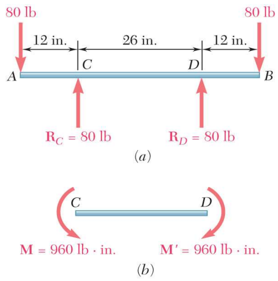
# ME 211 Statics and Strength of Materials

## Chapter 10

### Pure Bending








***Pure Bending*:**
Prismatic members subjected to equal and opposite couples acting in the same longitudinal plane

**Fig. 4.2** (a) Free-body diagram of the barbell pictured in the chapter opening photo and (b) Free-body diagram of the center bar portion showing pure bending.

# Other Loading Types

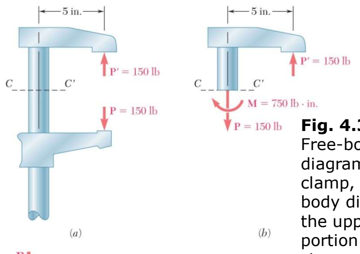



**Fig. 4.3** (a) Free-body diagram of a clamp, (b) free-body diagram of the upper portion of the clamp.

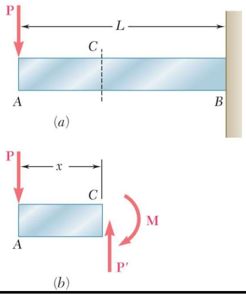



**Fig. 4.4** (a) Cantilevered beam with end loading. (b) As portion *AC* shows, beam is not in pure bending.

*Eccentric Loading*: Axial loading which does not pass through section centroid produces internal forces equivalent to an axial force and a couple

*Transverse Loading*: Concentrated or distributed transverse load produces internal forces equivalent to a shear force and a couple

*Principle of Superposition*: The normal stress due to pure bending may be combined with the normal stress due to axial loading and shear stress due to shear loading to find the complete state of stress.

# Symmetric Member in Pure Bending

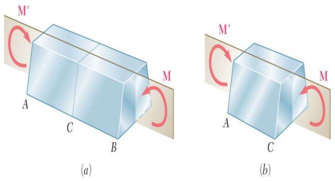



**Fig. 4.5** (a) A member in a state of pure bending. (b) Any intermediate portion of *AB* will also be in pure bending.

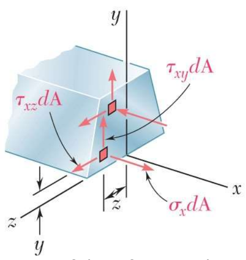



**Fig. 4.6** Summation of the infinitesimal stress elements must produce the equivalent pure-bending moment.

Internal forces in any cross section are equivalent to a couple. The moment of the couple is the section *bending moment*.

From statics, a couple **M** consists of two equal and opposite forces.

The sum of the components of the forces in any direction is zero.

The moment is the same about *any* axis perpendicular to the plane of the couple and zero about any axis contained in the plane.

These requirements may be applied to the sums of the components and moments of the statically indeterminate elementary internal forces.

$$F_x = \int \sigma_x \, dA = 0$$
$$M_y = \int z\sigma_x \, dA = 0$$
$$M_z = \int -y\sigma_x \, dA = M$$

# Bending Deformations

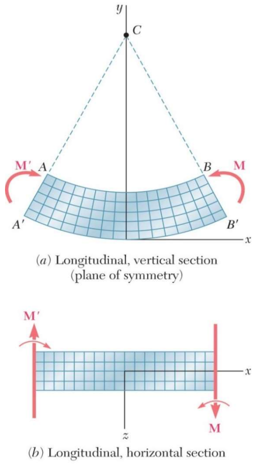



**Fig. 4.9** Member subject to pure bending shown in two views. (a) Longitudinal, vertical view (plane of symmetry) and (b) Longitudinal, horizontal view.

Beam with a plane of symmetry in pure bending:

member remains symmetric

bends uniformly to form a circular arc

cross-sectional plane passes through arc center and remains planar

length of top decreases and length of bottom increases

a *neutral surface* must exist that is parallel to the upper and lower surfaces and for which the length does not change

stresses and strains are negative (compressive) above the neutral plane and positive (tension) below it

# Strain Due to Bending

Consider a beam segment of length $L$.
After deformation, the length of the neutral surface remains $L$. At other sections,

$$L' = (\rho - y)\theta$$

$$\delta = L' - L = (\rho - y)\theta - \rho\theta = -y\theta$$

$$\varepsilon_x = \frac{\delta}{L} = -\frac{y\theta}{\rho\theta} = -\frac{y}{\rho} \quad \text{(strain varies linearly)}$$

$$\varepsilon_m = \frac{c}{\rho} \quad \text{or} \quad \rho = \frac{c}{\varepsilon_m}$$

$$\varepsilon_x = -\frac{y}{c}\varepsilon_m$$

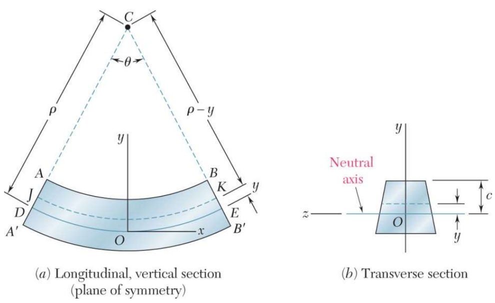
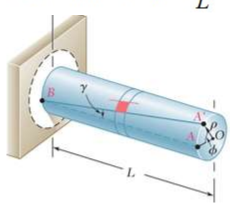



**Fig. 4.10** Kinematic definitions for pure bending. (a) Longitudinal-vertical view and (b) Transverse section at origin.

$$L\gamma = \rho\phi \quad \text{or} \quad \gamma = \frac{\rho\phi}{L}$$




$$T = \int \rho\tau\, dA = \frac{\tau_{max}}{c}\int \rho^2\, dA = \frac{\tau_{max}}{c}J$$

