Conversion of Numbers in Standard Form to Normal Numbers

Example 1 Convert 1.98×10^5 into a normal number. $1{,}98 \times 100000$
198000

The power to which the 10 is raised (called the ***index*** incidentally) is 5,

so the ***decimal point*** has to be moved 5 places to the ***right***..

$1{\cdot}98 \times 10^5$

1 · 9 8 0 0 0

i.e. 198000

Example 2 Convert 3.26×10^{-4} into a normal number. 0,000326

This time the power to which the 10 is raised (***the index***) is -4, so the decimal point has to be moved 4 places to the ***left***.

$3{\cdot}26 \times 10^{-4}$

0 0 0 3 · 2 6

i.e. 0.000326

NB. Always put a zero in front of the decimal point in these cases.

Addition and Subtraction of Numbers in Standard Form

To achieve this without the aid of a calculator the standard form numbers should be converted into 'normal' numbers, added/subtracted and then converted back into standard form.

Example 1

Evaluate $1.25 \times 10^3 + 3.27 \times 10^4$, giving your answer in standard form.

Now $1.25 \times 10^3 = 1250$ $\Rightarrow 1{,}25 \times 1000 = 1250$

and $3.27 \times 10^4 = 32700$ $\Rightarrow 3{,}27 \times 10000 = 32700$

So $1.25 \times 10^3 + 3.27 \times 10^4$

$= 1250 + 32700$

$= 33950$

$= 3.3950 \times 10^4$

$$\begin{array}{r} 32700 \\ +1250 \\ \hline 33950 \end{array}$$

$3{,}3950 \times 10^4 = 3{,}3950000 \times 10^4$
$= 3{,}395 \times 10^4$

$1{\cdot}25 \times 10^3 + 3{\cdot}27 \times 10^4$

$\downarrow \qquad \downarrow$

$1250 + 32700 = 33950$

$\downarrow$

As standard form: $3{\cdot}395 \times 10^4$

Example 1

Evaluate $1.25 \times 10^3 + 3.27 \times 10^4$, giving your answer in standard form.

$$1.25\times10^3+3.27\times10^4=1.25\times10^3+3.27\times10^3\times10^1=1.25\times10^3+32.7\times10^3$$

$$=(1.25+32.7)\times10^3=33.95\times10^3=3.395\times10\times10^3$$

$$=3.395\times10^4$$

$$\begin{array}{r} 32.70 \\ +\ 1.25 \\ \hline 33.95 \end{array} = 3.395\times10$$

Example 2

Evaluate $3.56 \times 10^5 - 1.98 \times 10^4$, giving your answer in standard form.

$$3.56\times10^5-1.98\times10^4=35.6\times10^4-1.98\times10^4=33.62\times10^4=3.362\times10^5$$

$$\begin{array}{r} 35.60 \\ -1.98 \\ \hline 33.62 \end{array}$$

Example 3

Evaluate $4.27 \times 10^{-3} + 2.18 \times 10^{-4}$, giving your answer in standard form.

$$4.27\times10^{-3}+2.18\times10^{-4}=4.27\times10^{-3}+2.18\times10^{-4}\times10^{1}\times10^{-1}$$

$$10^{-4}\times10^{1}=10^{-3}$$

$$=4.27\times10^{-3}+0.218\times10^{-3}=4.488\times10^{-3}$$

$$\begin{array}{r} 4.270 \\ +0.218 \\ \hline 4.488 \end{array}$$

Example 4

Evaluate $5.68 \times 10^{-3} - 3.21 \times 10^{-4}$, giving your answer in standard form.

$$5.68\times10^{-3}-3.21\times10^{-4}=5.68\times10^{-3}-0.321\times10^{-3}=5.359\times10^{-3}$$

$$\begin{array}{r} 5.680 \\ -0.321 \\ \hline 5.359 \end{array}$$

Example 2

Evaluate $3.56 \times 10^5 - 1.98 \times 10^4$, giving your answer in standard form.

Now $3.56 \times 10^5 = 356000$

And $1.98 \times 10^4 = 19800$

So $3.56 \times 10^5 - 1.98 \times 10^4$

$= 356000 - 19800$

$= 336200$

$= 3.362 \times 10^5$

$3{\cdot}56 \times 10^5 - 1{\cdot}98 \times 10^4$

$\downarrow \qquad \downarrow$

$356000 - 19800 = 336200$

$\downarrow$

As standard form: $3{\cdot}362 \times 10^5$

Example 3

Evaluate $4.27 \times 10^{-3} + 2.18 \times 10^{-4}$, giving your answer in standard form.

Now $4.27 \times 10^{-3} = 0.004270$

and $2.18 \times 10^{-4} = 0.000218$

So $4.27 \times 10^{-3} + 2.18 \times 10^{-4}$

$= 0.004270 + 0.000218$

$= 0.004488$

$= 4.488 \times 10^{-3}$

$4{\cdot}27 \times 10^{-3} + 2{\cdot}18 \times 10^{-4}$

$\downarrow \qquad \downarrow$

$0.004270 + 0.000218 = 0.004488$

$\downarrow$

As standard form: $4{\cdot}488 \times 10^{-3}$

Example 4

Evaluate $5.68 \times 10^{-3} - 3.21 \times 10^{-4}$, giving your answer in standard form.

Now $5.68 \times 10^{-3} = 0.005680$

and $3.21 \times 10^{-4} = 0.000321$

So $5.68 \times 10^{-3} - 3.21 \times 10^{-4}$

$= 0.005680 - 0.000321$

$= 0.005359$

$= 5.359 \times 10^{-3}$

$5{\cdot}68 \times 10^{-3} - 3{\cdot}21 \times 10^{-4}$

$\downarrow \qquad \downarrow$

$0.005680 - 0.000321 = 0.005359$

$\downarrow$

As standard form: $5{\cdot}359 \times 10^{-3}$

Multiplication and Division of Numbers in Standard Form

Multiplication

Example 1

Evaluate $3.2 \times 10^5 \times 1.6 \times 10^3$, giving your answer in standard form.

