# ME 211 Statics and Strength of Materials

## Chapter 7

### Introduction - Concept of Stress

## Concept of Stress

The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.

Both the analysis and design of a given structure involve the determination of *stresses* and *deformations*. This chapter is devoted to the concept of stress.

## Review of Statics

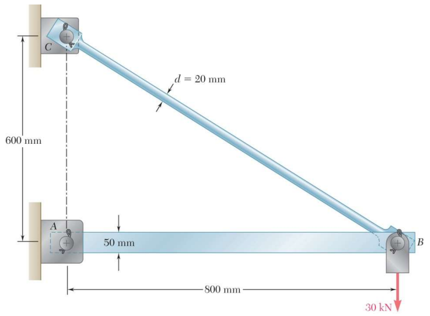


**Fig. 1.1** Boom used to support a 30-kN load.

The structure is designed to support a 30 kN load

The structure consists of a boom *AB* and rod *BC* joined by pins (zero moment connections) at the junctions and supports

Perform a static analysis to determine the reaction forces at the supports and the internal force in each structural member

## Structure Free-Body Diagram

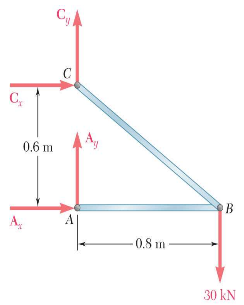


**Fig. 1.2** Free-body diagram of boom showing Applied load and reaction forces.

Structure is detached from supports and the loads and reaction forces are indicated to produce a free-body diagram

Conditions for static equilibrium:

$$+\circlearrowleft \quad \sum M_C = 0 = A_x(0.6\,\text{m}) - (30\,\text{kN})(0.8\,\text{m})$$
$$A_x = +40\,\text{kN}$$
$$\overset{+}{\rightarrow} \quad \sum F_x = 0 = A_x + C_x$$
$$C_x = -A_x = -40\,\text{kN}$$
$$+\uparrow \quad \sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$
$$A_y + C_y = +30\,\text{kN}$$

$A_y$ and $C_y$ cannot be determined from these equations

# Component Free-Body Diagram

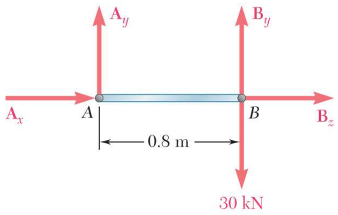



**Fig. 1.3** Free-body diagram of member *AB* freed from structure.

In addition to the complete structure, each component must satisfy the conditions for static equilibrium

Consider a free-body diagram of the boom *AB*:

$$+\circlearrowleft \sum M_B = 0 = -A_y(0.8\,\text{m})$$

$$A_y = 0$$

substitute into the structure equilibrium equation

$$C_y = +30\,\text{kN}$$

Results:

$$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

Reaction forces are directed along the boom and rod

# Method of Joints

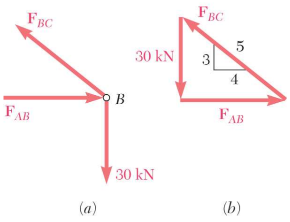



**Fig. 1.4** Free-body diagram of boom's joint B and associated force triangle.

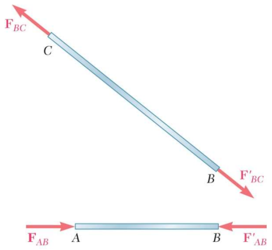



**Fig. 1.5** Free-body diagrams of two-force members *AB* and *BC*.

Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

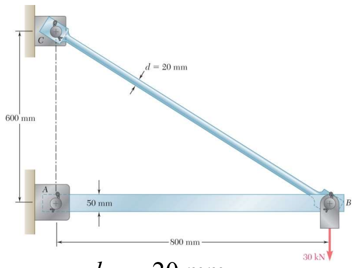
$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at the ends of the members

For equilibrium, the forces must be parallel to an axis between the force application points, equal in magnitude, and in opposite directions

## Stress Analysis




$d_{BC} = 20$ mm

**Fig. 1.1** Boom used to support a 30-kN load.

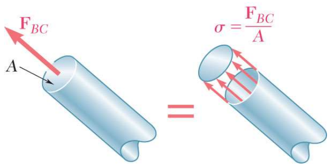



**Fig. 1.7** Axial force represents the resultant of distributed elementary forces.

Can the structure safely support the 30 kN load if rod *BC* has a diameter of 20 mm?