## Stress Due to Bending

For a linearly elastic and homogeneous material,

$$\sigma_x = E\varepsilon_x = -\frac{y}{c}E\varepsilon_m$$

$$= -\frac{y}{c}\sigma_m \quad \text{(stress varies linearly)}$$

For static equilibrium,

$$F_x = 0 = \int \sigma_x \, dA = \int -\frac{y}{c}\sigma_m \, dA$$

$$0 = -\frac{\sigma_m}{c}\int y \, dA$$

**First moment with respect to neutral axis is zero. Therefore, the neutral axis must pass through the section centroid.**

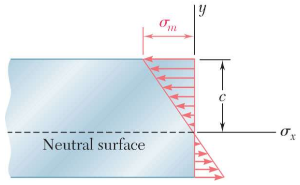



**Fig. 4.11** Bending stresses vary linearly with distance from the neutral axis.

For static equilibrium,

$$M = \int(-y\sigma_x \, dA) = \int(-y)\left(-\frac{y}{c}\sigma_m\right) dA$$

$$M = \frac{\sigma_m}{c}\int y^2 \, dA = \frac{\sigma_m I}{c}$$

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

Substituti ng $\sigma_x = -\frac{y}{c}\sigma_m$

$$\sigma_x = -\frac{My}{I}$$

## Beam Section Properties

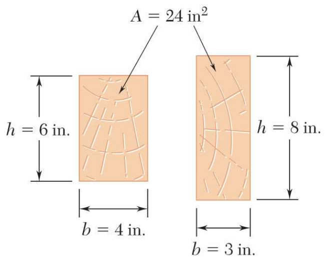



**Fig. 4.12** Wood beam cross sections.

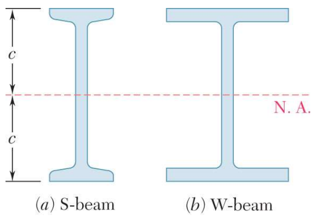



**Fig. 4.13** Two type of steel beam cross sections. (a) S-beam and (b) W-beam

The maximum normal stress due to bending,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

$I$ = section moment of inertia

$S = \frac{I}{c}$ = section modulus

**A beam section with a larger section modulus will have a lower maximum stress**

Consider a rectangular beam cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^3 = \frac{1}{6}Ah$$

Between two beams with the same cross sectional area, the beam with the larger depth $h$ will be more effective in resisting bending.

Structural steel beams are designed to have a large section modulus.

# Properties of American Standard Shapes

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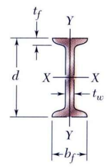
**Appendix C.** Properties of Rolled-Steel Shapes
(SI Units)

**S Shapes**
(American Standard Shapes)




| | | | Flange | | | Axis X-X | | | Axis Y-Y | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Designation† | Area $A$, mm$^2$ | Depth $d$, mm | Width $b_f$, mm | Thickness $t_f$, mm | Web Thickness $t_w$, mm | $I_x$ $10^6$ mm$^4$ | $S_x$ $10^3$ mm$^3$ | $r_x$ mm | $I_y$ $10^6$ mm$^4$ | $S_y$ $10^3$ mm$^3$ | $r_y$ mm |
| S610 × 180 | 22900 | 622 | 204 | 27.7 | 20.3 | 1320 | 4240 | 240 | 34.9 | 341 | 39.0 |
| 158 | 20100 | 622 | 200 | 27.7 | 15.7 | 1230 | 3950 | 247 | 32.5 | 321 | 39.9 |
| 149 | 19000 | 610 | 184 | 22.1 | 18.9 | 995 | 3260 | 229 | 20.2 | 215 | 32.3 |
| 134 | 17100 | 610 | 181 | 22.1 | 15.9 | 938 | 3080 | 234 | 19.0 | 206 | 33.0 |
| 119 | 15200 | 610 | 178 | 22.1 | 12.7 | 878 | 2880 | 240 | 17.9 | 198 | 34.0 |
| S510 × 143 | 18200 | 516 | 183 | 23.4 | 20.3 | 700 | 2710 | 196 | 21.3 | 228 | 33.9 |
| 128 | 16400 | 516 | 179 | 23.4 | 16.8 | 658 | 2550 | 200 | 19.7 | 216 | 34.4 |
| 112 | 14200 | 508 | 162 | 20.2 | 16.1 | 530 | 2090 | 193 | 12.6 | 152 | 29.5 |
| 98.3 | 12500 | 508 | 159 | 20.2 | 12.8 | 495 | 1950 | 199 | 11.8 | 145 | 30.4 |
| S460 × 104 | 13300 | 457 | 159 | 17.6 | 18.1 | 385 | 1685 | 170 | 10.4 | 127 | 27.5 |
| 81.4 | 10400 | 457 | 152 | 17.6 | 11.7 | 333 | 1460 | 179 | 8.83 | 113 | 28.8 |
| S380 × 74 | 9500 | 381 | 143 | 15.6 | 14.0 | 201 | 1060 | 145 | 6.65 | 90.8 | 26.1 |
| 64 | 8150 | 381 | 140 | 15.8 | 10.4 | 185 | 971 | 151 | 6.15 | 85.7 | 27.1 |

# Deformations in a Transverse Cross Section

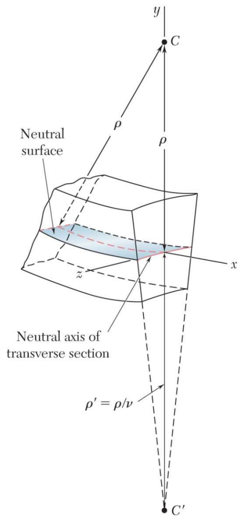



**Fig. 4.16** Deformation of a transverse cross section.

Deformation due to bending moment $M$ is quantified by the curvature of the neutral surface

