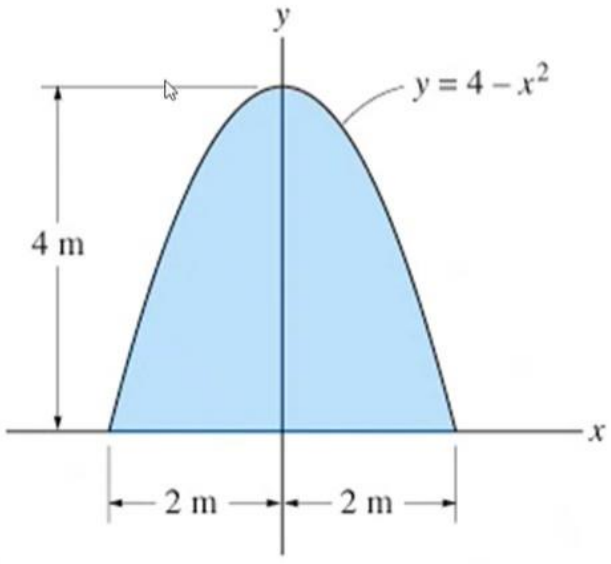
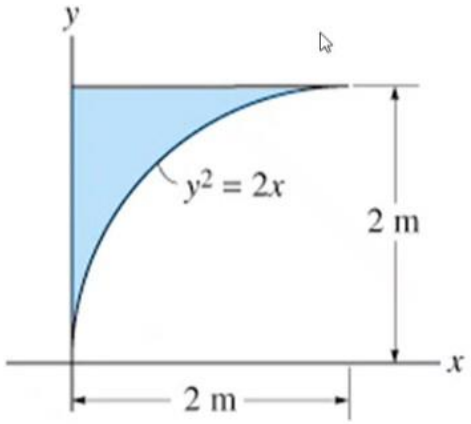
Example: 1

Calculate the moment of inertia of the shaded area about the y-axis.



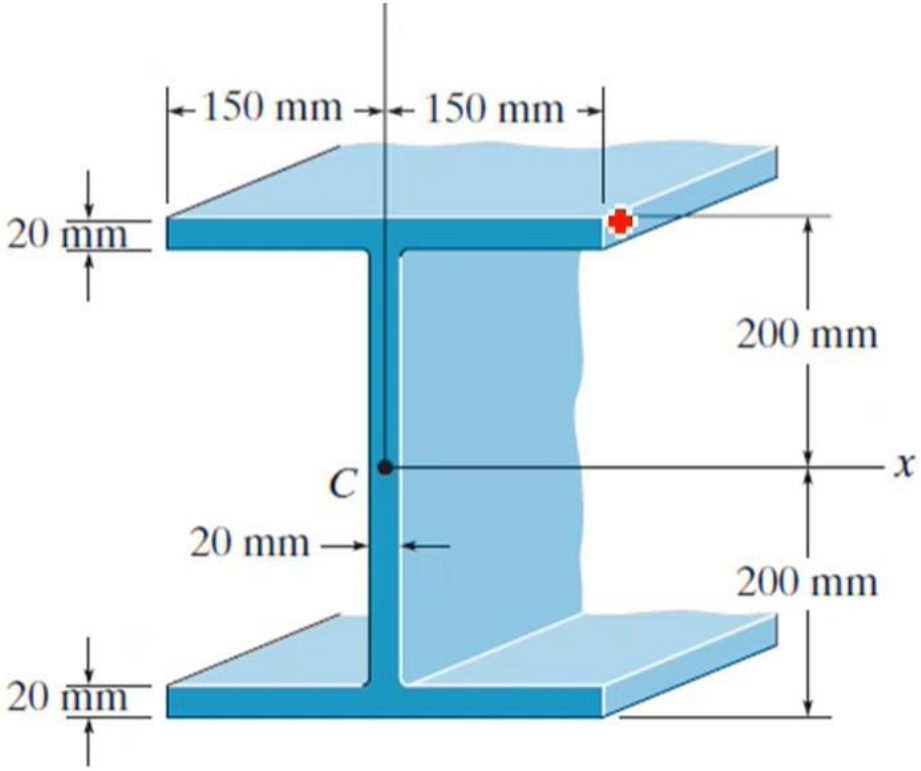
Example: 2

Calculate the moment of inertia of the shaded area about the x-axis.

