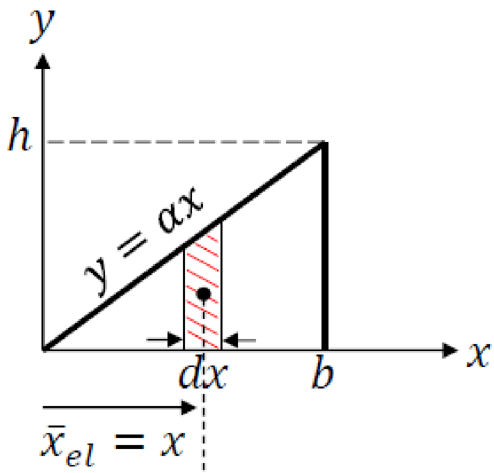
Example 1

Derive the centroid of a right triangle.



$$\bar{x}A = \int_0^b \bar{x}_{el}\, dA \qquad (dA = ydx)$$

$$\bar{x}A = \int_0^b x\, y\, dx$$

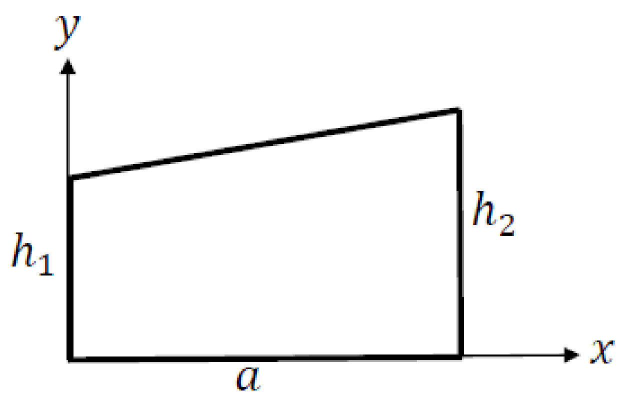
$$\bar{x}\frac{1}{2}\alpha b^2 = \int_0^b x\, \alpha x\, dx$$

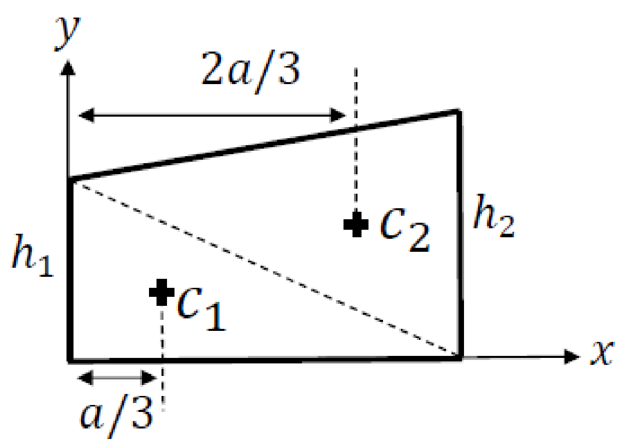
$$A = \frac{1}{2}bh = \frac{1}{2}b(\alpha b)$$
$$= \frac{1}{2}\alpha b^2$$

$$\bar{x}\frac{1}{2}\alpha b^2 = \alpha\frac{x^3}{3}\Big|_0^b = \alpha\frac{b^3}{3} \Longrightarrow \bar{x}\frac{1}{2}b^2 = \frac{b^3}{3}$$

$$\boxed{\bar{x} = \frac{2}{3}b}$$

Example 2





Determine the x coordinate of the centroid of the trapezoid in terms of h_1, h_2 and a.