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{\sigma_m}{Ec} = \frac{1}{Ec}\frac{Mc}{I}$$

$$= \frac{M}{EI}$$

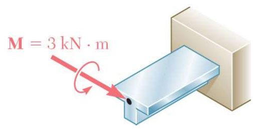
Although transverse cross sectional planes remain planar when subjected to bending moments, in-plane deformations are nonzero,

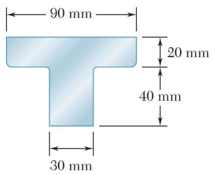
$$\varepsilon_y = -\nu\varepsilon_x = \frac{\nu y}{\rho} \qquad \varepsilon_z = -\nu\varepsilon_x = \frac{\nu y}{\rho}$$

Expansion above the neutral surface and contraction below it cause an in-plane curvature,

$$\frac{1}{\rho'} = \frac{\nu}{\rho} = \text{anticlastic curvature}$$

# Sample Problem 4.2




A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing $E$ = 165 GPa and neglecting the effects of fillets, determine (a) the maximum tensile and compressive stresses, (b) the radius of curvature.

SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} \qquad I_{x'} = \sum\left(\bar{I} + Ad^2\right)$$

Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

# Sample Problem 4.2

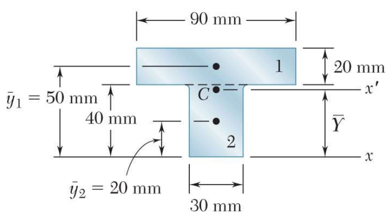



**Fig. 1** Composite areas for calculating centroid.

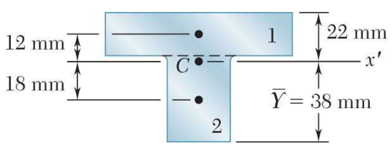



**Fig. 2** Composite sections for calculating moment of inertia.

SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

| | Area, mm$^2$ | $\bar{y}$, mm | $\bar{y}A$, mm$^3$ |
|---|---|---|---|
| 1 | $20\times90=1800$ | 50 | $90\times10^3$ |
| 2 | $40\times30=1200$ | 20 | $24\times10^3$ |
| | $\sum A = 3000$ | | $\sum \bar{y}A = 114\times10^3$ |

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{114\times10^3}{3000} = 38\text{ mm}$$

$$I_{x'} = \sum\left(\bar{I} + Ad^2\right) = \sum\left(\tfrac{1}{12}bh^3 + Ad^2\right)$$

$$= \left(\tfrac{1}{12}90\times20^3 + 1800\times12^2\right) + \left(\tfrac{1}{12}30\times40^3 + 1200\times18^2\right)$$

$$I = 868\times10^3\text{ mm}^4 = 868\times10^{-9}\text{ m}^4$$

## Sample Problem 4.2

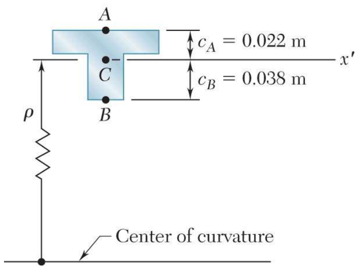



**Fig. 3** Deformed radius of curvature is measured to the centroid of the cross sections.

Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M\,c_A}{I} = \frac{3\,\text{kN}\cdot\text{m} \times 0.022\,\text{m}}{868\times10^{-9}\,\text{m}^4}$$

$$\boxed{\sigma_A = +76.0\,\text{MPa}}$$

$$\sigma_B = -\frac{M\,c_B}{I} = -\frac{3\,\text{kN}\cdot\text{m} \times 0.038\,\text{m}}{868\times10^{-9}\,\text{m}^4}$$

$$\boxed{\sigma_B = -131.3\,\text{MPa}}$$

Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$= \frac{3\,\text{kN}\cdot\text{m}}{(165\,\text{GPa})(868\times10^{-9}\,\text{m}^4)}$$

$$\boxed{\frac{1}{\rho} = 20.95\times10^{-3}\,\text{m}^{-1} \qquad \rho = 47.7\,\text{m}}$$

## Stress Concentrations

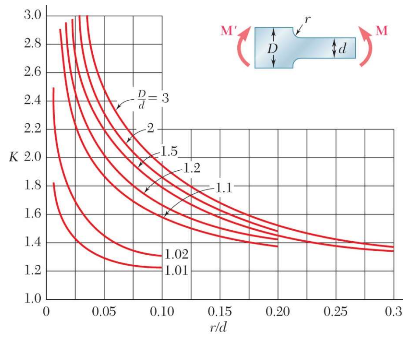



**Fig. 4.24** Stress-concentration factors for flat bars with fillets under pure bending.

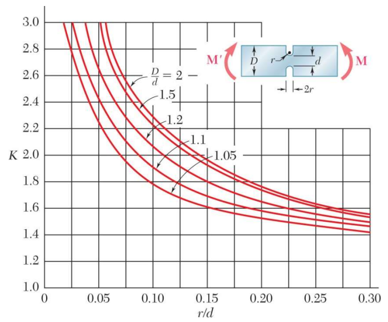



**Fig. 4.25** Stress-concentration factors for flat bars with grooves (notches) under pure bending.

Stress concentrations may occur:

in the vicinity of points where the loads are applied

in the vicinity of abrupt changes in cross section

Maximum stress:

$$\sigma_m = K\frac{Mc}{I}$$

# Eccentric Axial Loading in a Plane of Symmetry

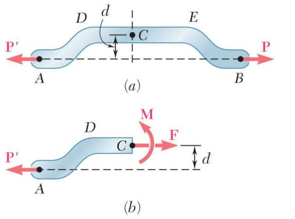



**Fig. 4.39** (a) Member with eccentric loading. (b) Free-body diagram of a member with internal loads at section C.

Eccentric loading

$$F = P$$
$$M = Pd$$

Stress due to eccentric loading found by superposing the uniform stress due to a centric load and linear stress distribution due to a pure bending moment

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$
$$= \frac{P}{A} - \frac{My}{I}$$

Result are valid if stresses do not exceed the proportional limit, deformations have negligible effect on geometry, and stresses are not evaluated near points of load application.

