = -16 \quad M_C = +92 \text{ kip} \cdot \text{ft}$$

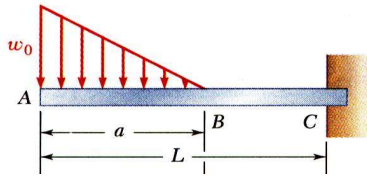
$$M_D - M_C = -140 \quad M_D = -48 \text{ kip} \cdot \text{ft}$$

$$M_E - M_D = +48 \quad M_E = 0$$

- With a linear shear variation between D and E, the bending moment diagram is a parabola.

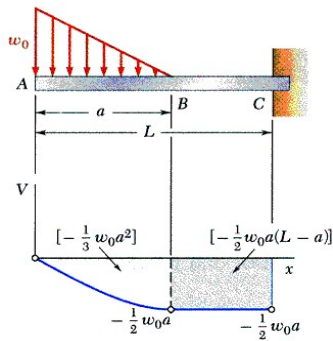
Sample Problem 7.6

SOLUTION:



Sketch the shear and bending-moment diagrams for the cantilever beam and loading shown.

- The change in shear between A and B is equal to the negative of area under load curve between points. The linear load curve results in a parabolic shear curve.
- With zero load, change in shear between B and C is zero.
- The change in moment between A and B is equal to area under shear curve between points. The parabolic shear curve results in a cubic moment curve.
- The change in moment between B and C is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.



SOLUTION:

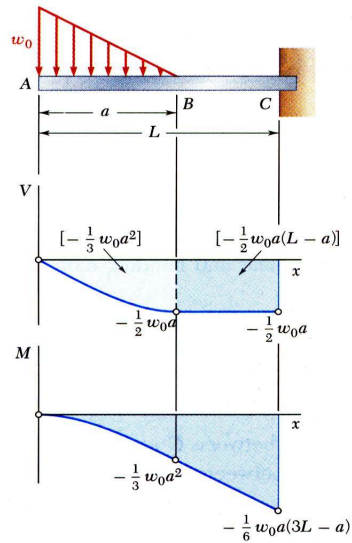
- The change in shear between A and B is equal to negative of area under load curve between points. The linear load curve results in a parabolic shear curve.

$$\text{at } A, \quad V_A = 0, \quad \frac{dV}{dx} = -w = -w_0$$

$$V_B - V_A = -\frac{1}{2} w_0 a \quad V_B = -\frac{1}{2} w_0 a$$

$$\text{at } B, \quad \frac{dV}{dx} = -w = 0$$

- With zero load, change in shear between B and C is zero.



- The change in moment between A and B is equal to area under shear curve between the points. The parabolic shear curve results in a cubic moment curve.

$$\text{at } A, \quad M_A = 0, \quad \frac{dM}{dx} = V = 0$$

$$M_B - M_A = -\frac{1}{3}w_0a^2 \quad M_B = -\frac{1}{3}w_0a^2$$

$$M_C - M_B = -\frac{1}{2}w_0a(L-a) \quad M_C = -\frac{1}{6}w_0a(3L-a)$$

- The change in moment between B and C is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.