To do this without a calculator, we must remember that when multiplying powers of the same base, we ***add the indices***.

Method: $3.2 \times 10^5 \times 1.6 \times 10^3 = 32 \times 10^4 \times 16 \times 10^2 = 32 \times 16 \times 10^6$

$= 512 \times 10^6$

$= 5.12 \times 10^2 \times 10^6$

$= 5.12 \times 10^8$

1) Multiply the two numbers together $3.2 \times 1.6 = 5.12$

2) Add the indices $10^5 \times 10^3 = 10^{5+3} = 10^8$

$$\begin{array}{r} 32 \\ \times 16 \\ \hline 192 \\ +320 \\ \hline 512 \end{array}$$

3) Multiply (1) and (2) to give 5.12×10^8

Example 2

Evaluate $2.8 \times 10^4 \times 6 \times 10^{-2}$ giving your answer in standard form.

$2.8 \times 10^2 \times 6 = 28 \times 6 \times 10 = 168 \times 10 = 1.68 \times 10^2 \times 10$

$= 1.68 \times 10^3$

As before:

1) Multiply the two numbers together $2.8 \times 6 = 16.8$

2) Add the indices $10^4 \times 10^{-2} = 10^2$

3) Multiply (1) by (2) to give 16.8×10^2

This answer is not in standard form because the definition says ***'Any number between 1 and 10 multiplied by a power of 10'.*** Remember?

So we have to modify: $16.8 \times 10^2 = (1.68 \times 10^1) \times 10^2 = 1.68 \times 10^3$

Example 3

Evaluate $3 \times 10^{-2} \times 2 \times 10^{-3}$ giving your answer in standard form.

$6 \times 10^{-2} \times 10^{-3} = 6 \times 10^{-5} = 6.0 \times 10^{-5}$

(1) Multiply the two numbers together $3 \times 2 = 6$

(2) Add the indices $10^{-2} \times 10^{-3} = 10^{-2-3} = 10^{-5}$

(3) Multiply (1) and (2) to give 6×10^{-5}

Division

Example 1 Evaluate $\frac{3.6\times10^4}{1.2\times10^2} = \frac{3.6\times10^4\times10^{-2}}{1.2} = \frac{3.6\times10^2}{1.2} = \frac{36\times10^2}{12}$

$= 3\times10^2$

To do this without a calculator, we must remember that when dividing powers of 10, we must ***subtract the index*** of the ***denominator*** (bottom) from that of the ***numerator*** (top).

$\frac{3.6}{1.2} = \frac{3.6}{1.2}\times\frac{10}{10} = \frac{36}{12}$

(1) Divide the numbers $3.6 \div 1.2 = 3$

(2) Subtract the indices $10^4 \div 10^2 = 10^{4-2} = 10^2$

(3) Multiply (1) and (2) to give 3×10^2

$5)\overline{27}$ = 5.4
25
20

Example 2 Evaluate $\frac{2.7\times10^5}{5\times10^{-3}} = \frac{2.7}{5}\times10^8 = \frac{27}{5}\times10^7$

As before: $\frac{10^5}{10^{-3}} = 10^5\times10^3 = 10^8$ $= 5.4\times10^7$

(1) Divide the numbers $2.7 \div 5 = 0.54$

(2) Subtract the indices $10^5 \div 10^{-3} = 10^{5--3} = 10^8$

(3) Multiply (1) and (2) to give 0.54×10^8

This is not standard form. The number must be between 1 and 10. You must take one more step, as before:

(4) $0.54 \times 10^8 = (5.4 \times 10^{-1}) \times 10^8 = 5.4 \times 10^7$

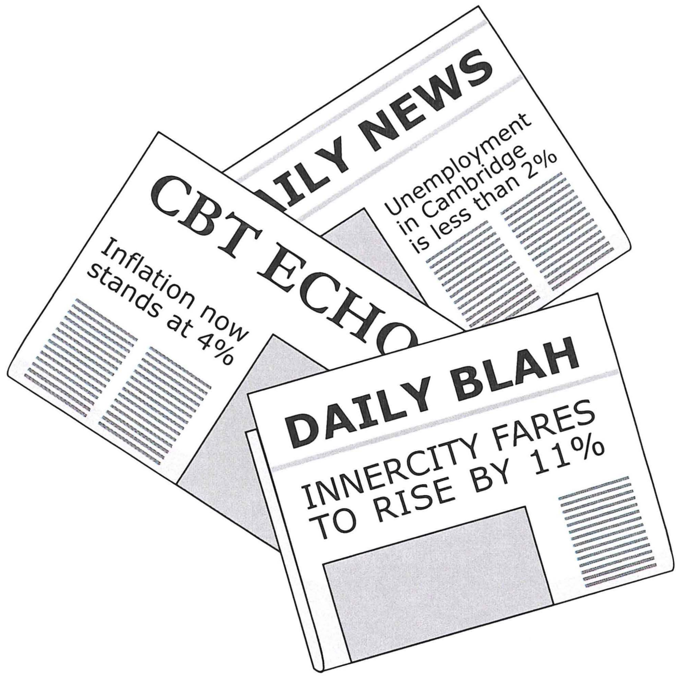
Percentages

We often hear on the TV or read statements in the newspapers such as:

'Inflation now stands at 4%'

'Intercity fares are to rise by 11%'

'Unemployment in Cambridge is less than 2%'.



These are all examples of percentages which are used in everyday life.

Percentages are also continually used in Banks and Building Societies to express rates of interest and in Schools and Colleges to publish examination results.

