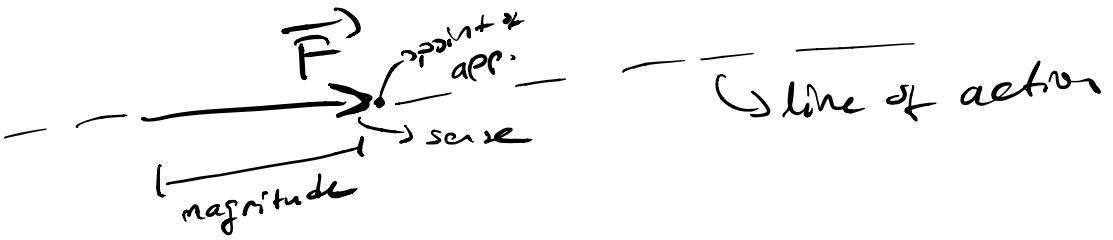
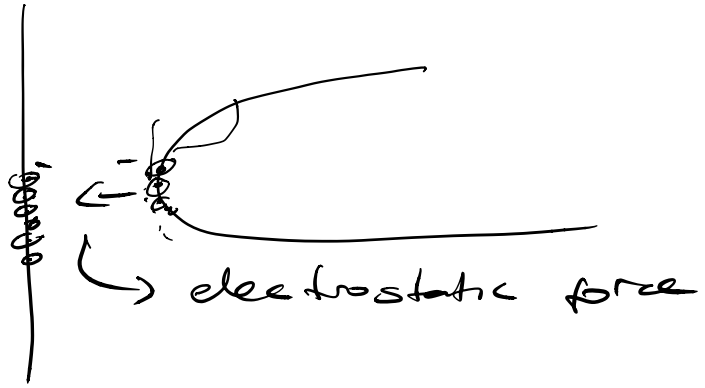
ME 211
Statics & Strength of Materials

Fundamental Definitions in Mechanics

Force: Action of one body on another; characterized by its point of application, magnitude, line of action and sense.



* Force is a vector quantity ∇



In mechanics physical quantities are expressed by using scalar, vectors and tensors.

Scalar: A scalar is a physical quantity which expresses only a magnitude

Eg: Mass, temperature, length, volume time, speed.

Note: they're generally represented by lowercase letters.

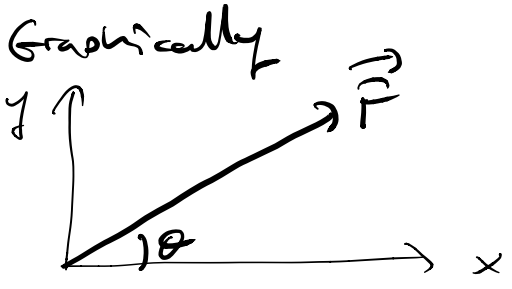
Vector: A vector is a physical quantity which carries both a magnitude and direction as information

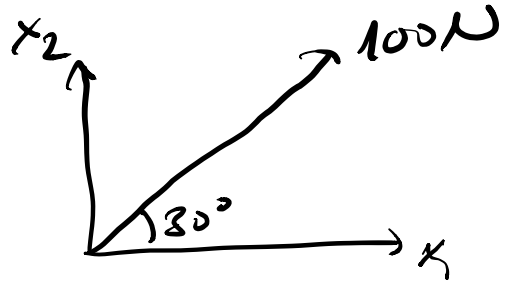
Eg. Force, velocity, position, acc, mag. field. momentum.

They are expressed by letters in bold or lined.

$\vec{A}$, $\vec{C}$, $\vec{B}$

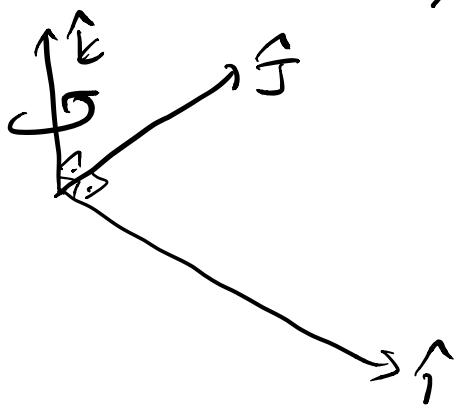
Graphically



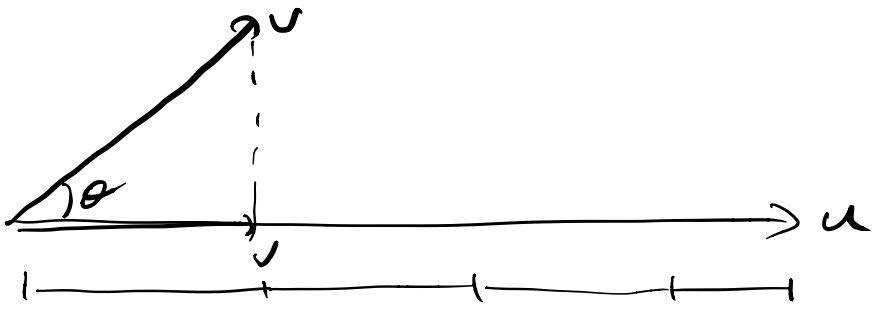


Cartesian Vectors

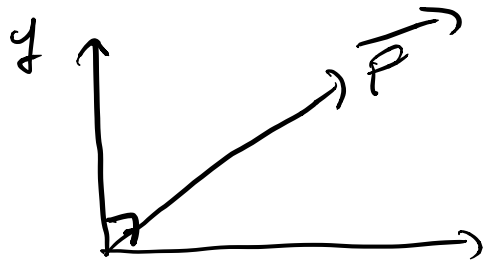
Every vector defined in 3D space can be expressed in terms of cartesian unit vectors $\hat{i}, \hat{j}, \hat{k}$