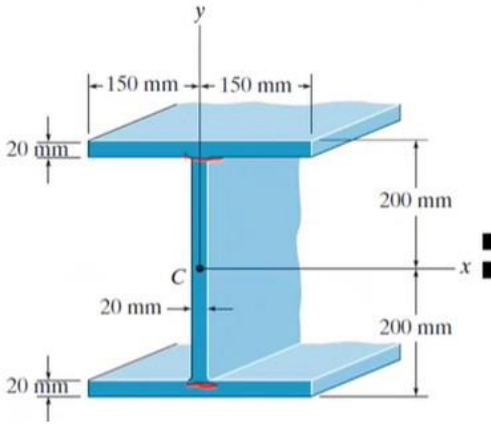


Example: 3

Calculate the moment of inertia of area of the I beam about the x and y-axis.



Solution:



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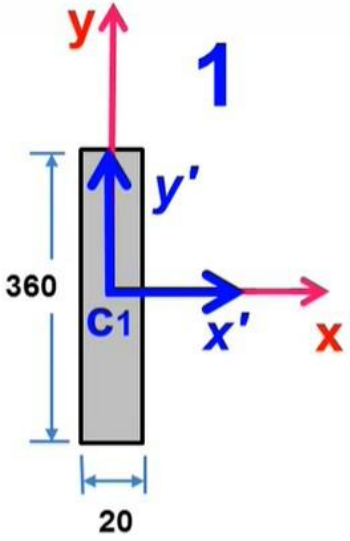

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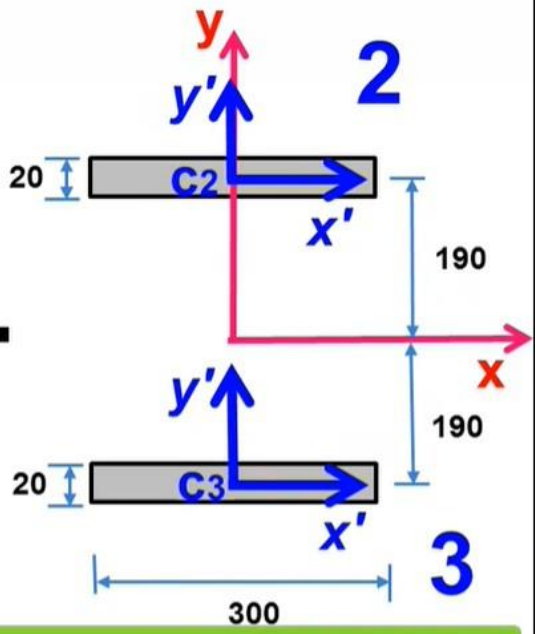
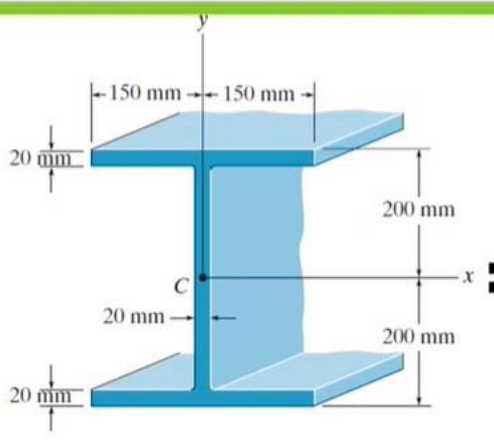


2nd way:



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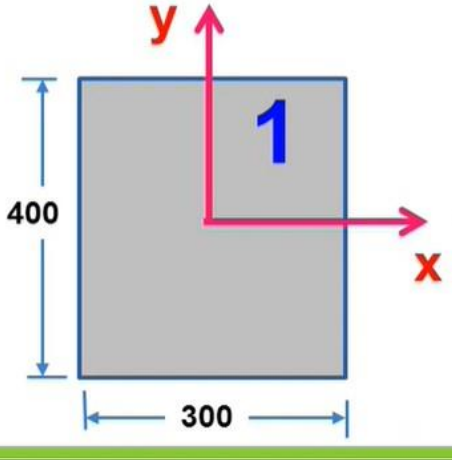