These examples are just the tip of the iceberg. There are many, many more uses in everyday life and they are very often misunderstood by a large number of people. So let us try to alleviate this problem.

The word percentage originates from the Latin ***'per centum'***, which literally means ***'out of one hundred'***. This is represented by the **%** sign.

A statement such as 25% means 25 out of 100 and this can be written as $\frac{25}{100}$.

A percentage is just a fraction expressed with a denominator (bottom) of 100. Also since 25/100 means 25 ÷ 100 which is equal to 0.25, then percentages are just a simple and convenient way of expressing fractions and decimals.

Conversion of Fractions to Percentages

To convert a fraction or a decimal to a percentage we simply ***multiply by 100*** and ***tag on*** a ***%*** sign.

Example 1

1) $\frac{3}{4} \times 100 = \frac{300}{4} = 75\%$

2) $\frac{9}{50} \times 100 = \frac{900}{50} = 18\%$

3) $\frac{3}{16} \times 100 = \frac{300}{16} = 18.75\%$

When decimal fractions are involved, move the decimal point two places to the ***right***.

3) 0.3271 x100 = 32.71 %

1) 0.40 x 100 = 0.40 = 40%

2) 0.060 x 100 = 0.060 = 6.0%

Percentages are not necessarily whole numbers.

In Banks, Building Societies and other Finance Houses we often see rates of interest such as 8.5% 7.75% and 5.625% for example, or if they were quoted as fractions as $8\frac{1}{2}\%$ $7\frac{3}{4}\%$ 8.15% .

Example 2

$$\frac{3}{16} \times 100 = \frac{300}{16} = 18.75\%$$

Example 3

0.0575 x 100 = 0.0575 = 5.75%

Conversion of Percentages to Fractions and Decimals

When we converted fractions and decimals to percentages we multiplied by 100 and tagged on a % sign. Now we employ the reverse procedure, we ***drop*** the % sign and ***divide by 100***.

Example 1

$$50\% = \frac{50}{100} = \frac{1}{2}$$

(by dividing top and bottom by 50)

Example 2

$$65\% = \frac{65}{100} = \frac{13}{20}$$

(by dividing top & bottom by 5)

Example 3

92.5% = 92.5 ÷ 100 = 0.925 (moving the decimal point 2 places to the left)

$\frac{92.5}{100} = \frac{925}{1000} = \frac{185}{200} = \frac{37}{40}$ in fraction.

Example 4

36.75% = 36.75 ÷ 100 = 0.3675

$36\% = 0.36 = \frac{36}{100} = \frac{9}{25}$

Since fractions and decimals are interchangeable, there is a convenient method of converting a percentage into both. Look at the examples below:

Fraction Route: $65\% \rightarrow \frac{65}{100} = \frac{13}{20}$

Decimal Route: $65\% \rightarrow 65.0 \div 100 = 0.65$

So $\frac{13}{20} = 0.65$

$\frac{1}{2} = 0.5$

Example 5

Fraction Route: $\frac{37\frac{1}{2}}{100} \frac{\times 2}{\times 2} = \frac{75}{200} = \frac{3}{8}$

$37\frac{1}{2}\%$ $= 37.5$

$37 + \frac{1}{2} = 37.5$

Decimal Route: $37.5 \div 100 = 0.375$

So $\frac{3}{8} = 0.375$

Example 6

$63 + \frac{2}{3} = \frac{63 \times 3 + 2}{3} = \frac{191}{3} = 63.666666\ldots = 63.67\%$

$= 0.6367$

Fraction Route: $\frac{63\frac{2}{3}}{100} \frac{\times 3}{\times 3} = \frac{191}{300}$

$63\frac{2}{3}\%$

Decimal Route: $63.67 \div 100 = 0.6367$

So $\frac{191}{300} = 0.6367$

Calculating a Percentage of a Quantity

To do this first change the percentage to a fraction or decimal and then multiply the quantity by this.

Example 1

$$10\% \text{ of } 70 = \frac{10}{100} \times 70 = \frac{700}{100} = 7$$

Example 2

$$12\% \text{ of } £84 = \frac{12}{100} \times 84 = \frac{1008}{100} = £10.08$$

Example 3

$$15\% \text{ of } 120\text{kg} = \frac{15}{100} \times 120 = 18\text{kg}$$

$\frac{15}{10} \times 12 = \frac{3}{2} \times 12$
$= 18$

Increasing An Amount By a Given Percentage

If the fare from Heathrow to New York is £180 one-way, how much would it cost after a rise of 16%?

There are two methods that can be used: -

Method A

Find 16% of £180 and then add this to £180.

$$16\% \text{ of } £180 = \frac{16}{100} \times 180 = £28.80$$

Therefore the new fare will be: £180 + £28.80 = £208.80

Method B

Let the original fare of £180 be equivalent to 100%, so increasing the fare by 16% is equivalent of finding 116% of the original price.

$$\text{Now } 116\% = \frac{116}{100} = 1.16$$

So multiply the original fare of £180 by 1.16 to give £208.80.

Example: A worker whose salary is 6000 TL has a salary increment by 15% so find his updated salary in two ways.

1. 15% of 6000

$\frac{15}{100} \times 6000 = 900$

$6000 + 900 = 6900$ TL

2. 115% of 6000

$\frac{115}{100} \times 6000 = 115 \times 60 = 6900$ TL

Decreasing an Amount by a Given Percentage

A shop window has a notice that reads: 'SALE 30% OFF ALL ITEMS'

If the marked price of an item is £50, we could again use two methods to find the sale price.