Right Handed System



$3,\vec{r}(v\cos\theta)\cdot\vec{u} = u$



↳ 2D cartesian system

$\cos 90° \Rightarrow$
$f(y) \neq x$

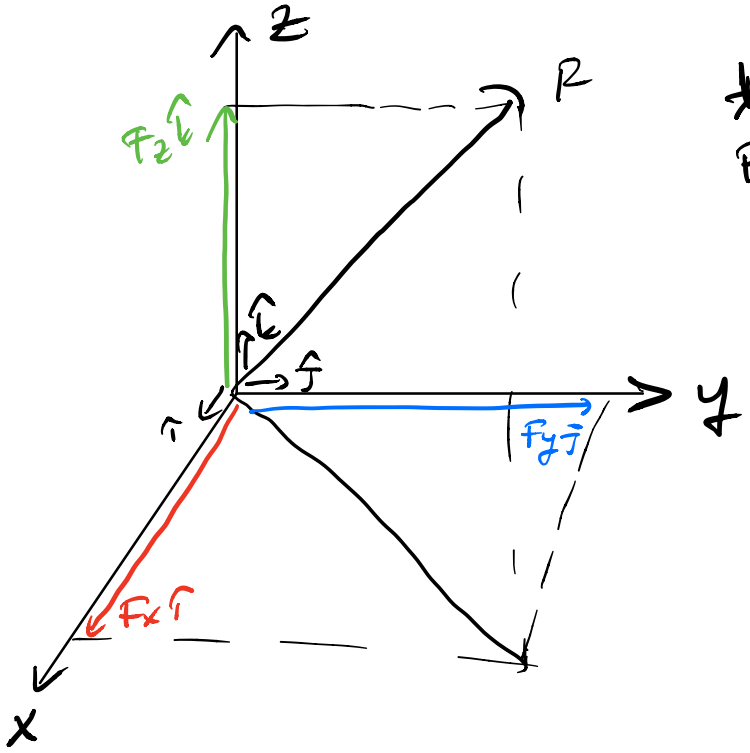
Cartesian unit vectors are linearly independent unit vectors which designates the cartesian coordinate system. Any vector can be decomposed into its cartesian components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

scalars

unit vectors

Cartesian notation is especially useful for 3D statics problems



* Decomposition of F in cartesian coordinates

CHAPTER - 3

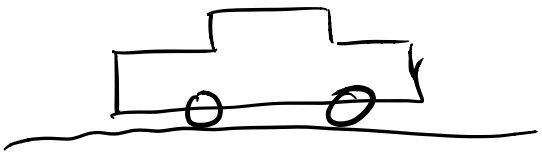
Equilibrium of a Particle

In the scope of statics, the term "equilibrium" always stands for "static equilibrium". i.e. the resultant force acting on a particle or a body is equal to zero. The object is at rest

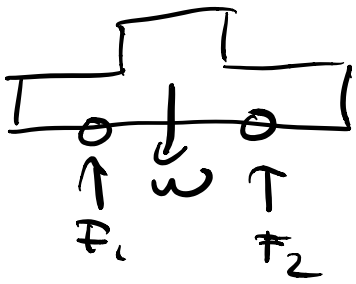
$$\Sigma F = 0$$

↳ vector sum of all forces

To ensure that a point or a body is at rest with zero resultant one must draw a free body diagram (FBD)



FBD



$\Sigma F = 0$

$F_1 + F_2 - W = 0$ ✓

Moment of a Force: A measure of the tendency of the force to cause the body to rotate about an axis

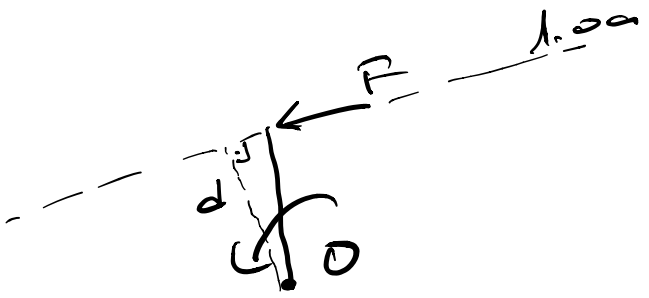
∴ Moment is a vector and so has both magnitude and a direction