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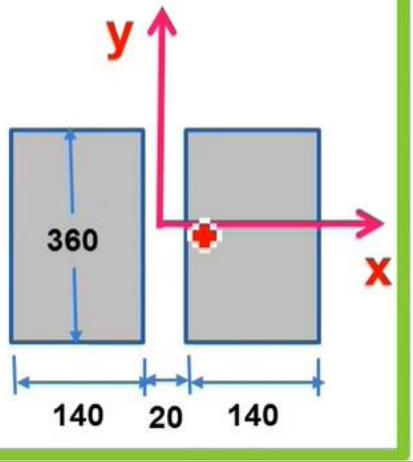
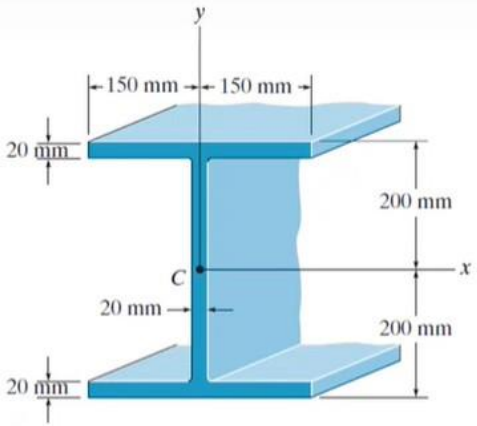


Solution:



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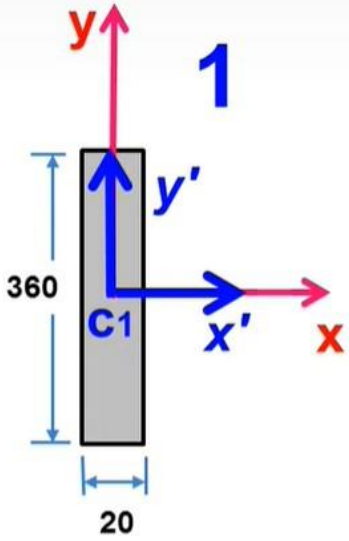

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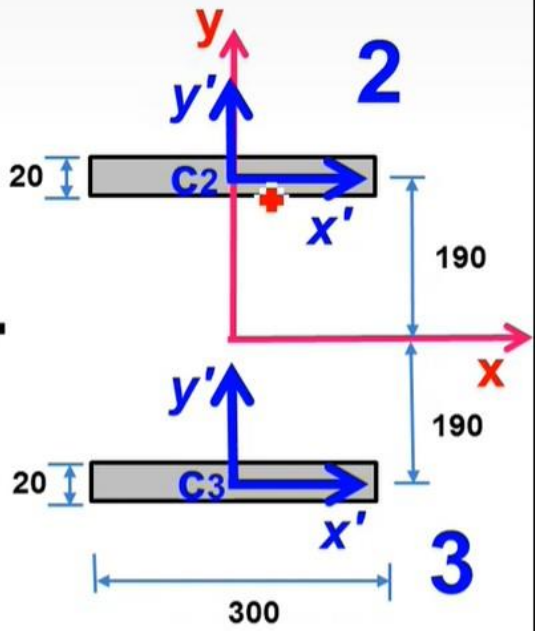
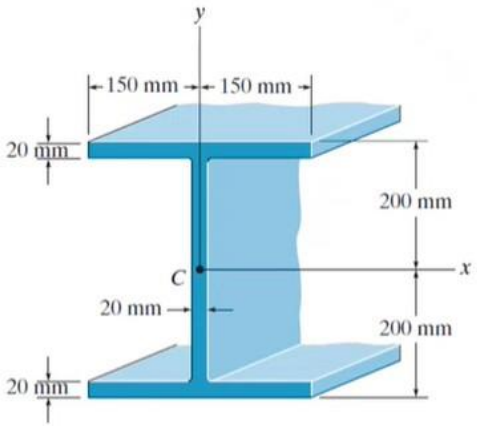
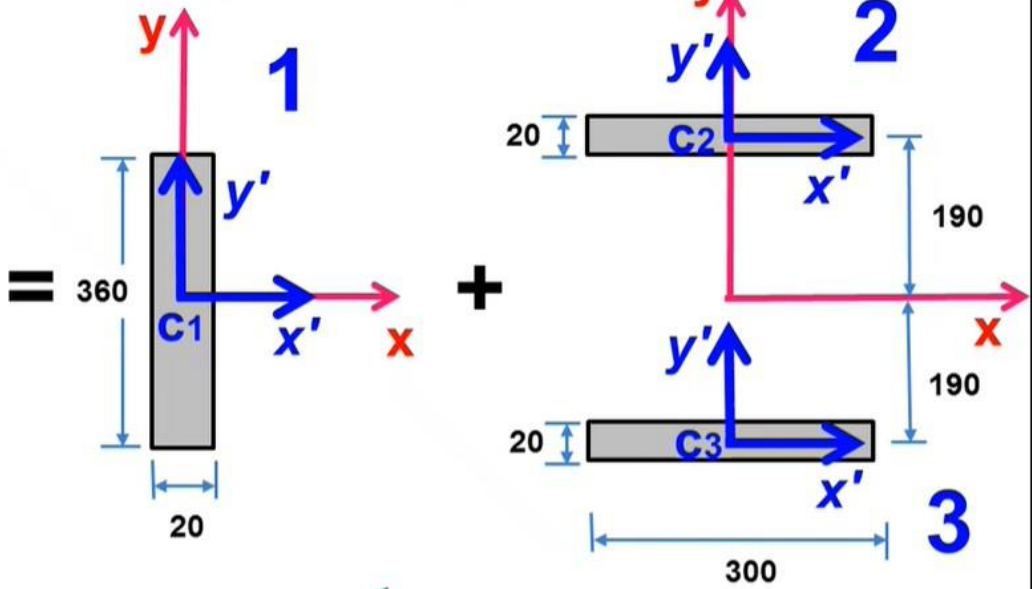