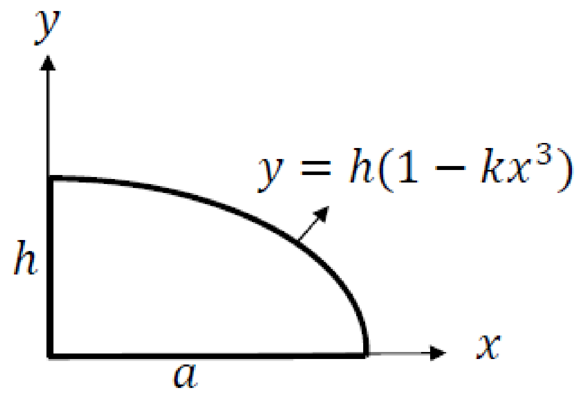
	A	$\bar{x}$	$A\bar{x}$
1	$\frac{1}{2}h_1 a$	$\frac{1}{3}a$	$\frac{1}{6}h_1 a^2$
2	$\frac{1}{2}h_2 a$	$\frac{2}{3}a$	$\frac{2}{6}h_2 a^2$

$$\bar{x} = \frac{\sum \bar{x}A}{\sum A} = \frac{\frac{1}{6}a^2(h_1 + 2h_2)}{\frac{1}{2}a\,(h_1 + h_2)}$$

$$\boxed{\bar{x} = \frac{1}{3}a\,\frac{h_1 + 2h_2}{h_1 + h_2}}$$

Example 3

Determine the x and y coordinate of the centroid of the given shape in terms of h and a.

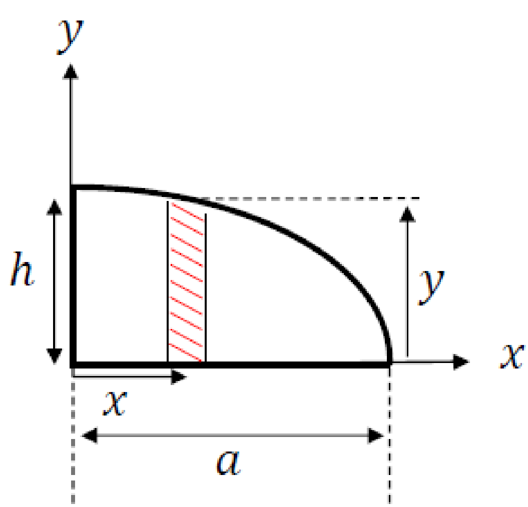


$y = h(1 - kx^3)$

For $x = a$, $y = 0$

$0 = h(1 - kx^3)$ $\Rightarrow$ $k = 1/a^3$ which gives,

$$y = h\left(1 - \frac{x^3}{a^3}\right)$$



$\bar{x}_{el} = x$, $\bar{y}_{el} = \frac{y}{2}$, $dA = ydx$

$$A = \int_0^a dA = \int_0^a ydx = \int_0^a h\left(1 - \frac{x^3}{a^3}\right)\, dx = \left[x - \frac{x^4}{4a^3}\right]\Big|_0^a$$

$$A = \frac{3}{4}ah$$

$$\bar{x}A = \int_0^a \bar{x}_{el} dA = \int_0^a xh\left(1 - \frac{x^3}{a^3}\right) dA = \int_0^a h\left(x - \frac{x^4}{a^3}\right) dA$$

$$= h\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]\Big|_0^a = \frac{3}{10} a^2 h$$

$$\bar{x}A = \bar{x}\frac{3}{4}ah = \frac{3}{10}a^2 h \quad \Rightarrow \quad \boxed{\bar{x} = \frac{2}{5}a}$$

$$\bar{y}_{el} = \frac{y}{2},\ dA = ydx$$

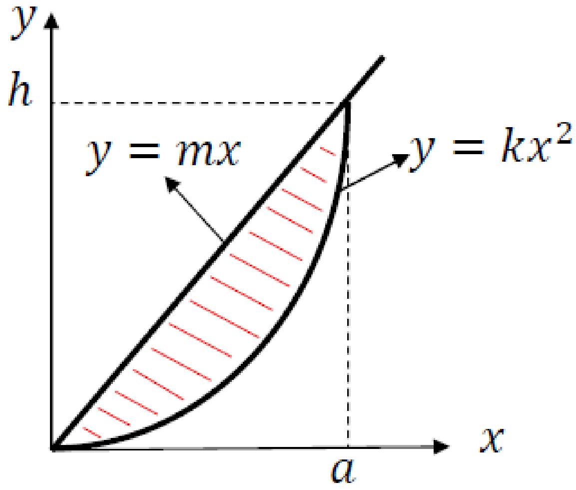
$$\bar{y}A = \int_0^a \bar{y}_{el} dA = \int_0^a \frac{y}{2} ydx = \frac{1}{2}\int_0^a y^2 dx = \frac{1}{2}\int_0^a h^2\left(1 - \frac{x^3}{a^3}\right)^2 dx$$