Scalar Formulation: The magnitude of the moment is determined from

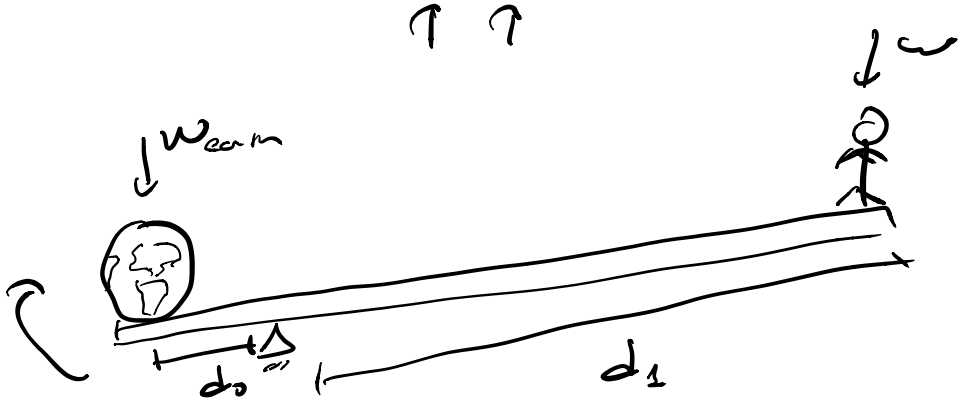
$$M_o = F \cdot d$$

d: the shortest distance from point "O" to the l.o.a

F: the acting force



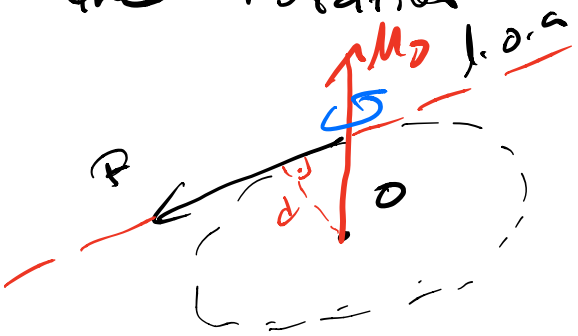
$$M_o = F \cdot d$$
$$\uparrow \quad \uparrow$$



$W_{earth.\ do} = W \cdot d_1$

moment created by you

The direction of the rotation is indicated by RHR the fingers show the rotation

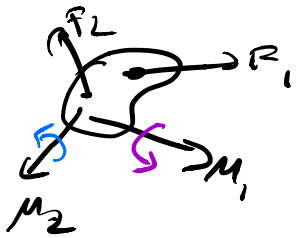


The resultant moment M_{R_o} of the system can be found by simply adding the moments of all forces algebraically (All moments are in z direction for 2D prob.)

Eqb. of a particle

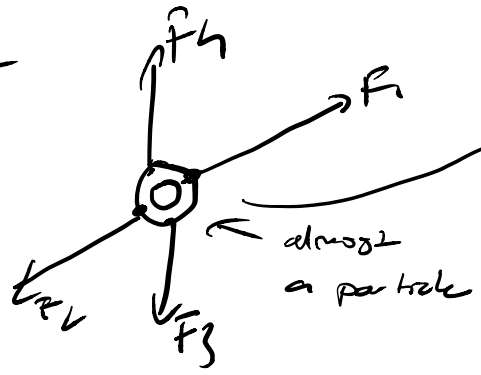
$\Sigma F = 0$

Eqb. of a Body



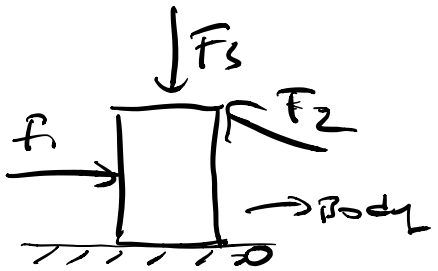
$\Sigma F = 0$ $\quad \Sigma M = 0$

Ex:



Only check $\Sigma F = 0$

Ex



$\Sigma F = 0$

$\Sigma M = 0$

$\Sigma F = 0$ → $\Sigma F_x = 0$, $\Sigma F_y = 0$

$\Sigma M_o = 0$

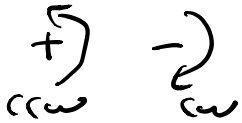
∴ for a 2D problem we have 3 equilibrium equations →

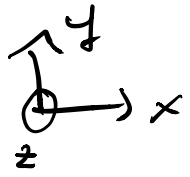
$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_o = 0$

Note: For positive z-axis the rotation is counter clockwise. Clockwise rotation indicates a negative moment.

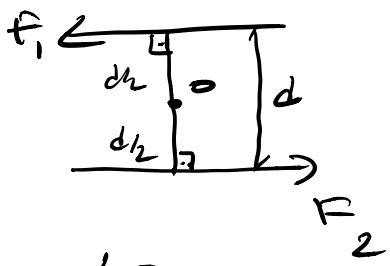




$\circlearrowleft \Sigma M_o = 0$

Special Case: Moment of a Couple

A couple is defined as two parallel forces that have the same magnitude but opposite directions.