$$I_x = I_{x1} + I_{x2} + I_{x3} = I_{x1} + 2I_{x2}$$

Solution:

$$I_x = I_{x'} + Ad_y^2 \qquad I_y = I_{y'} + Ad_x^2$$





$$I_{x1} = \frac{1}{12}20(360^3) = 77.7(10^6)\ mm^4$$

$$I_{x2} = \frac{1}{12}300(20^3) + (190^2)[(20)(300)] = 216.8(10^6)\ mm^4$$

$$I_{y1} = \frac{1}{12}360(20^3) = 0.24(10^6)\ mm^4$$

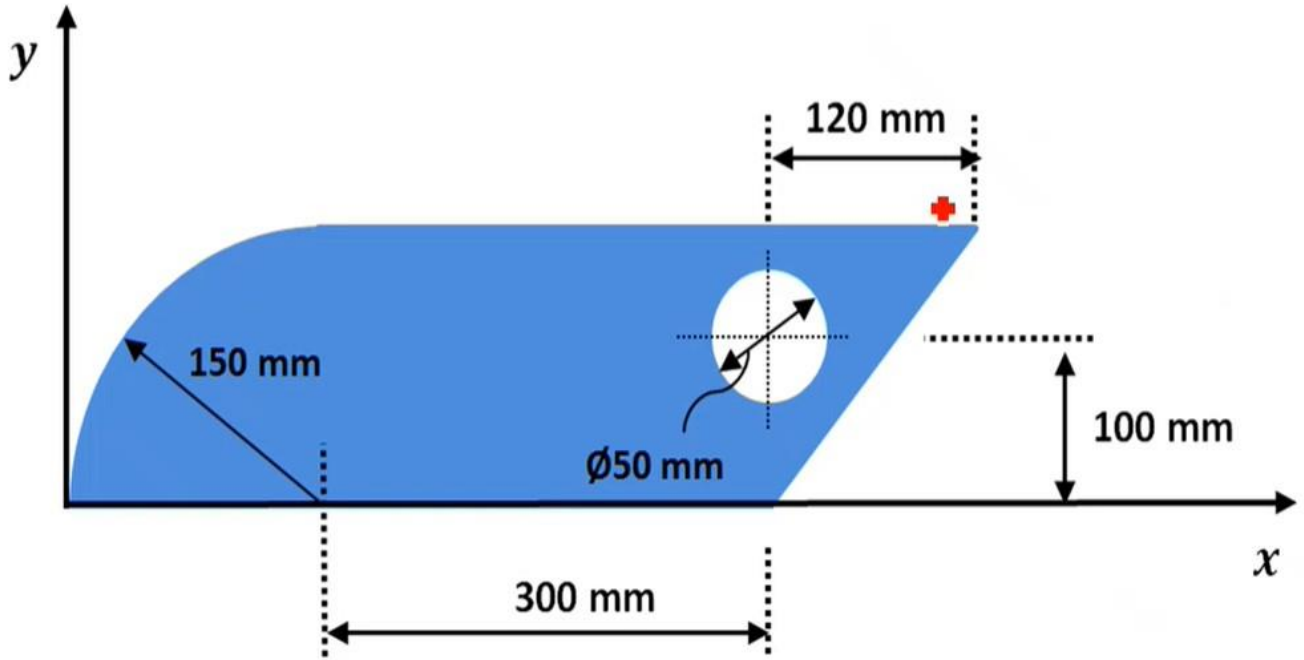
$$I_{y2} = \frac{1}{12}20(300^3) = 45(10^6)\ mm^4$$