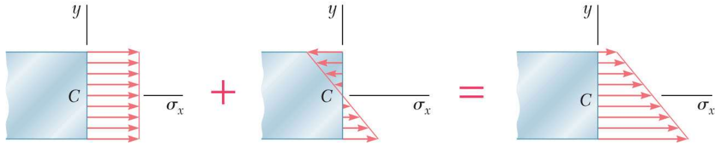



**Fig. 4.41** Stress distribution for eccentric loading is obtained by superposing the axial and pure bending distributions.

# Concept Application 4.7

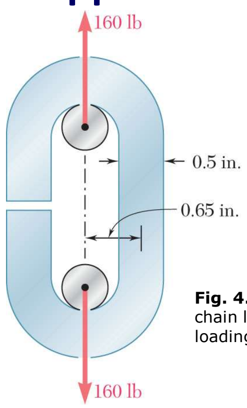



**Fig. 4.43** Open chain link under loading.

An open-link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine (a) maximum tensile and compressive stresses, (b) distance between section centroid and neutral axis

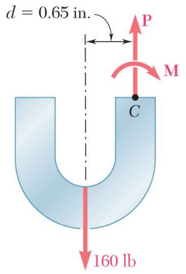
SOLUTION:

Find the **equivalent** centric load and bending moment

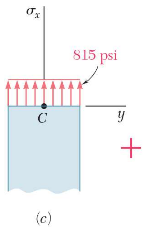
Superpose the uniform stress due to the centric load and the linear stress due to the bending moment.

Evaluate the maximum tensile and compressive stresses at the inner and outer edges, respectively, of the superposed stress distribution.

Find the neutral axis by determining the location where the normal stress is zero.





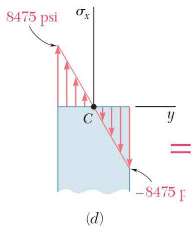




Normal stress due to a centric load

$$A = \pi c^2 = \pi(0.25\,\text{in})^2 = 0.1963\,\text{in}^2$$

$$\sigma_0 = \frac{P}{A} = \frac{160\,\text{lb}}{0.1963\,\text{in}^2} = 815\,\text{psi}$$

**Fig. 4.43** Free-body diagram for section at C to find axial force and moment. Stress at section C is superposed axial and bending stresses.




Equivalent centric load and bending moment

$$P = 160\,\text{lb}$$

$$M = Pd = (160\,\text{lb})(0.65\,\text{in}) = 104\,\text{lb}\cdot\text{in}$$

Normal stress due to bending moment

$$I = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.25)^4 = 3.068\times10^{-3}\,\text{in}^4$$

$$\sigma_m = \frac{Mc}{I} = \frac{(104\,\text{lb}\cdot\text{in})(0.25\,\text{in})}{3.068\times10^{-3}\,\text{in}^4} = 8475\,\text{psi}$$

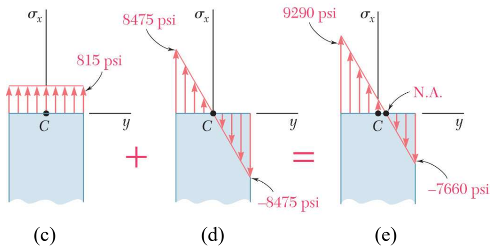



**Fig. 4.43** (c) Axial stress at section C. (d) Bending stress at C. (e) Superposition of stresses.

Maximum tensile and compressive stresses

$$\sigma_t = \sigma_0 + \sigma_m = 815 + 8475 \qquad \boxed{\sigma_t = 9260\,\text{psi}}$$

$$\sigma_c = \sigma_0 - \sigma_m = 815 - 8475 \qquad \boxed{\sigma_c = -7660\,\text{psi}}$$

Neutral axis location

$$0 = \frac{P}{A} - \frac{My_0}{I}$$

$$y_0 = \frac{P}{A}\frac{I}{M} = (815\,\text{psi})\frac{3.068\times10^{-3}\,\text{in}^4}{105\,\text{lb}\cdot\text{in}}$$

$$\boxed{y_0 = 0.0240\,\text{in}}$$

# Sample Problem 4.8

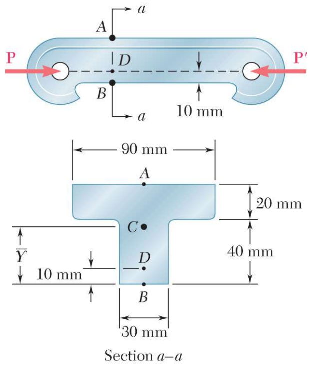



**Fig. 1** Section geometry to find centroid location.

From Sample Problem 4.2,

$$A = 3\times10^{-3}\,\text{m}^2$$
$$\bar{Y} = 0.038\,\text{m}$$
$$I = 868\times10^{-9}\,\text{m}^4$$

The largest allowable stresses for the cast iron link are 30 MPa in tension and 120 MPa in compression. Determine the largest force *P* which can be applied to the link.

SOLUTION:

Determine equivalent centric load and bending moment.

Superpose the stress due to a centric load and the stress due to bending.

Evaluate the critical loads for the allowable tensile and compressive stresses.

The largest allowable load is the smallest of the two critical loads.