$F_1 = -F_2$

$\sum F_x = 0$

$-F_1 + F_2 = 0$ ✓

$+\circlearrowleft \sum M_o \neq 0$

$\sum F_y = 0$ ✓

$\sum F = 0$

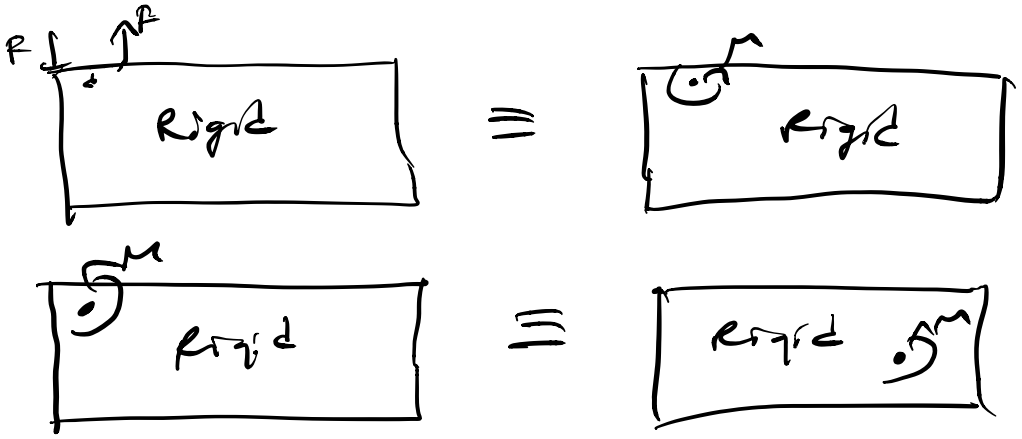
$+\circlearrowleft \sum M_o = ?$

$+F_1 \cdot d_2 + F_2 \cdot d_2$

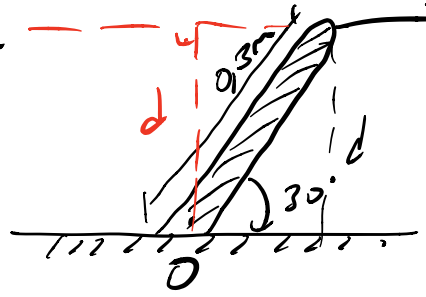
$= F \cdot d_2 + F \cdot d_2 = F \cdot d$

Moment Couple $= F \cdot d$

⇒ A couple system can only produce a couple moment on any point. The magnitude and the effect of the moment is <u>same</u> everywhere. (if it is a rigid body)



Ex:



Determine the moment of the force about point O.

$$+\circlearrowleft \Sigma M_O = F \cdot d = 50N \cdot d$$

$$d = \frac{0,3}{2} = 0,15m \Rightarrow M_O = -50N \cdot 0,15m$$

$$= -7,5\,N.m$$

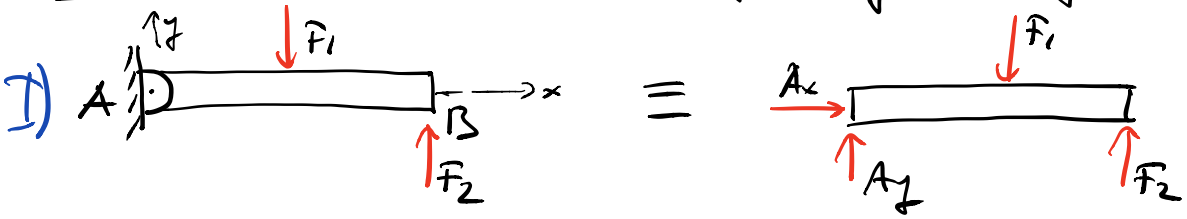
The unit of Moment is N.m

Fundamentals of FBD

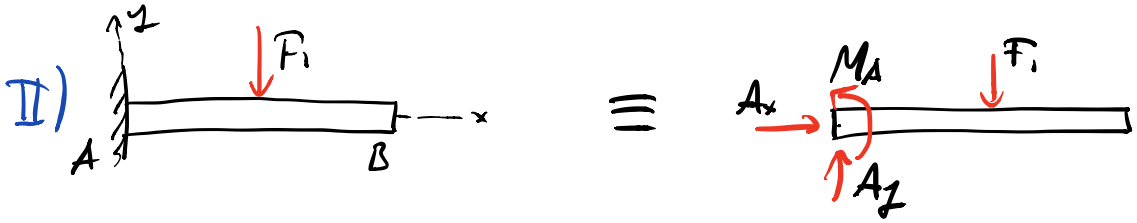
* A support in a system can be replaced either by a force or a moment or both.
* If a support prevents the system from translation then it exerts a force on the system
* If a support prevents the system from rotation then it exerts a couple moment on the system
* Internal forces are not shown on FBD
* The weight of a body exerts a force through the center of gravity.
* Couple moments can be placed anywhere on the system, since, they are free vectors
* Forces can act at any point along their line of action since they are sliding vectors

Ex: Draw the FBD of the given systems

I)



II)