$$I_x = I_{x1} + 2I_{x2} = (77.2 + 2(216.8))(10^6) = 510.8\ (10^6)\ mm^4$$

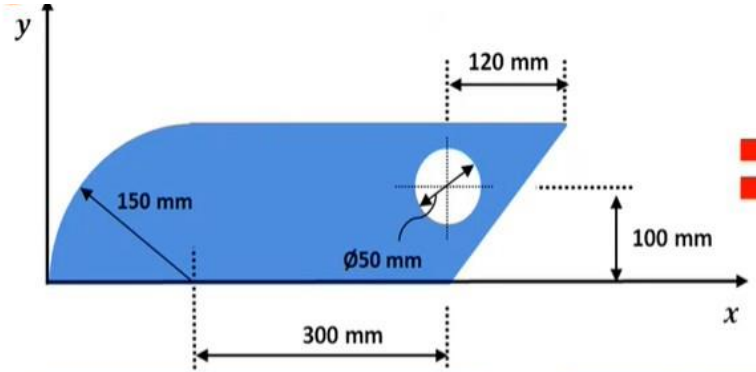
$$I_y = I_{y1} + 2I_{y2} = (0.24 + 2(45))(10^6) = 90.24(10^6)\ mm^4$$

Example: 4

Calculate the moment of inertia of area of the shape about the x and y-axis.



Solution:



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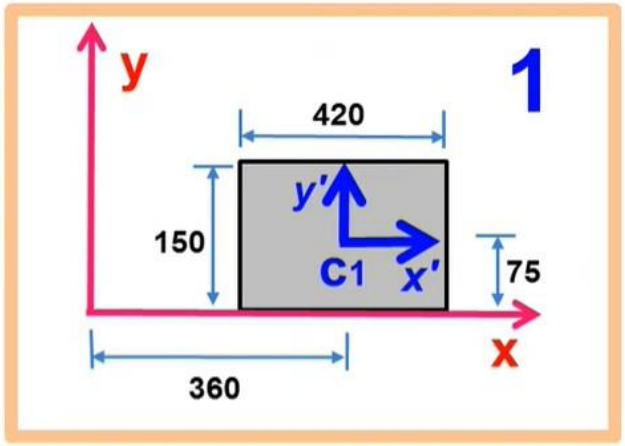

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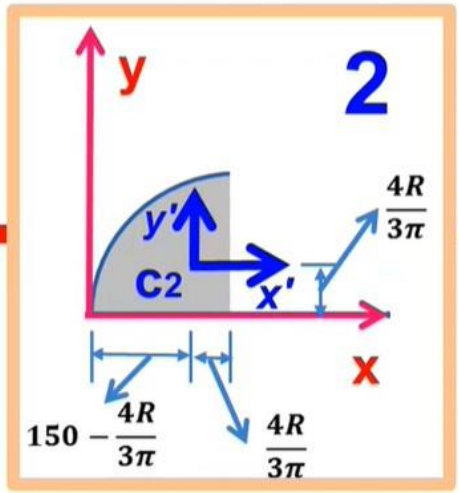

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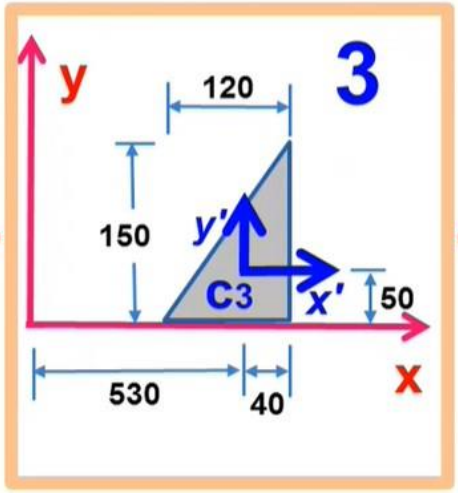