# Sample Problem 4.8

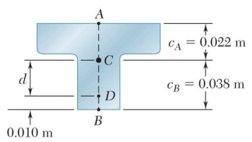
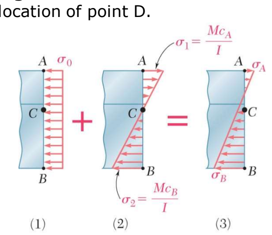



**Fig. 2** Section dimensions for finding location of point D.




**Figs. 4** Stress distribution at section C is superposition of axial and bending distributions acting at centroid.

Determine equivalent centric and bending loads.

$$d = 0.038 - 0.010 = 0.028\,\text{m}$$
$$P = \text{centric load}$$
$$M = Pd = 0.028P = \text{bending moment}$$

Superpose stresses due to centric and bending loads

$$\sigma_A = -\frac{P}{A} + \frac{Mc_A}{I} = -\frac{P}{3\times10^{-3}} + \frac{(0.028P)(0.022)}{868\times10^{-9}} = +377P$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc_A}{I} = -\frac{P}{3\times10^{-3}} - \frac{(0.028P)(0.022)}{868\times10^{-9}} = -1559P$$

Evaluate critical loads for allowable stresses.

$$\sigma_A = +377P = 30\,\text{MPa} \qquad P = 79.6\,\text{kN}$$
$$\sigma_B = -1559P = -120\,\text{MPa} \qquad P = 77.0\,\text{kN}$$

The largest allowable load $\boxed{P = 77.0\ \text{kN}}$

## Unsymmetric Bending

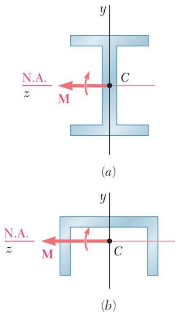



**Fig. 4.44** Moment in plane of symmetry.

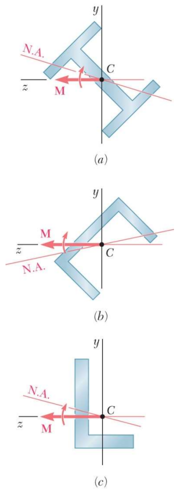



**Fig. 4.45** Moment not in plane of symmetry.

Analysis of pure bending has been limited to members subjected to bending couples acting in a plane of symmetry.

Members remain symmetric and bend in the plane of symmetry.

The neutral axis of the cross section coincides with the axis of the couple.

Will now consider situations in which the bending couples do *not* act in a plane of symmetry.

Cannot assume that the member will bend in the plane of the couples.

In general, the neutral axis of the section will not coincide with the axis of the couple.

## Unsymmetric Bending

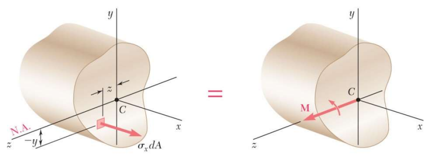



**Fig. 4.46** Section of arbitrary shape where the neutral axis coincides with the axis of couple **M**.

Wish to determine the conditions under which the neutral axis of a cross section of arbitrary shape coincides with the axis of the couple as shown.

The resultant force and moment from the distribution of elementary forces in the section must satisfy

$$F_x = 0 = M_y \quad M_z = M = applied \quad couple$$

$$0 = F_x = \int \sigma_x dA = \int \left( -\frac{y}{c}\sigma_m \right) dA$$

or $0 = \int y\,dA$

neutral axis passes through centroid

$$M = M_z = -\int y\left( -\frac{y}{c}\sigma_m \right) dA$$

or $M = \dfrac{\sigma_m I}{c} \quad I = I_z = moment\ of\ inertia$

defines stress distribution

$$0 = M_y = \int z\sigma_x dA = \int z\left( -\frac{y}{c}\sigma_m \right) dA$$

or $0 = \int yz\,dA = I_{yz} = product\ of\ inertia$

couple vector must be directed along a principal centroidal axis

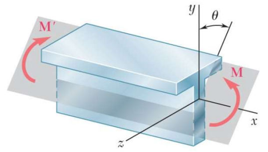


**Fig. 4.49** Unsymmetric bending, with bending moment not in a plane of symmetry.

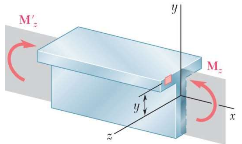


**Fig. 4.51** $\mathbf{M}_z$ acts in a plane that includes a principal centroidal axis, bending the member in the vertical plane.

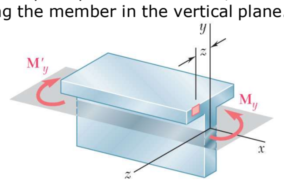


**Fig. 4.52** My acts in a plane that includes a principal centroidal axis, bending the member in the horizontal plane.

Superposition is applied to determine stresses in the most general case of unsymmetric bending.

Resolve the couple vector into components along the principle centroidal axes.

$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

Superpose the component stress distributions

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y y}{I_y}$$

Along the neutral axis,

$$\sigma_x = 0 = -\frac{M_z y}{I_z} + \frac{M_y y}{I_y} = -\frac{(M\cos\theta)y}{I_z} + \frac{(M\sin\theta)y}{I_y}$$

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$



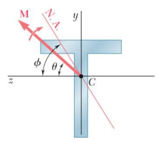
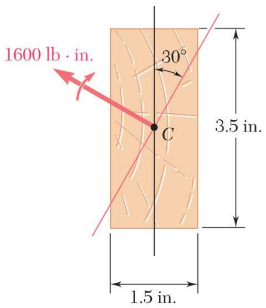


**Fig. 4.54** Neutral axis for unsymmetric bending.

# Concept Application 4.8




A 1600 lb-in couple is applied to a rectangular wooden beam in a plane forming an angle of 30° with the vertical. Determine (a) the maximum stress in the beam, (b) the angle that the neutral axis forms with the horizontal plane.

SOLUTION:

Resolve the couple vector into components along the principle centroidal axes and calculate the corresponding maximum stresses.