$$= \frac{h^2}{2}\int_0^a \left(\frac{x^6}{a^6} - \frac{2x^3}{a^3} + 1\right) dx = \frac{h^2}{2}\left[\frac{x^7}{7a^6} - \frac{x^4}{2a^3} + x\right]\Big|_0^a = \frac{h^2}{2}\left(\frac{a}{7} - \frac{a}{2} + a\right) = \frac{h^2}{2}\left(\frac{9a}{14}\right)$$

$$= \frac{9ah^2}{14} \quad \Rightarrow \quad \bar{y}A = \bar{y}\frac{3}{4}ah = \frac{9ah^2}{14}$$

$$\boxed{\bar{y} = \frac{3}{7}h}$$

Example 4



Determine the centroid of the area by direct integration in terms of a and h.

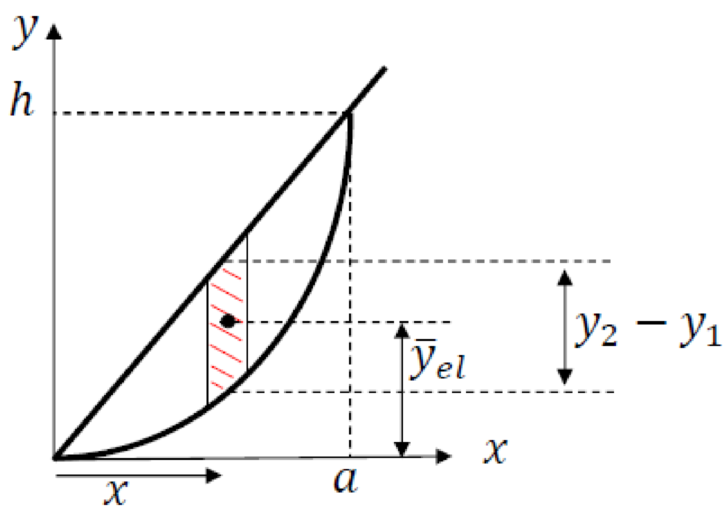
$y_1 = kx^2$ $\quad$ $y_2 = mx$

$@\ x = a$ $\quad$ $@\ x = a$

$y = h$ $\quad$ $y = h$

$k = \dfrac{h}{a^2}$ $\quad$ $m = \dfrac{h}{a}$

$y_1 = \dfrac{h}{a^2}x^2$ $\quad$ $y_2 = \dfrac{h}{a}x$



$$\bar{x}_{el} = x, \quad \bar{y}_{el} = \frac{y_2 + y_1}{2}$$

$$dA = (y_2 - y_1)dx = \left(\frac{h}{a}x - \frac{h}{a^2}x^2\right)dx$$

$$= \frac{h}{a^2}(ax - x^2)dx$$

$$A = \int dA = \int_0^a \frac{h}{a^2}(ax - x^2)dx$$

$$= \frac{h}{a^2}\left[\frac{ax^2}{2} - \frac{x^3}{3}\right]\Big|_0^a = \frac{h}{a^2}\left[\frac{a^3}{2} - \frac{a^3}{3}\right] = \frac{h}{a^2}\frac{a^3}{6} \Rightarrow A = \frac{ah}{6}$$

$$Q_y = \int \bar{x}_{el}\, dA = \int_0^a x\frac{h}{a^2}(ax - x^2)dx$$

$$= \frac{h}{a^2}\int_0^a (ax^2 - x^3)dx = \frac{h}{a^2}\left[\frac{ax^3}{3} - \frac{x^4}{4}\right]\Big|_0^a$$