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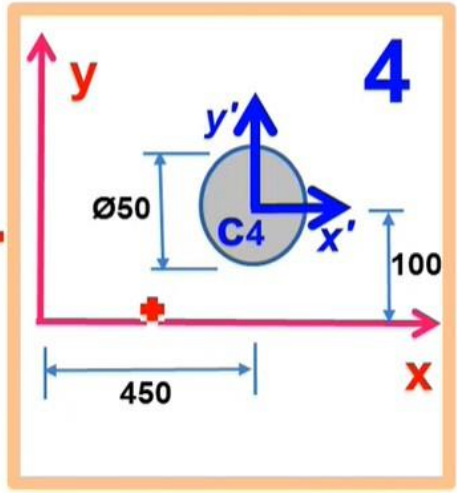
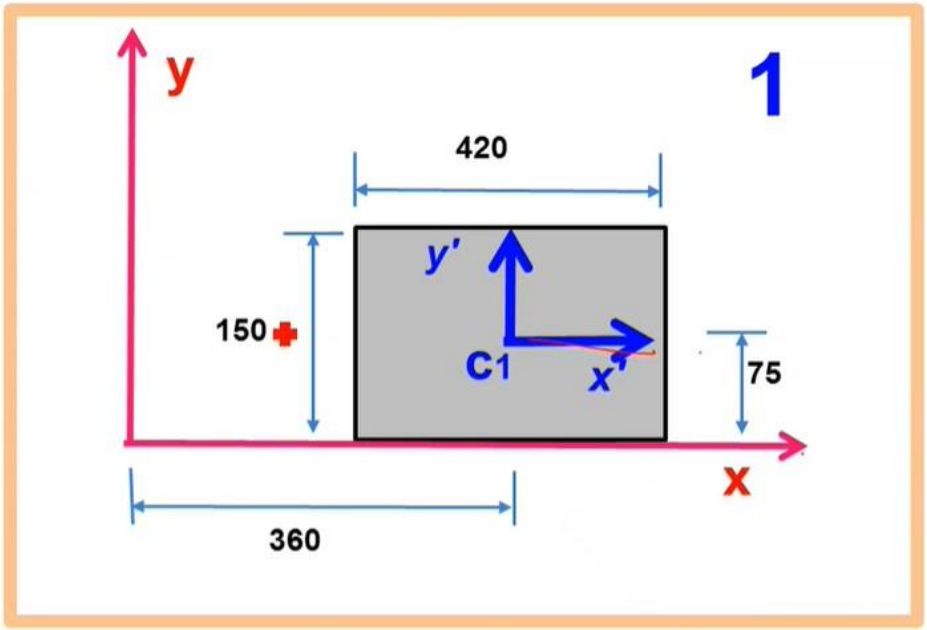


$$I_x = I_{x1} + I_{x2} - I_{x3} - I_{x4}$$

$$I_y = I_{y1} + I_{y2} - I_{y3} - I_{y4}$$

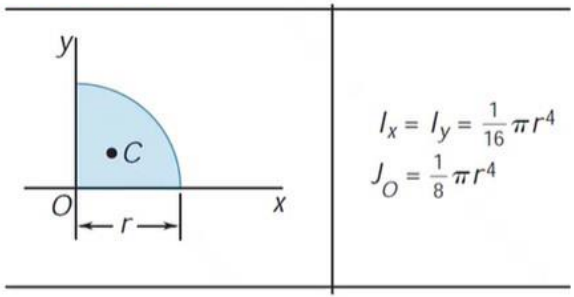
Solution:



$$I_{x1} = \frac{1}{12}420(150^3) + (75^2)[(420)(150)] = 472.5(10^6)\ mm^4$$

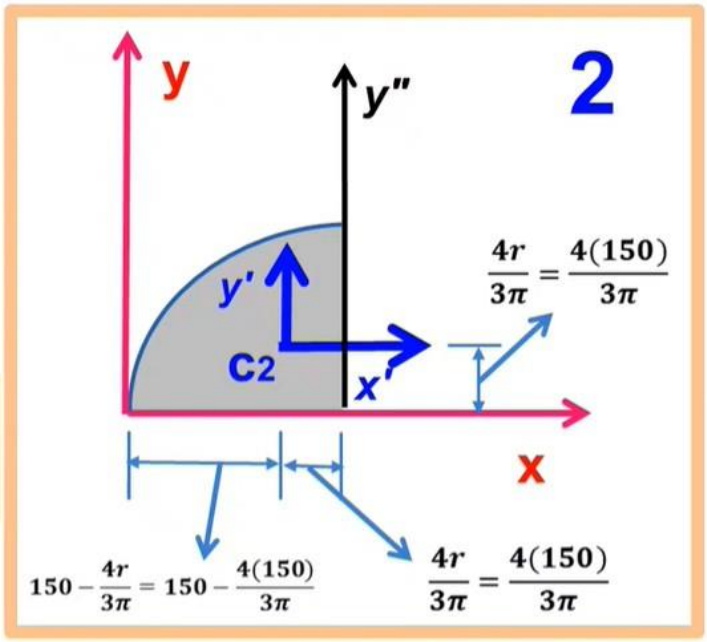
$$I_{y1} = \frac{1}{12}150(420^3) + (360^2)[(420)(150)] = 9090.9(10^6)\ mm^4$$

Solution:



$$I_x = I_y = \frac{1}{16}\pi r^4$$

$$J_O = \frac{1}{8}\pi r^4$$



$$I_{y2} = I_{y'} + \left(150 - \frac{4(150)}{3\pi}\right)^2 \left(\frac{\pi 150^2}{4}\right)$$

$I_{y'} = ?$

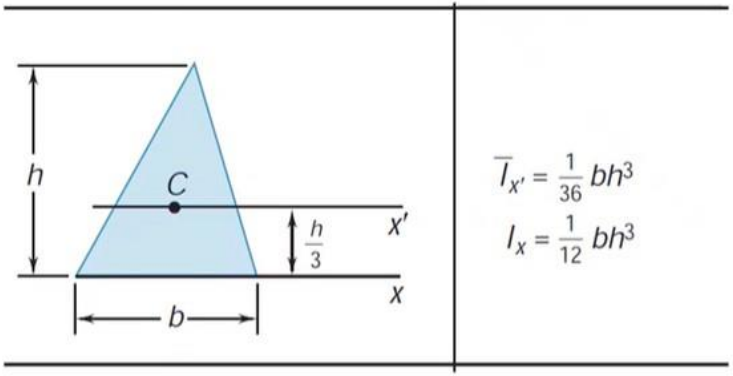
$$I_{y''} = \frac{\pi r^4}{16}$$

$$I_{y''} = I_{y'} + \left(\frac{4r}{3\pi}\right)^2 \left(\frac{\pi r^2}{4}\right) \Rightarrow I_{y'} = \frac{\pi r^4}{16} - \frac{4r^4}{9\pi}$$

$$I_{x2} = \frac{\pi 150^4}{16} = 99.4(10^6)\ mm^4$$

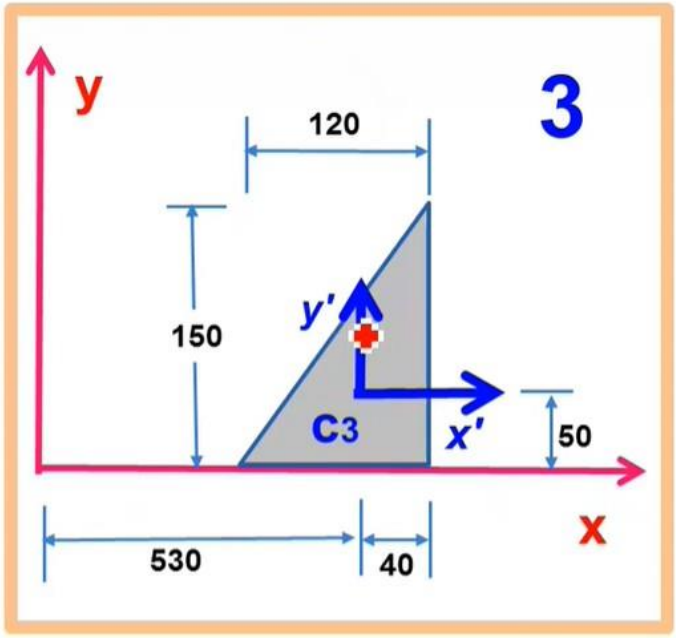
$$I_{y2} = \left(\frac{\pi 150^4}{16} - \frac{4(150^4)}{9\pi}\right) + \left(150 - \frac{4(150)}{3\pi}\right)^2 \left(\frac{\pi 150^2}{4}\right) = 159.51(10^6)\ mm^4$$