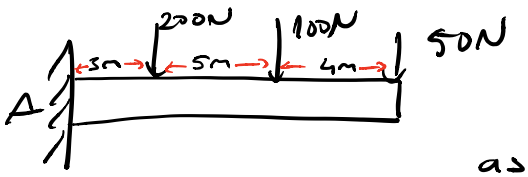
I) → 2 Unknowns A_x & A_y

2 Eqb. Eqns. $\sum F_x = 0$, $\sum F_y = 0$ ($\sum M = 0$) ← trivial (No moment on the sys.) ↳ useless

II) → 3 Unknowns A_x, A_y & M_A

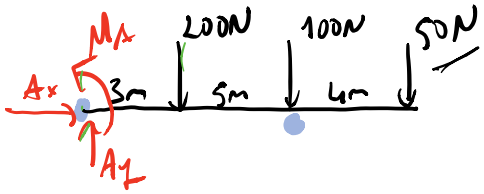
3 Eqb Eqns $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$

Ex:



A cantilever beam is loaded as shown. The beam is fixed at A and free at the right end. Determine the reactions at the fixed support.

FBD



3 Unknowns : A_x , A_y , M_A

3 Eqb. Eqns: $\sum F_x = 0$, $\sum F_y = 0$, $\circlearrowleft \sum M = 0$

We can solve for unknowns

$\sum F_x = 0$

$A_x = 0$ ✓

$\sum F_y = 0$

$A_y - 200N - 100N - 50N = 0$

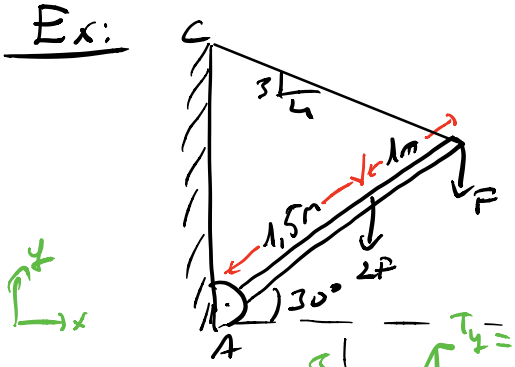
$A_y = (200 + 100 + 50)N = 350N$ // ✓

$\circlearrowleft \sum M_A = 0$

$M_A - 200N \cdot 3m - 100N \cdot 9m - 50N \cdot 12m = 0$

$M_A = 2000N.m = 2kN.m$ // ✓

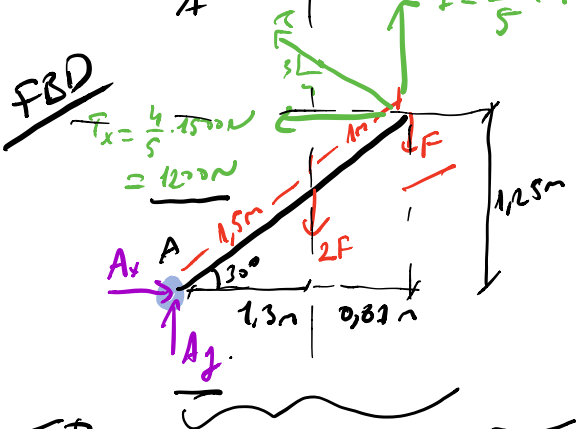
Ex:



If the tension in the cable is 1500N. Determine the magnitude of F and the reaction at pin A

$T_y = \frac{3}{5} \cdot 1500N = 900N$

FBD



3 Unknowns: A_x, A_y, F

3 Eqs Eqn: $\sum F_x = 0$, $\sum F_y = 0$, $+\circlearrowleft \sum M_o = 0$

Determined ✓

$\sum F_x = 0$

$A_x - 1200N = 0$

$A_x = 1200N$ //

$+\circlearrowleft \sum M_A = 0$ (2,17m)

$-2F \cdot 1,3m - F(1,3 + 0,87)m + 1200N \cdot 1,25m + 900N \cdot 2,17m = 0$

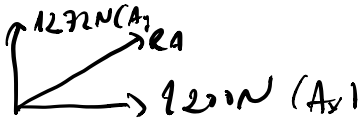
$\Rightarrow F = 724N$ // ✓

$\sum F_y = 0$

$A_y - 2 \cdot 724N - 724N + 900N = 0$

$\Rightarrow A_y = 1272N$ // ✓

$R_A = ?$



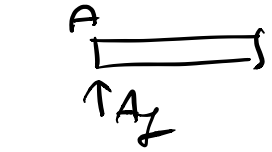
$R_A = \sqrt{1272^2 + 1200^2} = 1749N \approx 1,75kN$

Most common Support Types

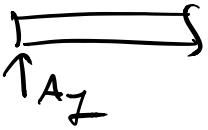
2D

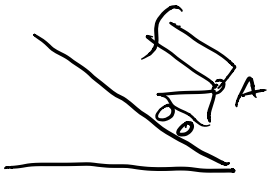
Roller

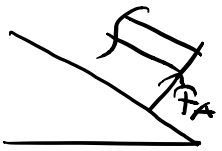
FBD



FBD







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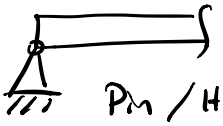