Method A

Find 30% of the original price and then subtract this from £50.

$$30\% \text{ of } £50 = \frac{30}{100} \times 50 = £15$$

Therefore the new price will be: £50 - £15 = £35.00

Method B

Let the original price of £50 be equivalent to 100%, so decreasing the price by 30% is equivalent to finding 70% of the original price.

$$\text{Now, } 70\% = \frac{70}{100} = 0.7$$

Multiply the original price of £50 by 0.7 to give £35

Expressing a Quantity as a Percentage of Another

In a College math's examination, the total marks available were 60. If a particular student scored 45 then his/her result would be 45/60. As you are aware most examination results are quoted as a percentage. So the question becomes: "Express 45 as a percentage of 60"

The first step is to write the mark as a fraction: $\frac{45}{60}$

This fraction is then multiplied by 100 to change it into a percentage.

$$\frac{45}{60} \times 100 = 75\%$$

Therefore the student has scored 75% in the exam.

Example 1

Find 25 as a percentage of 150 giving your answer correct to two decimal places.

First express 25 as a fraction of 150: $\frac{25}{150}$

Then multiply this fraction by 100 to express it as a percentage

$$\frac{25}{150} \times 100 = 16.666 = 16.67 \text{ (2d.p)}$$

Example 2

Find 60 as a percentage of 180 giving your answer correct to three significant figures.

Again, first express 60 as a fraction of 180: $\frac{60}{180}$

Multiply this fraction by 100 to express it as a percentage.

$$\frac{1}{3} = \frac{60/60}{180/60} \times 100 = 33.333 = 33.3 \text{ (3s.f)} \qquad \frac{100}{3} = 33.3$$

Ratio and Proportion

Johns earns £120 per week in the local supermarket. His sister Ann earns £30 per week as a part-time waitress. So John earns four times as much as his sister.

We can say this in another way – 'the ***ratio*** of John's earnings to Ann's is 4 :1'.

Ratios can be written in general as ***First quantity: Second quantity***.

i.e. with a **colon** (:) between the amounts or they can be expressed as a fraction like this:

$$\frac{\text{First Quantity}}{\text{Second Quantity}}$$

The ratios 5 : 7 and 11 : 4 are said to be in their ***simplest forms***, because there is no whole number (other than 1) that will divide exactly into both sides.

However, the ratio 12 : 4 is ***not*** in its ***simplest form*** because 4 will divide into both sides to give 3 : 1. The ratios 12 : 4 and 3 : 1 are said to be ***equivalent***.

Example 1

The ratio 35 : 42 = 5 : 6 (by dividing 7 into both sides) or as a fraction: $\frac{5}{6}$

Example 2

The ratio 20 : 80 = 1 : 4 (by dividing 20 into both sides) or as a fraction $= \frac{1}{4}$

When we convert from the general form x : y into a fraction then the left hand number becomes the top of the fraction and the right hand number becomes the bottom of the fraction.

However before any of this is done, the quantities involved ***must be in the same units.***

In the following examples write each pair of the quantities as ratios in their ***simplest terms***.

Example 1 60m, 100m

You can see the units are the same so:

60m : 100m = 60 : 100 = 3 : 5 (dividing both sides by 20)

Example 2 £3, 20p

Here we have to change the pounds (£) into pence:

£3 : 20p = 300p : 20p = 300 : 20 = 15 : 1 (dividing both sides by 20)

Example 3 800g, 2kg

Now we have to convert the kilograms into grams.

800g : 2kg = 800g : 2000g = 800 : 2000 = 2 : 5 (dividing both sides by 400)

Division in a Given Ratio

If three people invest different amounts of money in a business then they cannot realistically expect to receive equal shares of any profit made. It is only fair that they receive shares of the profit in proportion to their individual investments.

5+6+3=14

Example 1

20:24:12 ⇒ 5:6:3

Alan, Brian and Charles invest £20000, £24000 and £12000 in a business venture. At the end of the first year of trading the profits from the business amount to £28700.

2050
14)28700

Alan 5x2050=£10250

If they each receive a share of the profits in proportion to their original investment, how much does each receive?

Brian 6x2050=£12300

The original investments are in the ratio 20000 : 24000 : 12000

Charles 3x2050=£6150

In its simplest form this ratio is 5 : 6 : 3 (dividing each by 4000)

Now the total number of parts = 5 + 6 + 3 = 14

$$\text{one part of the profit is} = \frac{£28,700}{14} = £2050$$

So Alan receives 5 parts $= £2050 \times 5 = £10250$

Brian receives 6 parts $= £2050 \times 6 = £12300$

And Charles receives 3 parts $= £2050 \times 3 = £6150$

We should always check to see if our calculation is correct by adding together the individual amounts to see if they equal the amount that was shared.

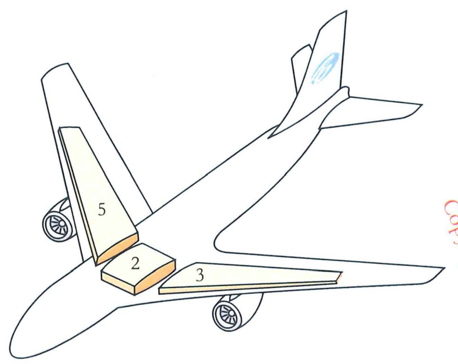
$£10250 + £12300 + £6150 = £28700$ (yes the calculation is correct!)

Example 2

An aircraft carries fuel in three tanks in the ratio 2 : 3 : 5. If the total amount carried is 2500 gallons, how much is carried in each tank.

Total number of parts = 2 + 3 + 5 = 10

Amount to be shared = 2500 gallons



Therefore one part $= \dfrac{2500}{10} = 250$ gallons

So two parts $= 2 \times 250 = 500$ gallons

three parts $= 3 \times 250 = 750$ gallons

And five parts $= 5 \times 250 = 1250$ gallons

Check: $500 + 750 + 1250 = 2500$. (Correct).

Example 3

A sum of money is shared three ways in the ratio 2 : 3 : 4. If the largest share is £160, what was the original sum?

The largest ratio part is 4 and this is equal to £160.