$$= \frac{h}{a^2}\left(\frac{a^4}{3} - \frac{a^4}{4}\right) = \frac{h}{a^2}\frac{a^4}{12}$$

$$Q_y = \frac{a^2h}{12}$$

$$Q_x = \int \bar{y}_{el}\, dA = \int_0^a \frac{y_1 + y_2}{2} \frac{h}{a^2} (ax - x^2) dx$$

$$= \int_0^a \frac{1}{2} \left(\frac{h}{a^2} x^2 + \frac{h}{a} x \right) \frac{h}{a^2} x^2 (ax^2 - x^3) dx = \frac{h^2}{2a^4} \int_0^a (x^2 + ax)(ax - x^2) dx$$

$$= \frac{h^2}{2a^4} \int_0^a (a^2x^2 - x^4) dx = \frac{h^2}{2a^4} \left[\frac{a^2x^3}{3} - \frac{x^5}{5} \right] \Big|_0^a$$

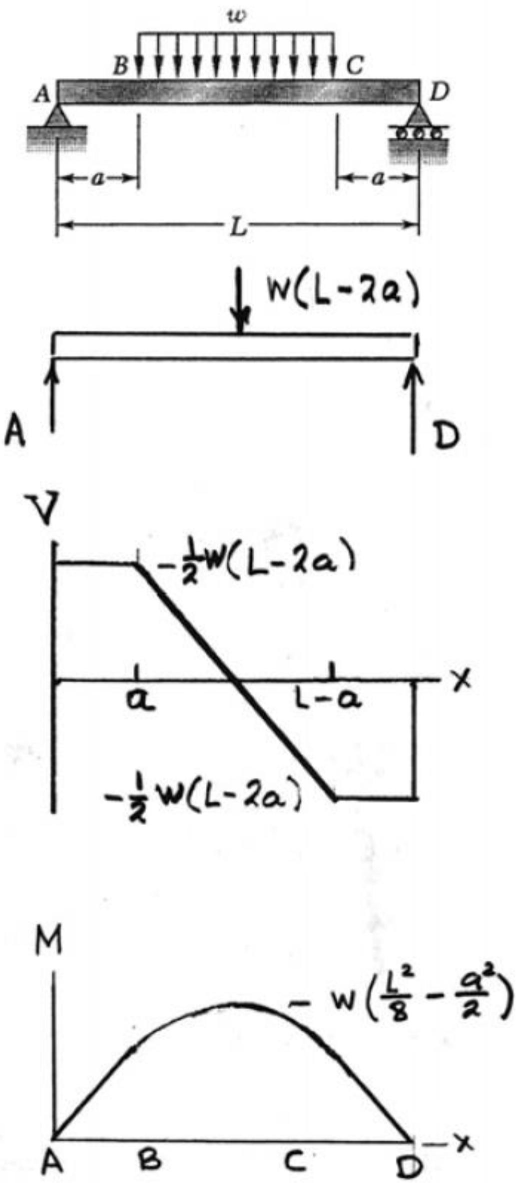
$$= \frac{h^2}{2a^4} \left(\frac{a^5}{3} - \frac{a^5}{5} \right) = \frac{h^2}{2a^4} \frac{2a^5}{15} \qquad Q_x = \frac{ah^2}{15}$$