$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

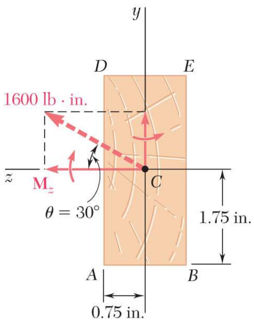
Combine the stresses from the component stress distributions.

$$\sigma_x = \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Determine the angle of the neutral axis.

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$

Resolve the couple vector into components and calculate the corresponding maximum stresses.




$$M_z = (1600\,\text{lb}\cdot\text{in})\cos 30 = 1386\,\text{lb}\cdot\text{in}$$
$$M_y = (1600\,\text{lb}\cdot\text{in})\sin 30 = 800\,\text{lb}\cdot\text{in}$$
$$I_z = \tfrac{1}{12}(1.5\,\text{in})(3.5\,\text{in})^3 = 5.359\,\text{in}^4$$
$$I_y = \tfrac{1}{12}(3.5\,\text{in})(1.5\,\text{in})^3 = 0.9844\,\text{in}^4$$

The largest tensile stress due to $M_z$ occurs along $AB$

$$\sigma_1 = \frac{M_z y}{I_z} = \frac{(1386\,\text{lb}\cdot\text{in})(1.75\,\text{in})}{5.359\,\text{in}^4} = 452.6\,\text{psi}$$

The largest tensile stress due to $M_z$ occurs along $AD$

$$\sigma_2 = \frac{M_y z}{I_y} = \frac{(800\,\text{lb}\cdot\text{in})(0.75\,\text{in})}{0.9844\,\text{in}^4} = 609.5\,\text{psi}$$

The largest tensile stress due to the combined loading occurs at $A$.

$$\sigma_{max} = \sigma_1 + \sigma_2 = 452.6 + 609.5 \qquad \boxed{\sigma_{max} = 1062\,\text{psi}}$$

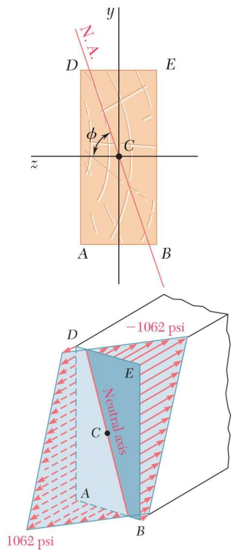



Fig. 4.55 Cross section with neutral axis and stress distribution.

Determine the angle of the neutral axis.

$$\tan\phi = \frac{I_z}{I_y}\tan\theta = \frac{5.359\,\text{in}^4}{0.9844\,\text{in}^4}\tan 30$$
$$= 3.143$$
$$\boxed{\phi = 72.4^o}$$

## General Case of Eccentric Axial Loading

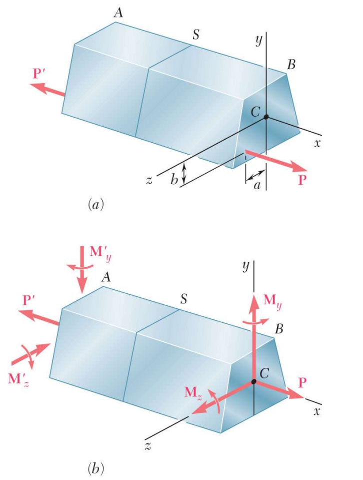



**Fig. 4.56** Eccentric axial loading. (a) Axial force applied away from section centroid. (b) Equivalent force-couple system acting at centroid.

Consider a straight member subject to equal and opposite eccentric forces.

The eccentric force is equivalent to the system of a centric force (**P**) and two couples ($\mathbf{M_x}$ and $\mathbf{M_y}$).

$$P = \text{centric force}$$

$$M_y = Pa \qquad M_z = Pb$$

By the principle of superposition, the combined stress distribution is

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

If the neutral axis lies on the section, it may be found from

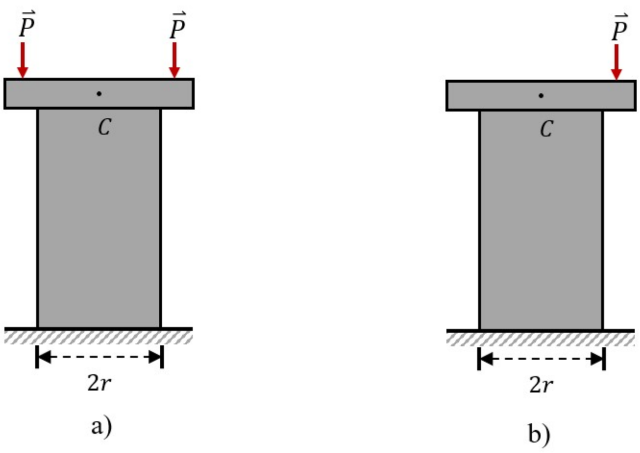
$$\frac{M_z}{I_z} y - \frac{M_y}{I_y} z = \frac{P}{A}$$

## Example 1

Determine the maximum compressive stress

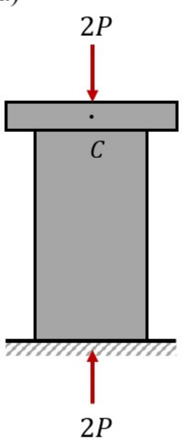
a) when both forces are applied

b) when only one force is applied



# Example 1

a)