Therefore one part must be equal to $\dfrac{£160}{4}$ which is £40

The total number of parts is $(2 + 3 + 4) = 9$

Therefore the original sum $= £40 \times 9 = £360$

So the general method is:

- Add the ratio parts together to give the total number of parts.
- Divide the amount to be shared by this total number of parts to find the value of one part.
- Multiply this one part amount by each of the parts making up the ratio to find each constituent part.

Direct Proportion (Direct Variation)

Two quantities are said to be directly proportional if they increase or decrease at the same rate. The symbol α is read as '***proportional to***'.

For example, if one litre of fuel costs 70p then we would expect to pay twice this amount (140p) for two litres. Again, if you double your speed you would expect to travel twice as far in the same period of time.

If Y is proportional to X (written Y α X) then Y = kX, where k is some positive number called the ***proportionality constant***.

If $Y = kX$,

then $k = \dfrac{Y}{X}$

(dividing both sides by X)

So if we know one pair of values for Y and X then we can find the value of k.

Direct variation means that if X is doubled then Y is doubled and if X is halved then Y is halved. The problems that we shall encounter involve finding the constant of proportionality from information given in particular problems.

Example 1

In the following table Y is proportional to X. Find the value of k, the proportionality constant and complete the table.

$k = \frac{Y}{X} = \frac{10.5}{7} = 1.5$ $\frac{10.5}{7}$ $Y = kX$

X	2	5	7	10	20
Y	1.5x2 3	7.5	10·5	15	30

Since Y α X

Then Y = k.X ———— (1)

i.e. $k = \frac{Y}{X}$

when X = 7

Y = 10.5

so $k = \frac{10 \cdot 5}{7}$

k = 1·5

The exact relationship between X and Y is therefore:

Y = 1.5X (from (1) above)

We can now use this equation to find the values of Y when X =2, 5, 10 and 20. The completed table is:

X	2	5	7	10	20	40	50
Y	**3**	**7.5**	10.5	**15**	**30**	60	75

Example 2

If the volume of fuel (V) flowing from a pump is 60 litres in 4 minutes, find the volume of fuel that would flow in 7 minutes.

Let the volume = V litres and let the time = T mins.

so we can write V α T

i.e. V = k.T

and $k = \frac{V}{T}$

$\frac{60}{4} = 15$

k = 15

V = 15 x 7 = 105 litres

Substituting these values in the equation we obtain:

$$k = \frac{60}{4}$$
$$= 15$$

The exact relationship between V and T becomes:

$$V = 15T$$

so when $T = 7$

$$V = 15 \times 7$$
$$V = 105 \text{ litres}$$

The same problem can be approached by the **unitary method**.

In 4 minutes the amount of fuel is	60 litres
So in 1 minute the amount would be	$\frac{60}{4}$ = 15 litres
Therefore in 7 minutes the amount is	15 x 7 = 105 litres

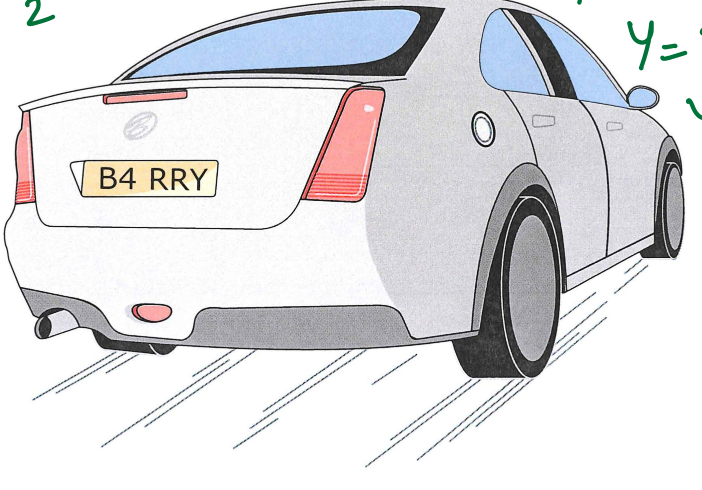
This method is called the unitary method because we find the volume flowing in 1 minute first.

Example 3

A car travels 75 miles on 2 gallons of fuel. How far would it travel on 7 gallons?

$\frac{75}{2} = 37.5 = k$

$y = kx$
$y = 37.5 \times 7$
$y = 262.5$ miles



In this case let M = number of miles and N = number of gallons.

$$M \propto N$$

$$M = k.N$$

$$\text{Therefore } k = \frac{M}{N}$$

$$\text{when } M = 75,\ N = 2$$

$$k = \frac{75}{2}$$

$$k = 37.5$$

The exact relationship between M and N becomes:

$$M = 37.5N$$

$$\text{when } N = 7$$

$$M = 37.5 \times 7$$

$$M = 262.5$$

The same problem via the unitary method is:

On 2 gallons the car travels	75 miles
So, on 1 gallon it would travel	$\frac{75}{2} = 37.5$
Therefore on 7 gallons it would travel	$37.5 \times 7 = 262.5$ miles

Indirect Proportion (Inverse Variation)

If an increase in one quantity gives rise to a decrease in another quantity in the same ratio or a decrease in one quantity gives rise to an increase in another quantity in the same ratio, then the two quantities are said to be in ***indirect proportion*** or ***inverse variation***.

If you travel a distance of 300 miles at 75mph, the time taken would be 4 hours. If you were able to travel the same distance at 150mph (some motor car!) the time taken would be 2 hours. So as you ***double*** the speed, you ***halve*** the time taken.

In both these cases the speed and the time taken are inversely or indirectly proportional to each other.

If Y is inversely proportional to X i.e. $Y \alpha \frac{1}{X}$

then $Y = \frac{k.1}{X}$

Multiplying both sides by X, we obtain $k = X.Y$

Once again if we know one pair of values for X and Y, we can find the actual value of k.