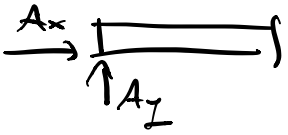

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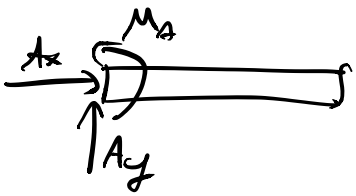

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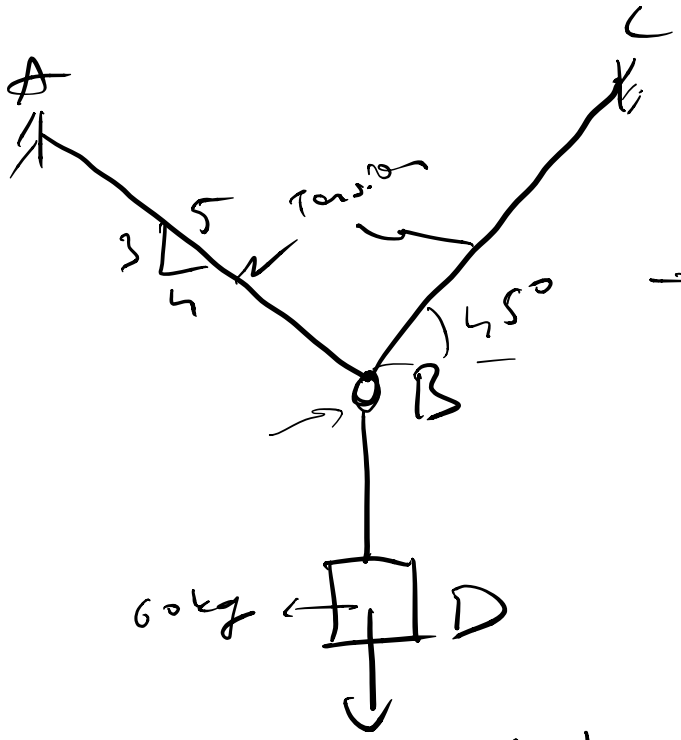


$\Sigma F_x = 0$

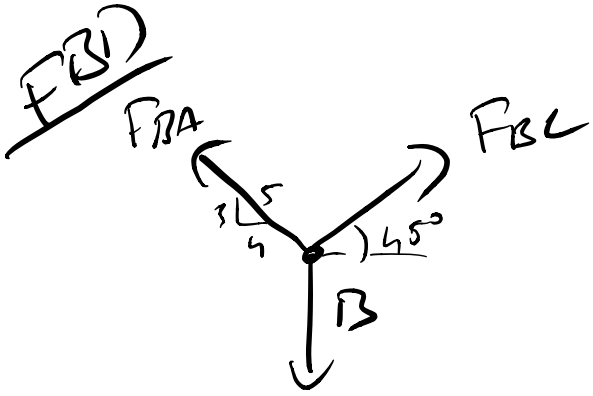
$\Sigma F_y = 0$

$\rightarrow \Sigma M = 0$

$W = 60\,kg \cdot 9{,}81\,N/kg$

$= 588{,}6\,N$

FBD



588,6 N

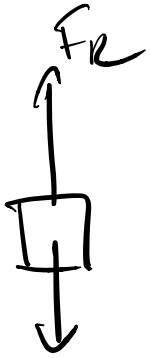
2 Unknowns F_{BA}, F_{BC}

2 Eqb Eqn $\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$ → dummy

FBD

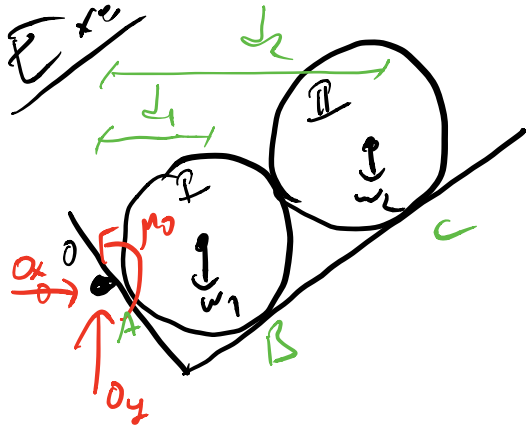


588,6 N

1 Unknown F_R

1 Eqb Eqn $\Sigma F_y = 0$

dummy { $\Sigma F_x = 0$, $\Sigma M = 0$



$\Sigma F_x = 0$

$O_x = 0$

$\Sigma F_y = 0$

$O_y - w_1 - w_2 = 0$

$O_y = w_1 + w_2$

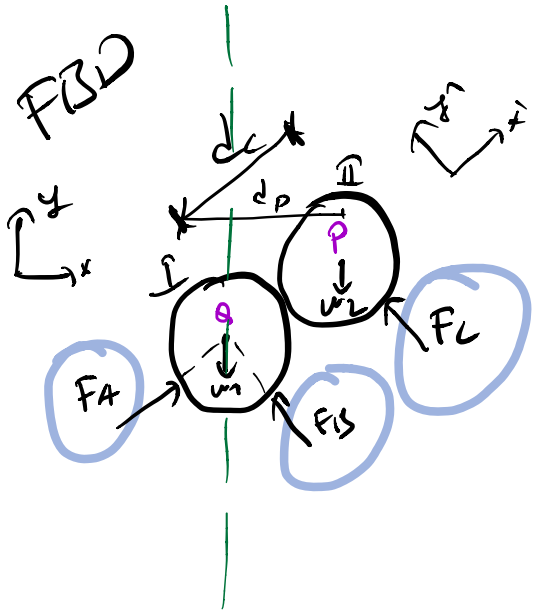
$+\circlearrowleft \Sigma M_O = 0$

$M_O - w_1 \cdot d_1 - w_2 \cdot d_2 = 0$

(known)