From a statics analysis

$F_{AB} = 40$ kN (compression)

$F_{BC} = 50$ kN (tension)

At any section through member BC, the internal force is 50 kN with a force intensity or <u>stress</u> of

$$\sigma_{BC} = \frac{P}{A} = \frac{50\times10^3\,\text{N}}{314\times10^{-6}\,\text{m}^2} = 159\ \text{MPa}$$

From the material properties for steel, the allowable stress is

$$\sigma_{\text{all}} = 165\,\text{MPa}$$

Conclusion: the strength of member *BC* is adequate

## Design

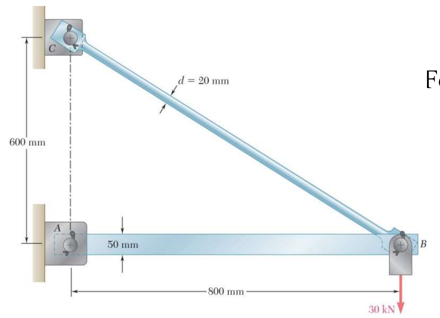



**Fig. 1.1** Boom used to support a 30-kN load.

Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum ($\sigma_{all}$= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^3\,\text{N}}{100\times10^6\,\text{Pa}} = 500\times10^{-6}\text{m}^2$$

$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\text{m}^2\right)}{\pi}} = 2.52\times10^{-2}\text{m} = 25.2\,\text{mm}$$

An aluminum rod 26 mm or more in diameter is adequate

# Axial Loading: Normal Stress

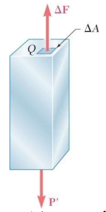


**Fig. 1.9** Small area ΔA, at an arbitrary cross section point carries/axial ΔF in this member.

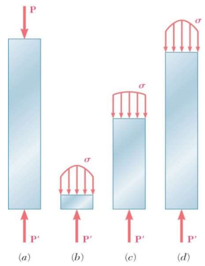


**Fig. 1.10** Stress distributions at different sections along axially loaded member.

The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.

The force intensity on that section is defined as the normal stress.

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

The actual distribution of stresses is statically indeterminate, i.e., can not be found from statics alone.

# Centric & Eccentric Loading

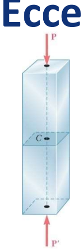


**Fig. 1.12** Centric loading having resultant forces passing through the centroid of the section.

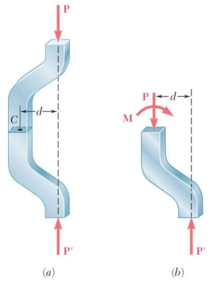


**Fig. 1.13** An example of simple eccentric loading.

A uniform distribution of stress in a section infers that the line of action for the resultant of the internal forces passes through the centroid of the section.

A uniform distribution of stress is only possible if the line of action of the concentrated loads **P** and **P'** passes through the centroid of the section considered. This is referred to as *centric loading*.

If a two-force member is *eccentrically loaded*, then the resultant of the stress distribution in a section must yield an axial force and a moment.

The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

## Shearing Stress

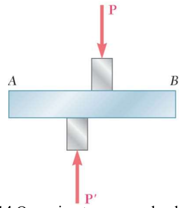



**Fig. 1.14** Opposing transverse loads creating shear on member *AB*.

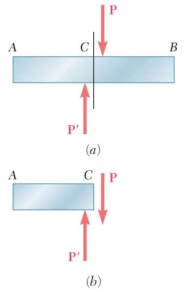



**Fig. 1.15** This shows the resulting internal shear force on a section between transverse forces.

Forces **P** and **P'** are applied transversely to the member *AB*.