Example 1

If 4 men take 15 hours to complete a job of work, how long will 5 men take?

In this case the time taken decreases as the number of men increases, so we can say that the time taken is inversely proportional to the number of men.

Let T = the time taken, and M = the number of men.

So $T \alpha \frac{1}{M}$

and $T = \frac{k.1}{M} = \frac{k}{M}$

when M = 4 and T = 15

$15 = \frac{k}{4}$

k = 60 (multiplying both sides by 4)

The exact relationship between T and M becomes:

$T = \frac{60}{M}$

when M = 5

$T = \frac{60}{5}$

T = 12 hours

Once again if we use the unitary method the:

4 men take	15 hours	
1 man would take	15 x 4	(multiply, one takes longer)
5 men will take	$\frac{15 \times 4}{5}$	(divide, 5 men take less time)
	= 12 hours	

Try the same calculation for 6, 12 and 15 men. You will find that the answers are as shown in the following table.

Number of Men	4	5	6	12	15	20	30
Time (in hours)	15	12	10	5	4	3	2
Men x Time	60	60	60	60	60	60	60

As you can see, when we multiply the number of men and the time taken together the answer is always the same – 60. We can state therefore that one number varies inversely to another when the product of a series of the two numbers gives the same answer – a ***constant***.

How many men would be required to complete the work in 2 hours?

$$2 = 60 \times \frac{1}{M}$$

$$2 = \frac{60}{M}$$

Multiplying both sides by M

$$2M = \frac{60 \times M}{M}$$

$$2M = 60$$

Dividing both sides by 2

$$M = 30$$

So 30 men would be required.

Example 2

The volume V of a given mass of gas varies inversely as the pressure P. When $V = 2m^3$, $P = 500Nm^2$ (a) Find the volume when the pressure is $400N/m^2$ (b) Find the pressure when the volume is $5m^3$.

$k = 2 \times 500 = 1000$

$\frac{1000}{400} = 2.5$ a)

$\frac{1000}{5} = 200 N_{m^2}$ b)

Now $V \alpha \frac{1}{P}$

So $V = k \cdot \frac{1}{P}$

Now when $V = 2m^3$

then $P = 500N/m^2$

Substituting these values into the equation we obtain

$$2 = k \times \frac{1}{500}$$

Multiplying both sides by 500

$$2 \times 500 = \frac{k}{500} \times 500$$

$$k = 1000$$

The equation becomes

$$V = 1000 \times \frac{1}{P}$$

(a) Determine the volume when $P = 400N/m^2$

$$V = 1000 \times \frac{1}{400}$$

$$V = 2.5m^3$$

(b) Determine the pressure when $V = 5m^3$

$$5 = 1000 \times \frac{1}{P}$$

$$5 = \frac{1000}{P}$$

Multiplying both sides by P

$$5P = 1000$$

Dividing both sides by 5

$$P = 200N/m^2$$

Convert Basic Units between Imperial and SI Units

Since 1971 when the British monetary system was decimalised, more and more units have changed from the Imperial system to the Metric system, but not all. There is still a necessity therefore to have a means of changing from one system to another so that comparisons can be made.

The most common equivalents are:

Imperial	Metric
1 Gallon	4.55 Litres
1 Inch	2.54 centimetres
1 Foot	30.48 centimetres
1 Yard	0.914 metres
1 Mile	1.609 Kilometres
1.76 Pints	1 Litre
2.205 lbs	1 Kilogramme
0.6215 miles	1 Kilometre

If a rough conversion only is requested then the above becomes:

Imperial	Metric
1 Gallon	4.6 Litres
1 Inch	2.5 centimetres
1 Foot	30 centimetres
1 Yard	1 metres
1 Mile	1.6 Kilometres
1.75 Pints	1 Litre
2.2 lbs	1 Kilogramme
0.62 miles	1 Kilometre

Example 1

Given that 1kg = 2.2lb, change 40 kg into pounds.

$$\text{If } 1 \text{ kg} = 2.2 \text{ lb}$$

$$\text{Then } 40 \text{ kg} = 40 \times 2.2 \text{ lb}$$

$$= 88 \text{ lb}$$

Example 2

Find the number of kilometres in 50 miles.

Since 1 mile = 1.6 km (approx.)

Then 50 miles = 50 × 1.6 km

= 80 km

Example 3

Convert 8 litres into pints.

The 1 litre = 1.75 pints (approx)

8 litres = 8 × 1.75 pints

= 14 pints

Example 4

Convert 50 litres into gallons.

Since 4.55 litres = 1 gallon

Then 1 litre = $\frac{1}{4.55}$ gallon

and 50 litres = $\frac{1}{4.55}$ x 50

= 10.99 gallons

Roughly 11 gallons

Example 5

Convert 9lb into kilograms.

Since 2.2 lb = 1 kilogram

Then 1 lb = $\frac{1}{2.2}$ kg

and 9 lb = $\frac{1}{2.2}$ x 9 kg

= 4.1 kg (1dp)

Example 6

Convert 60 cm to inches

Since 2.54 cm = 1 Inch

Then 1 cm = $\frac{1}{2.54}$ inches

and 60 cm = $\frac{1}{2.54}$ x 60 inches

= 23.6 inches

Example 7

Convert 120 km into miles.