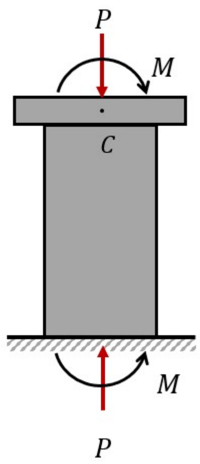

Equivalent forces is

$\vec{P} + \vec{P} = 2\vec{P}$

Moments cancel $(M = 0)$

$$\sigma_1 = -\frac{2P}{A} = -\frac{2P}{\pi r^2}$$

b) Equivalent force couple system at C is




$$|\vec{M}| = Pr$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I}$$

$$= -\frac{P}{A} - \frac{(Pr)r}{\frac{\pi}{4}r^4}$$

$$= -\frac{P}{\pi r^2} - \frac{4P}{\pi r^2}$$

$$\sigma_2 = -\frac{5P}{\pi r^2}$$

# Example 2

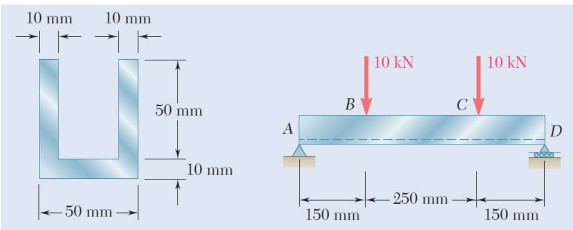
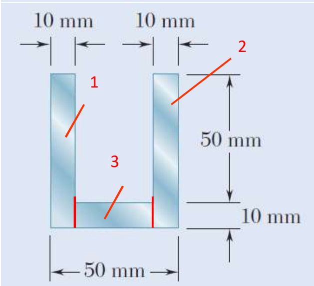



Fig. P4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

| | A $[mm^2]$ | $\bar{y}$ [mm] | $A\bar{y}$ $[mm^3]$ |
|---|---|---|---|
| 1 | 600 | 30 | 18000 |
| 2 | 600 | 30 | 18000 |
| 3 | 300 | 5 | 1500 |
| | 1500 | | 37500 |




$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{37500}{1500} = \boxed{25\ mm}$$

$$I_1 = I_2 = \frac{1}{12}(10mm)(60mm)^3 + (600)(5)^2 \qquad I_1 = I_2 = 195mm^4$$

Parallel axis theorem

$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(25-5)^2 \qquad I_3 = 122.5x10^3 mm^4$$

$$I = I_1 + I_2 + I_3 = 512.5x10^3 mm^4 \qquad \boxed{I = 512.5x10^{-9} m^4}$$

## Example 2

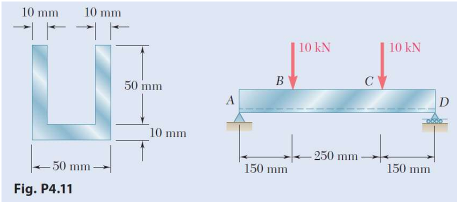
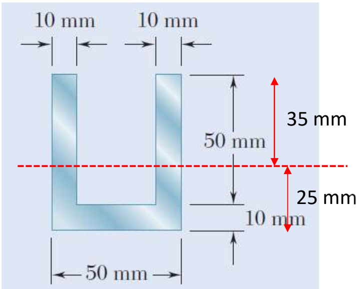
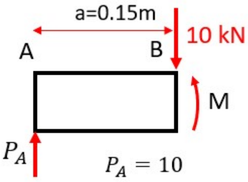


Fig. P4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.




$P_A = 10$

Moment about the load

$P_A a = (10)(0.15) = M$

$M = (10x10^3)(0.15)$

Maximum distance

$y_{top} = 0.035m$

$y_{bot} = -25mm = -0.025m$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5x10^3)(0.035)}{512.5x10^{-9}} = -102.4x10^6 Pa = -102.4\ MPa$$

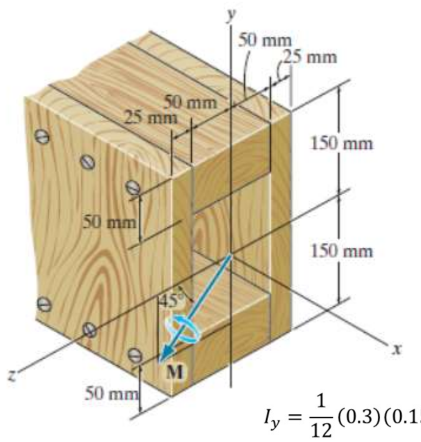
(compression)

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5x10^3)(-0.025)}{512.5x10^{-9}} = 73.2x10^6 Pa = 73.2\ MPa$$

(tension)

## Example 3

The box beam is subjected to the internal moment of M=4kN.m which is directed as shown. Determine the maximum bending stress developed in the beam and the orientation of the neutral axis.




y component of M (-y)
z component of M (+z)

$M_y = -4sin45 = -2.828kNm$

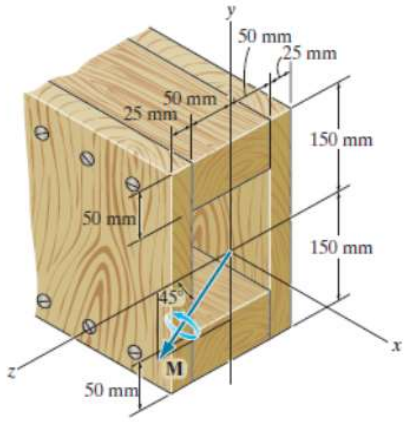
$M_z = 4cos45 = 2.828kNm$

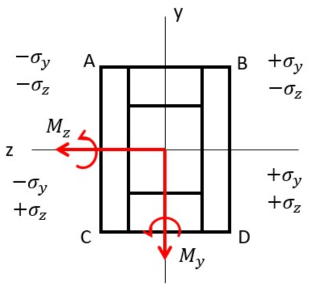
The moments of inertia of the cross section about the principal centroidal y and z axes:

$$I_y = \frac{1}{12}(0.3)(0.15)^3 - \frac{1}{12}(0.2)(0.1)^3 = 67.7083x10^{-6}\ m^4$$

$$I_z = \frac{1}{12}(0.15)(0.3)^3 - \frac{1}{12}(0.1)(0.2)^3 = 0.2708x10^{-3}\ m^4$$