Corresponding internal forces act in the plane of section *C* and are called *shearing* forces.

The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load **P**.

The corresponding average shear stress is,

$$\tau_{\text{ave}} = \frac{P}{A}$$

Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.

The shear stress distribution cannot be assumed to be uniform.

## Shearing Stress Examples

### Single Shear

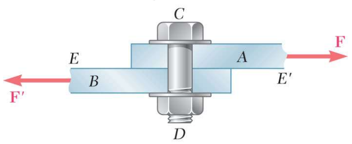



**Fig. 1.16** Bolt subject to single shear.

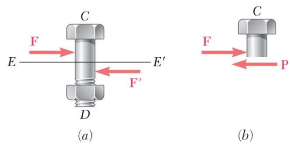



**Fig. 1.17** (a) Diagram of bolt in single shear; (b) section *E-E'* of the bolt

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

### Double Shear

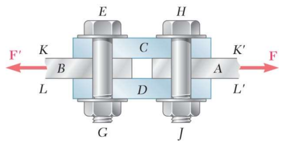



**Fig. 1.18** Bolt subject to double shear.

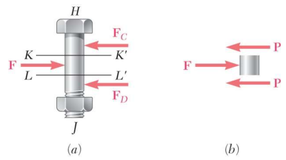



**Fig. 1.19** (a) Diagram of bolt in double shear; (b) section *K-K'* and *L-L'* of the bolt.

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

## Bearing Stress in Connections

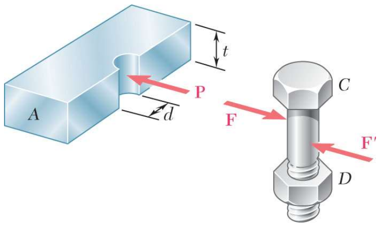



**Fig. 1.20** Equal and opposite forces between plate and bolt, exerted over bearing surfaces.

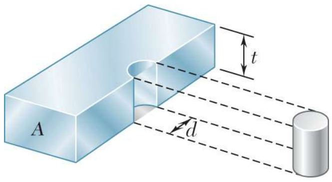



**Fig. 1.21** Dimensions for calculating bearing stress area.

Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.

The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.

Corresponding average force intensity is called the bearing stress,

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

## Stress in Two Force Members

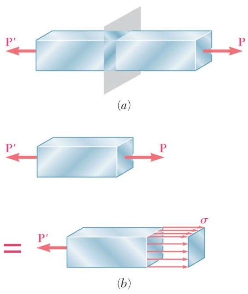



**Fig. 1.26** Axial forces on a two-force member. (a) Section plane perpendicular to member away from load application. (b) Equivalent force diagram models of resultant force acting at centroid and uniform normal stress.

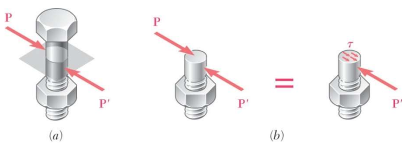



**Fig. 1.27** (a) Diagram of a bolt from a single shear joint with a section plane normal to the bolt. (b) Equivalent force diagram model of the resultant force acting at the section centroid and the uniform average shear stress.

Axial forces on a two force member result in only normal stresses on a plane cut perpendicular to the member axis.

Transverse forces on bolts and pins result in only shear stresses on the plane perpendicular to bolt or pin axis.

Axial or transverse forces may produce both normal and shear stresses with respect to a plane other than one cut perpendicular to the member axis.

# Stress on an Oblique Plane

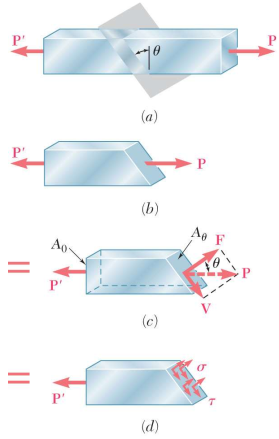



**Fig. 1.28** Oblique section through a two-force member. (a) Section plane made at an angle θ to the member normal plane, (b) Free-body diagram of left section with internal resultant force **P**. (c) Free-body diagram of resultant force resolved into components **F** and **V** along the section plane's normal and tangential directions, respectively. (d) Free-body diagram with equivalent as normal stress, σ, and shearing stress, τ.