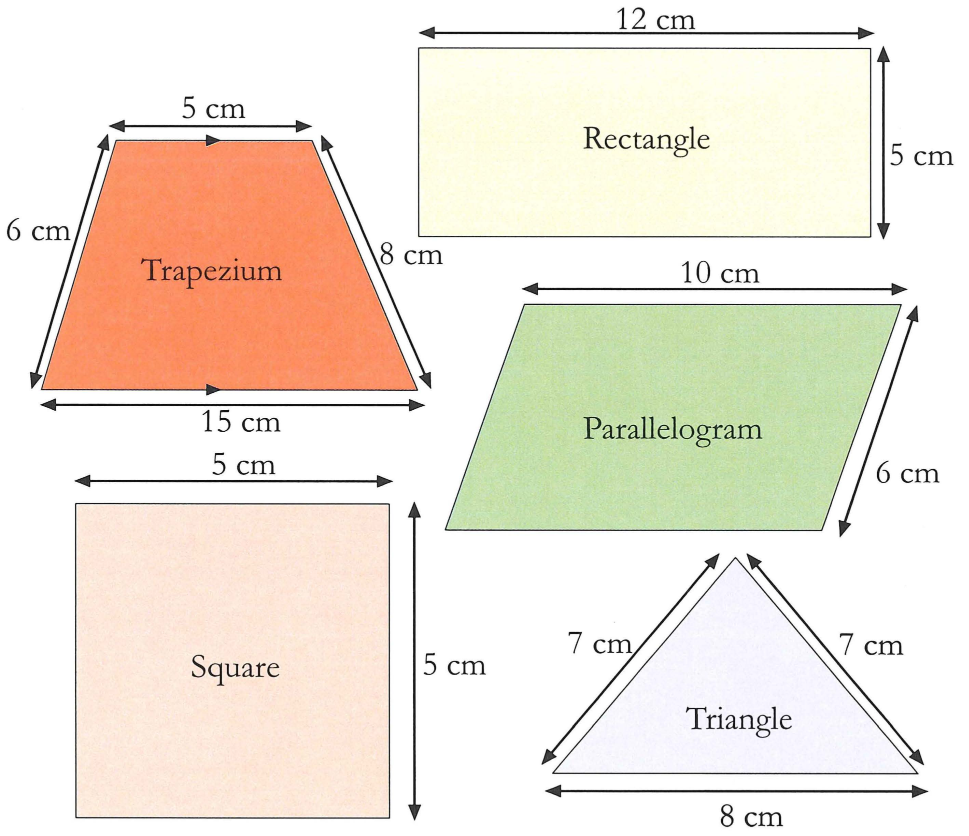
Since 1 km = 0.625 miles

Then 120 km = 0.625 x 120 miles

= 75 miles

Perimeters

A plane figure is a two-dimensional shape bounded by straight lines.



The perimeter of such shapes is the distance around the outside of the figure (i.e. the total length of its sides).

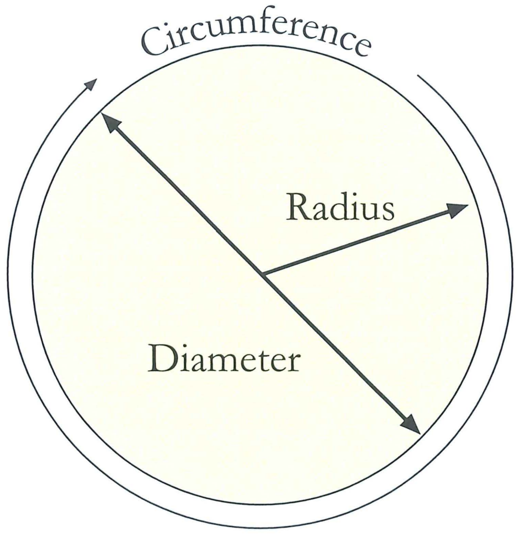
The units of perimeter are linear units - millimetres (mm), centimetres (cm) or metres (m).

So, from the diagrams:

a)	Perimeter of square	= (5 + 5 + 5 + 5) cm	= 20cm
b)	Perimeter of rectangle	= (12 + 5 + 12 +5) cm	= 34cm
c)	Perimeter of parallelogram	= (10 + 6 + 10 + 6) cm	= 32cm
d)	Perimeter of trapezium	= (5 + 8 + 15 + 6) cm	= 34cm
e)	Perimeter of triangle	= (7 + 7 + 8) cm	= 22cm

Circumference

This is the distance around the outside of a circle.



Circumference (C) $= 2 \times \pi \times r$ $\left[\pi = \frac{22}{7} \text{ or } 3.142\right]$

i.e. C $= 2\pi r$ or πd (since $2r = d$)

Example 1

Find the circumference of a circle that has a radius of 14 cm.

$$C = 2 \times \pi \times r$$
$$= 2 \times \frac{22}{7} \times 14$$
$$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{14}_2$$
$$= 2 \times 22 \times 2$$
$$= 88 \text{ cm}$$

Example 2

Find the circumference of a circle whose diameter is 7 cm.

$$C = \pi \times d$$
$$= \frac{22}{\cancel{7}_1} \times \cancel{7}_1$$
$$= 22 \text{ cm}$$

Example 3

Find the circumference of a circle that has a radius of 3.8cm.

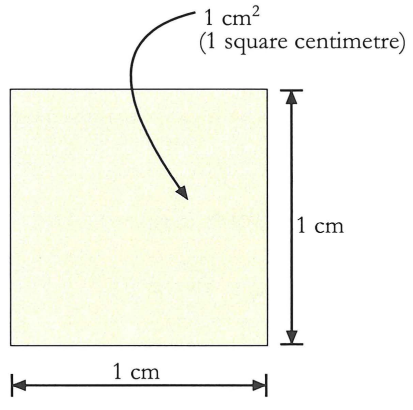
$C = 2 \times \pi \times r$

$C = 2 \times 3.142 \times 3.8$

$C = 23.88\text{cm}$ (2 dp)

Area

1 square centimetre (cm^2) is the **area** contained within a square that has a side of 1 centimetre (cm).



Similarly 1 square metre (m^2) is the area contained within a square that has a side of 1 metre (m) and similarly 1 square millimetre (mm^2) is the area contained within a square of side 1 millimetre (mm).

So, the area of a plane figure is found by measuring the number of mm^2 or cm^2 or m^2 within it.