# Example 3



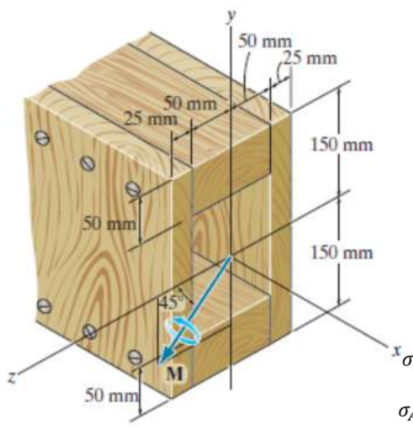

Maximum stress occurs at corners A and D

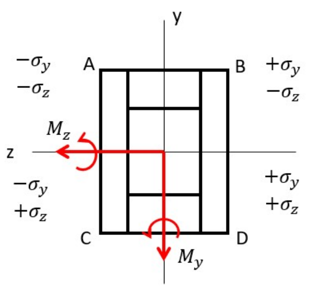
Bending stress: (+) (-)

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y}{I_y}$$

+y compression +z compression

# Example 3




$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y}{I_y}$$

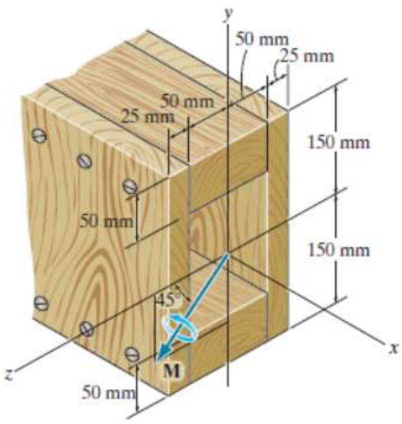
$$\sigma_{max} = \sigma_A = -\frac{2.828(10^3)(0.15)}{0.2708(10^{-3})} + \frac{(-2.828)(10^3)(0.075)}{67.7085(10^{-6})}$$

$$\sigma_A = -4.70\ MPa = 4.70\ MPa\ (C)$$

$$\sigma_{max} = \sigma_D = -\frac{2.828(10^3)(-0.15)}{0.2708(10^{-3})} + \frac{(-2.828)(10^3)(-0.075)}{67.7085(10^{-6})}$$

$$\sigma_D = 4.70\ MPa\ (T)$$

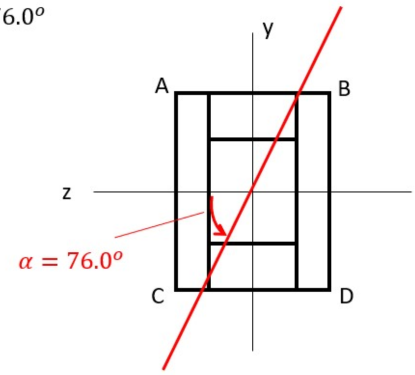
## Example 3



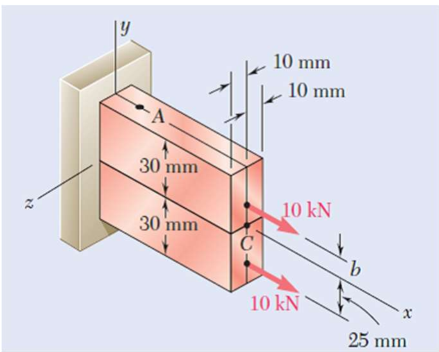

Orientation of Neutral Axis, here $\theta = -45$

$$tan\alpha = \frac{I_z}{I_y} tan\theta = \frac{0.2708(10^{-3})}{67.7083(10^{-6})} \tan(-45)$$

$$\alpha = -76.0^o$$




## Example 4




20x60 mm rectangular bar

Two 10 kN forces are applied.

Determine the stress at point A when b = 0

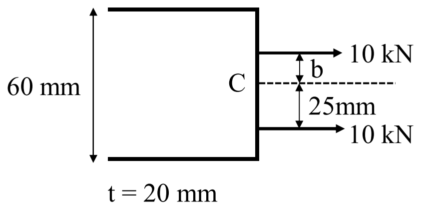
b = 15 mm

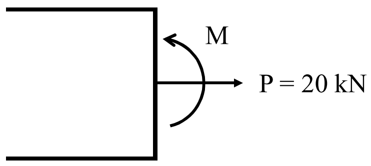
b = 25 mm

$$A = (0.06m)(0.02m) = 1.2\ x\ 10^{-3} m^2$$

$$S = \frac{I}{C} = \frac{1}{6} Ah = \frac{1}{6} bh^2 = \frac{1}{6}(0.02)(0.06)^2 = 12\ x\ 10^{-6} m^3$$

## Example 4




$$\sigma_A = \frac{P}{A} - \frac{M}{S}$$

$$M = 10kN\ x\ 25mm\ - 10kN\ x\ b$$

$$\boxed{M = 10kN\ \ (0.025m - b)}$$

## Example 4

$$\boxed{M = 10kN\ \ (0.025m - b)}$$

For b=0 ➜ $M = 0.25kNm = 250Nm$

$$\sigma_A = \frac{20kN}{1.2\ x\ 10^{-3}m} - \frac{250Nm}{12\ x\ 10^{-6}m^3} = 16.667MPa\ - 20.833\ MPa$$

$$\boxed{\sigma_A = -4.17MPa}$$

For b=15 mm ➜ $M = 10kN\ \ (0.025 - 0.015) = 100Nm$

$$\sigma_A = \frac{20kN}{1.2\ x\ 10^{-3}m} - \frac{100Nm}{12\ x\ 10^{-6}m^3} = 16.667MPa\ - 8.333\ MPa$$

$$\boxed{\sigma_A = 8.33MPa}$$

For b=25 mm ➜ $M = 0$

$$\sigma_A = \frac{20kN}{1.2\ x\ 10^{-3}m} \qquad \boxed{\sigma_A = 16.667MPa}$$