Pass a section through the member forming an angle $\theta$ with the normal plane.

From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force $P$.

Resolve $P$ into components normal and tangential to the oblique section,

$$F = P\cos\theta \qquad V = P\sin\theta$$

The average normal and shear stresses on the oblique plane are

$$\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{A_0 / \cos\theta} = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{A_0 / \cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

# Maximum Stresses

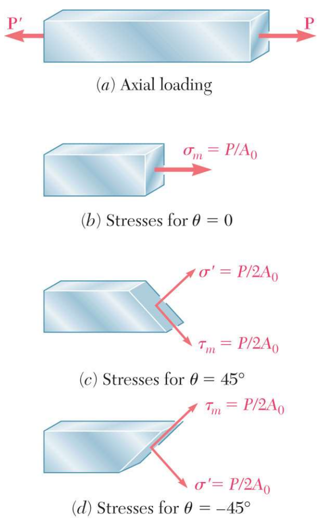



**Fig. 1.29** Selected stress results for axial loading.

Normal and shearing stresses on an oblique plane

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

$$\sigma_m = \frac{P}{A_0} \qquad \tau' = 0$$

The maximum shear stress occurs for a plane at ± 45° with respect to the axis,

$$\tau_m = \frac{P}{A_0}\sin 45 \cos 45 = \frac{P}{2A_0} = \sigma'$$

## Stress Under General Loadings

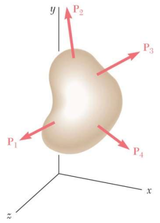



**Fig. 1.30** Multiple loads on a general body.

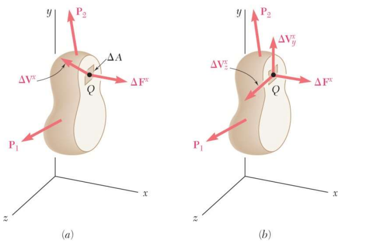


**Fig. 1.31** (a) Resultant shear and normal forces, $\Delta V^x$ and $\Delta F^x$, acting on small area ΔA at point Q. (b) Forces on ΔA resolved into force in coordinate directions.

A member subjected to a general combination of loads is cut into two segments by a plane passing through *Q*

The distribution of internal stress components may be defined as,

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

For equilibrium, an equal and opposite internal force and stress distribution must be exerted on the other segment of the member.

## State of Stress

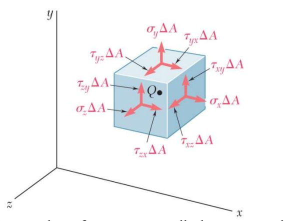


**Fig. 1.35** Positive resultant forces on a small element at point *Q* resulting from a state of general stress.

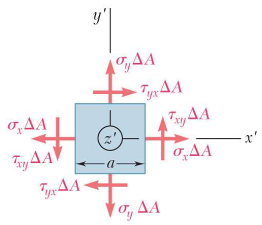


**Fig. 1.36** Free-body diagram of small element at *Q* viewed on projected plane perpendicular to *z*'-axis. Resultant forces on positive and negative *z*' faces (not shown) act through the *z*'-axis, thus do not contribute to the moment about that axis.

Stress components are defined for the planes cut parallel to the *x*, *y* and *z* axes. For equilibrium, equal and opposite stresses are exerted on the hidden planes.

The combination of forces generated by the stresses must satisfy the conditions for equilibrium:

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$

Consider the moments about the *z* axis:

$$\sum M_z = 0 = (\tau_{xy}\Delta A)a - (\tau_{yx}\Delta A)a$$

$$\tau_{xy} = \tau_{yx}$$

similarly, $\tau_{yz} = \tau_{zy}$ and $\tau_{yz} = \tau_{zy}$

Only six components of stress are required to define the complete state of stress

# Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

$FS =$ Factor of safety

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to integrity of whole structure
- risk to life and property
- influence on machine function