$M_O = w_1 \cdot d_1 + w_2 \cdot d_2$

FBD



$\Sigma F_x = 0$

(unknowns)

$F_{Ax} - F_{Bx} - F_{Cx} = 0$

$\Sigma F_y = 0$

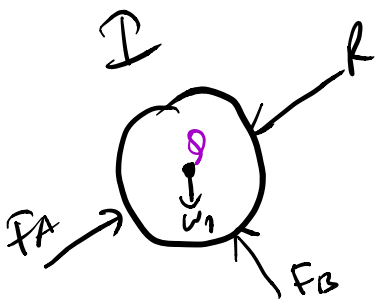
$F_{Ay} + F_{By} - F_{Cy} - w_1 - w_2 = 0$

$+\circlearrowleft \Sigma M_Q = 0$ ← First start with the moment eqn.

$F_C \cdot d_c - w_2 \cdot d_p = 0$

$\rightarrow F_C$ ✓

FBD



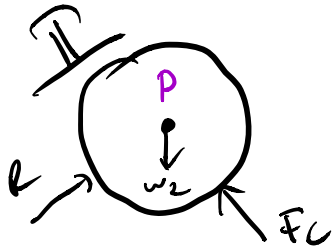
$+\circlearrowleft \sum M_Q = 0$ ✓ (dummy)

3 Unknowns $F_A, F_B, R_{@}$

2 Eqb Eqn $\sum F_x = 0$

$\sum F_y = 0$

FBD



$+\circlearrowleft \sum M_P = 0$ ✓ (dummy)

2 Unknowns R, F_C

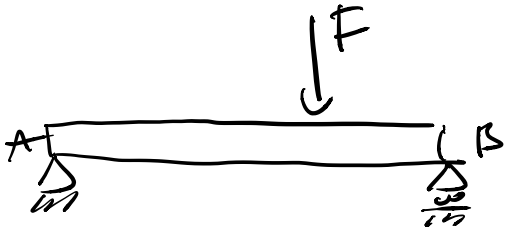
2 Eqb. Eqn $\sum F_x = 0$

$\sum F_y = 0$

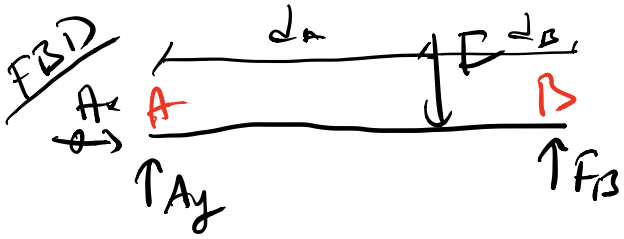
Solve for → R, F_C ✓

↳ Solve for F_A, F_B

Ex:



FBD



$\sum F_x = 0$

$A_x = 0$

$\sum F_y = 0$

↳ instead

$+\circlearrowleft \sum M_A = 0$ (You can solve for F_B)

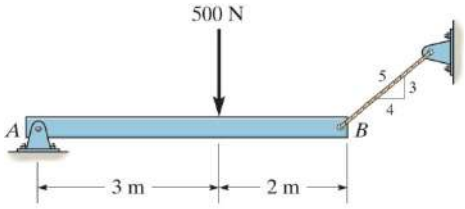
$-F \cdot d_A + F_B \cdot (d_A + d_B) = 0 \rightarrow F_B$ ✓

$+\circlearrowleft \sum M_B = 0$ (You can solve for A_y)

$-A_y \cdot (d_A + d_B) + F \cdot d_B = 0 \longrightarrow A_y$ ✓

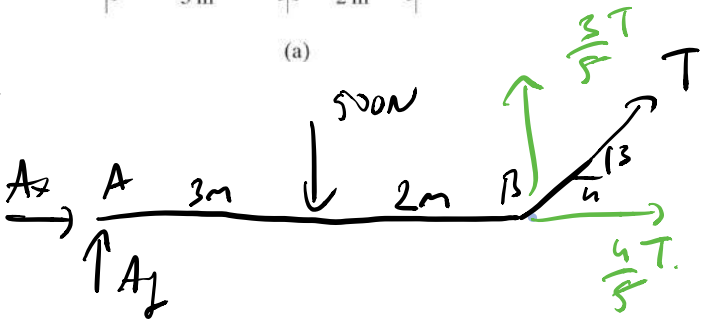
You don't need to use $\sum F_y = 0$

Ex's Determine the support reactions



(a)