Solution:



$$\bar{I}_{x'} = \frac{1}{36} bh^3$$

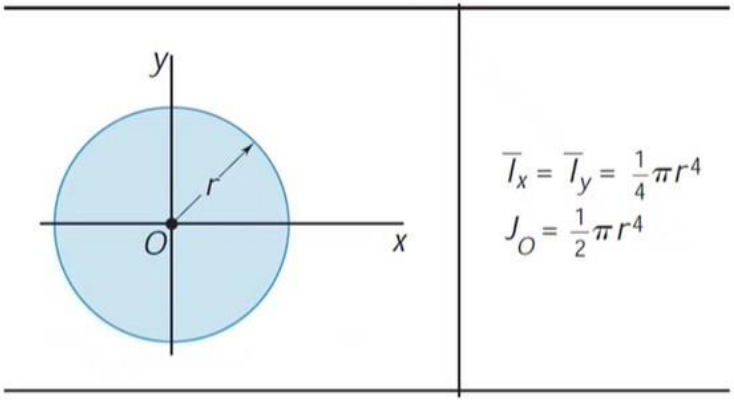
$$I_x = \frac{1}{12} bh^3$$



$$I_{x3} = \frac{1}{36}120(150^3) + (50^2)\left(\frac{(120)(150)}{2}\right) = 33.75(10^6)\ mm^4$$

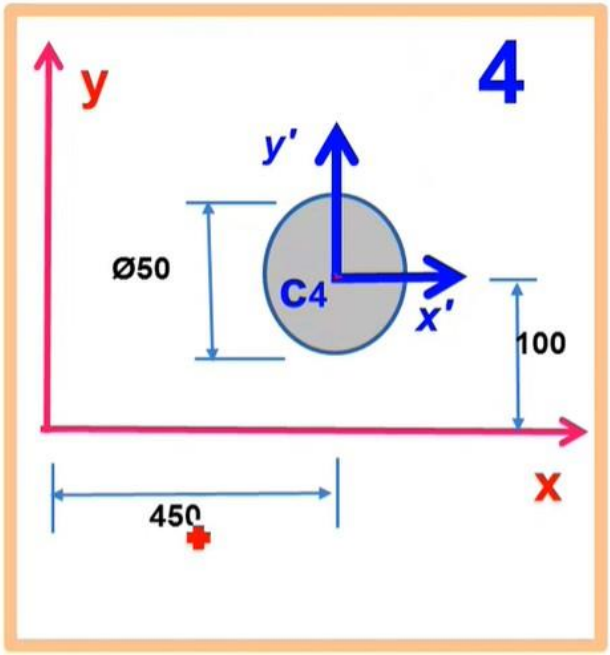
$$I_{y3} = \frac{1}{36}150(120^3) + (530^2)\left(\frac{(120)(150)}{2}\right) = 2535.3(10^6)\ mm^4$$

Solution:



$$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$$

$$J_O = \frac{1}{2}\pi r^4$$



$$I_{x4} = \frac{\pi 25^4}{4} + 100^2(\pi 25^2) = 19.94(10^6)\ mm^4$$

$$I_{y4} = \frac{\pi 25^4}{4} + 450^2(\pi 25^2) = 379.91(10^6)\ mm^4$$

Solution:

$$I_x = I_{x1} + I_{x2} - I_{x3} - I_{x4}$$

$$I_x = (472.5 + 99.4 - 33.75 - 19.94) = 518.21\,(10^6)\,mm^4$$

$$I_y = I_{y1} + I_{y2} - I_{y3} - I_{y4}$$

$$I_y = (9090.9 + 159.51 - 2535.3 - 379.91)(10^6)$$
$$= 6335.2\,(10^6)\,mm^4$$