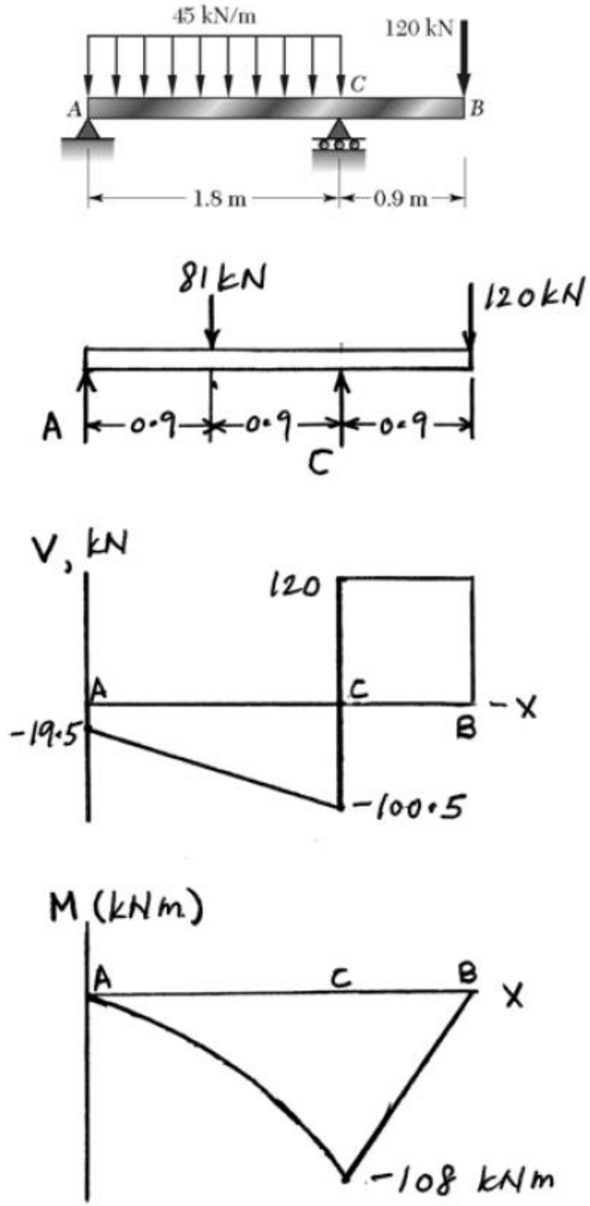
$$Q_x = \bar{y} A$$

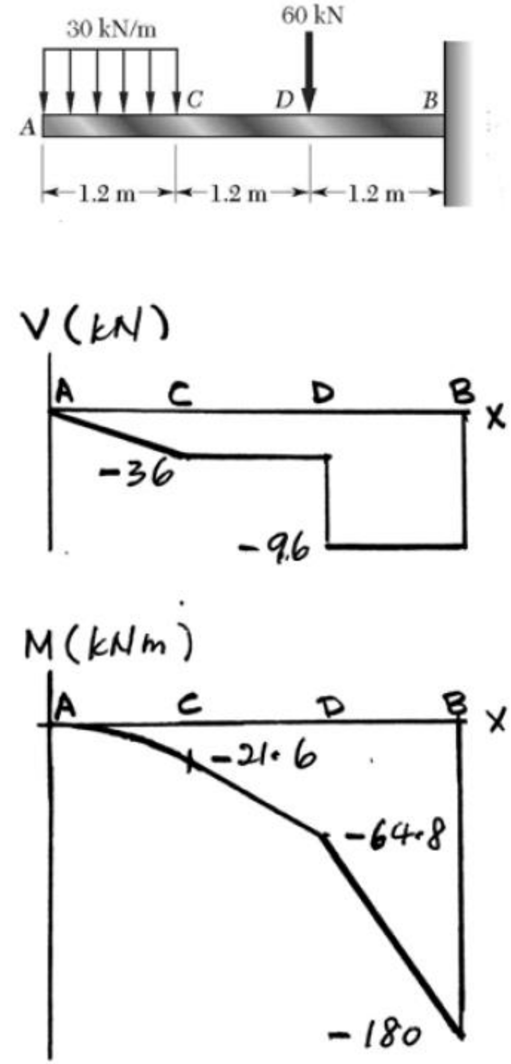
$$\frac{ah^2}{15} = \bar{y} \frac{ah}{6} \qquad \boxed{\bar{y} = \frac{2h}{5}}$$