FBD



Unknowns

A_x, A_y, T

Equ. Eqn

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M = 0$

$+\circlearrowleft \sum M_A = 0$

$-500N \cdot 3m + \frac{3}{5} T \cdot 5m = 0$

$\Rightarrow T = 500N$

$\sum F_x = 0$ $\quad$ $\frac{4}{5}\uparrow$

$A_x + \frac{4}{5} \cdot 500N = 0$

$A_x = -400N$

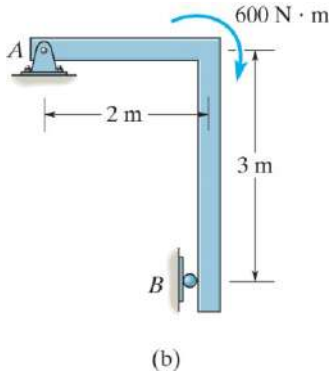
$= 400N\ (\leftarrow)$ //

$\sum F_y = 0$

$A_y - 500N + 300N = 0$

$A_y = 200N$ //

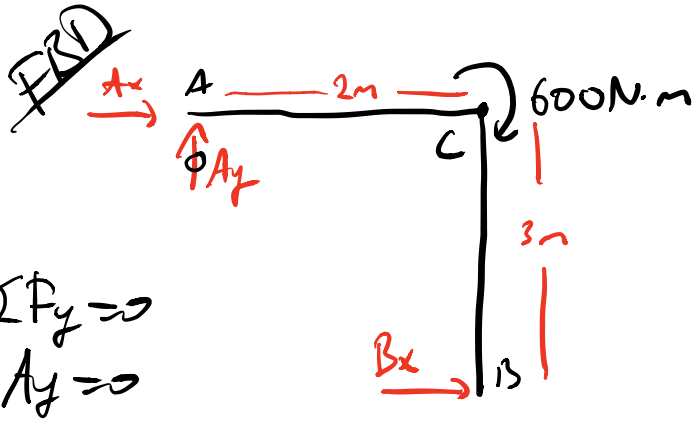
EX 3



(b)

Determine the support reactions

FBD



Unknown

A_x, A_y, B_x

Eqs. Eqn

$\sum F_x = 0$ $\sum F_y = 0$ $\sum M_{A=0}$

$\sum F_y = 0$

$A_y = 0$

$\sum F_x = 0$

$A_x + 200N = 0$

$A_x = -200N$

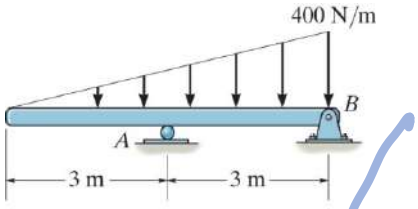
$= 200N\ (\leftarrow)$

$+\circlearrowleft \sum M_A = 0$

$-600N\cdot m + B_x \cdot 3m = 0$

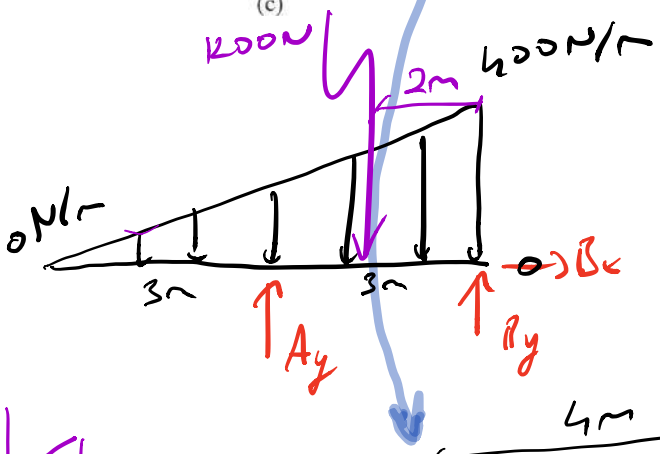
$\Rightarrow B_x = 200N$ //

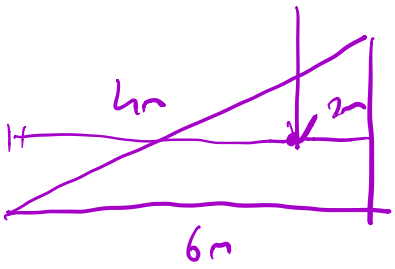
Ex:



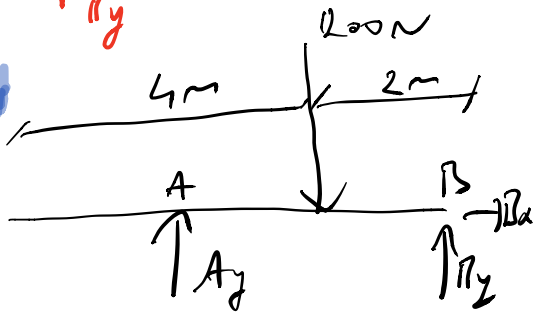
(c)

FBD





FBD



$+\circlearrowleft \sum M_B = 0$

$-A_y \cdot 3m + 1200N \cdot 2m = 0$

$A_y = 800N \uparrow$

$\sum F_y = 0 \qquad 800N - 1200N + B_y = 0$

$\Rightarrow B_y = 400N \uparrow$