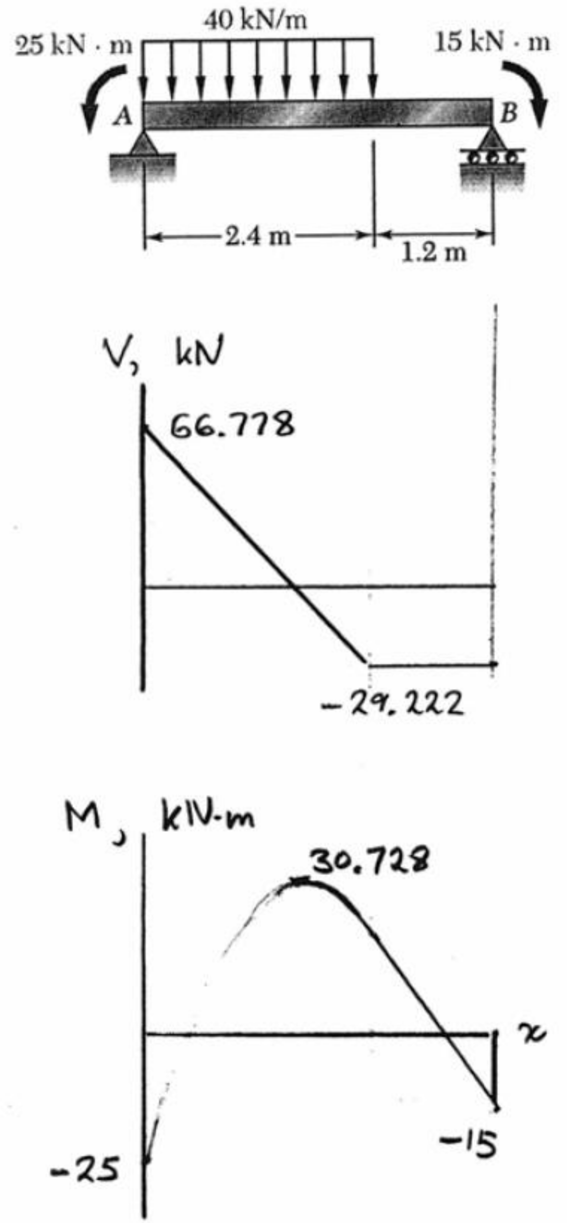
$$Q_y = \bar{x} A$$

$$\frac{a^2 h}{12} = \bar{y} \frac{ah}{6} \qquad \boxed{\bar{x} = \frac{a}{2}}$$

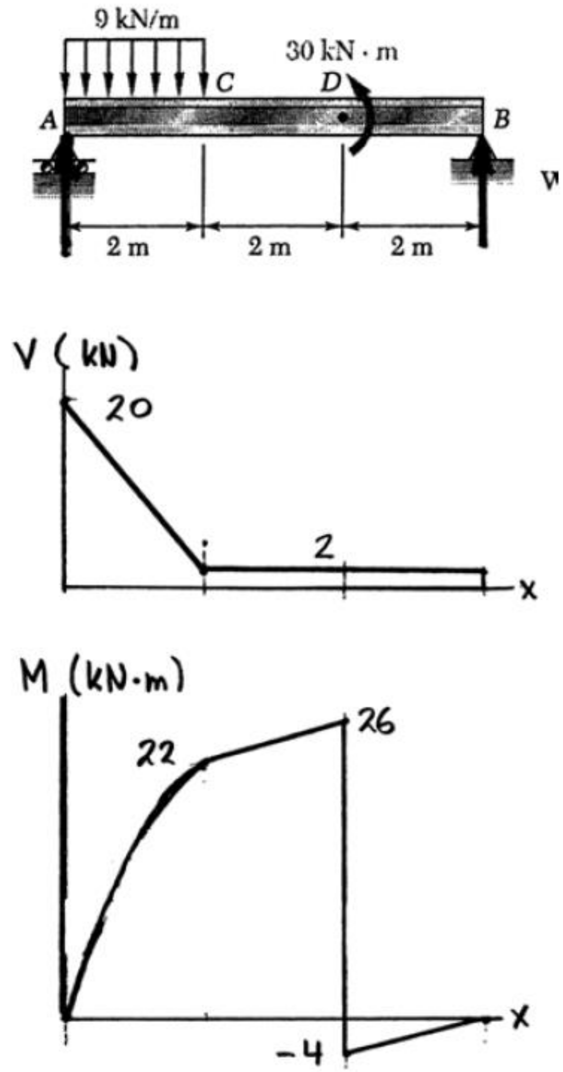

w
A
B
C
D
a
a
L
W(L-2a)
A
D
V
$-\frac{1}{2}W(L-2a)$
a
L-a
X
$-\frac{1}{2}W(L-2a)$
M
$-W\left(\frac{L^2}{8}-\frac{a^2}{2}\right)$
A
B
C
D
X


45 kN/m
120 kN
A
C
B
1.8 m
0.9 m
81 kN
120 kN
A
0.9
0.9
0.9
C
V, kN
120
A
C
X
B
-19.5
-100.5
M (kNm)
A
C
B
X
-108 kNm

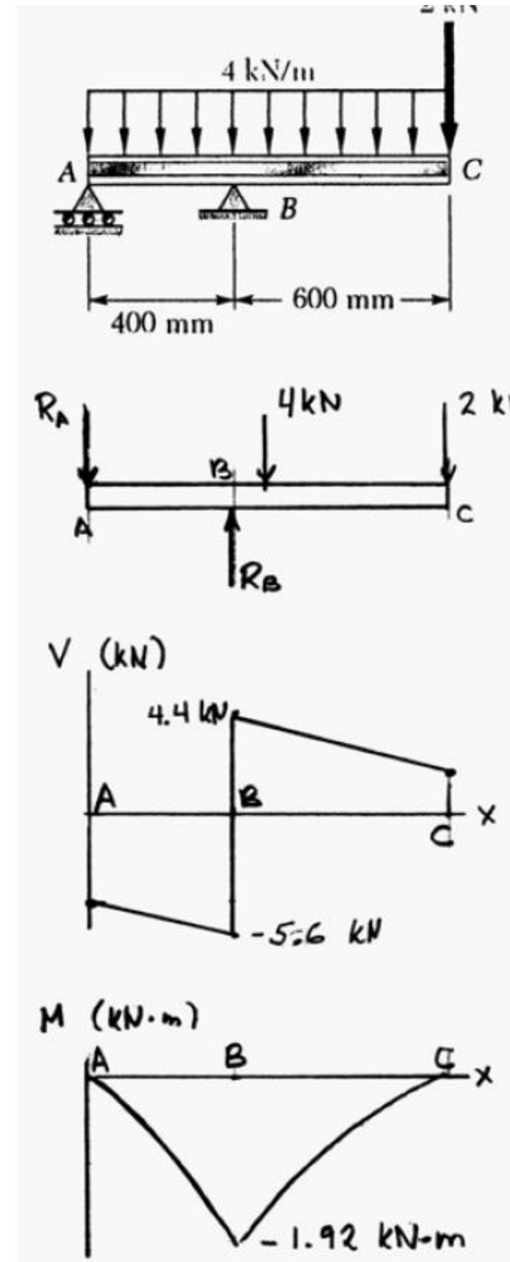

30 kN/m
60 kN
A
C
D
B
1.2 m
1.2 m
1.2 m
V (kN)
A
C
D
B
X
-36
-96
M (kNm)
A
C
D
B
X
-21.6
-64.8
-180



40 kN/m
25 kN · m
15 kN · m
A
B
2.4 m
1.2 m
V, kN
66.778
-29.222
M, kN-m
30.728
x
-15
-25



9 kN/m
30 kN · m
C
D
A
B
2 m
2 m
2 m
V (kN)
20
2
X
M (kN·m)
22
26
X
-4



4 kN/m
A
C
B
400 mm
600 mm
R_A
4kN
2 kN
B
A
C
R_B
V (kN)
4.4 kN
A
B
X
C
-5.6 kN
M (kN·m)
A
B
C
X
-1.92 kN-m