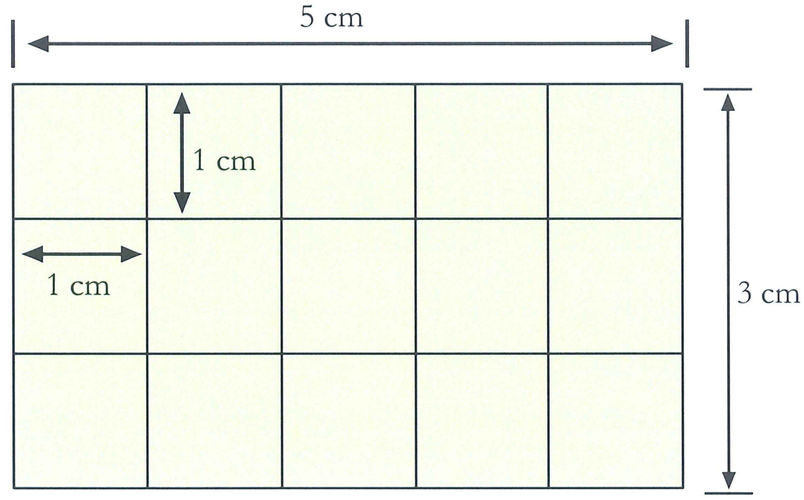
Now 10 mm = 1 cm

100 cm = 1 metre

(from which we see 1000mm = 1 metre)

The Rectangle

Consider a rectangle that has a length of 5cms and a width (breadth) of 3cm.



Now each small square within the rectangle has an area of $1cm^2$. Since there are 15 of these, the area within this rectangle must be $15 \times 1cm^2 = 15cm^2$

The same result could have been achieved by multiplying $5cm \times 3cm = 15cm^2$

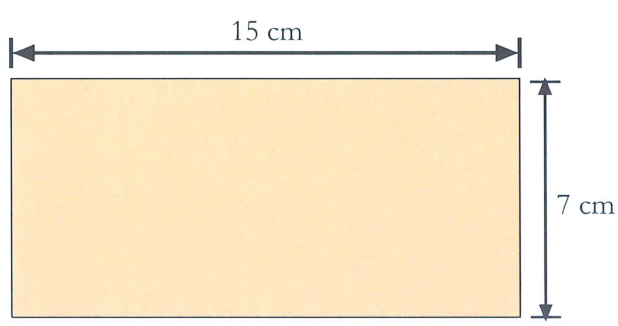
So

Area of a rectangle = length x width (breadth)

Note: The units of length and width must be the same. They must both be in, mms, cm, or m.

Example 1

Find the area of the following rectangle.

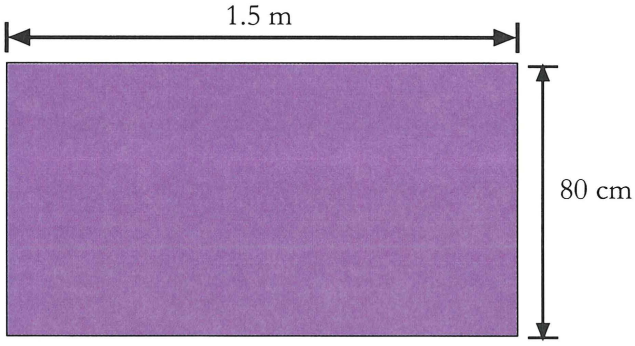


Area $= \text{length} \times \text{width}$

$= 15cm \times 7cm$

$= 105\ cm^2$

Example 2 Find the area of the rectangle.

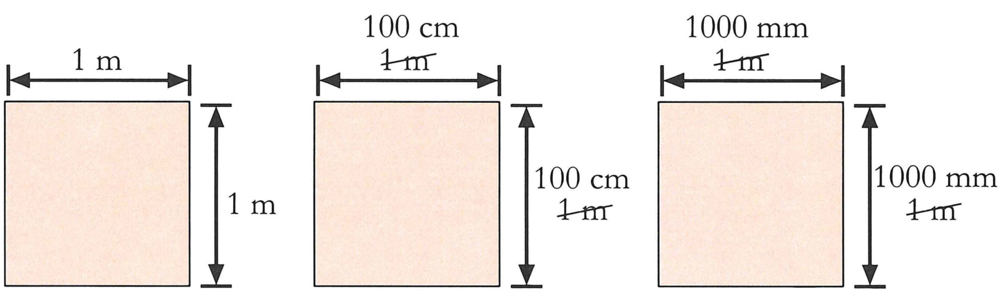


Since the units are different, we must change the metres to centimetres or the centimetres to metres before proceeding.

i.e. Area = $1.5\text{m} \times 0.8\text{m}$ or Area = $150\text{cm} \times 80\text{cm}$

$= 1.2\text{m}^2$ $= 12000\text{cm}^2$

Students still make the mistake of assuming that since there are 100cm in a metre there must be 100cm^2 in 1m^2. This is incorrect. To explain this, consider three identical squares of side 1 metre.



(a) (b) (c)

As shown, square (a) has its sides in metres, but square (b) could have its sides in centimetres (since 1m = 100cm) and square (c) could have its sides in millimetres (since 1m=1000mm).

When we find the area of each:-

a) $1\text{m} \times 1\text{m}$ b) $100\text{cm} \times 100\text{cm}$ c) $1000\text{mm} \times 1000\text{mm}$

$= 1\text{m}^2$ $=10000\text{cm}^2$ $=1000000\text{mm}^2$

These must be equal so:

To change 1m^2 to cm^2 we multiply by 10,000 (100×100)

To change 1m^2 to mm^2 we multiply by 1000,000 (1000×1000)

To change 1cm^2 to mm^2 we multiply by 100. (10×10)

As you can see these conversion factors are the squares of the linear ones (see brackets).

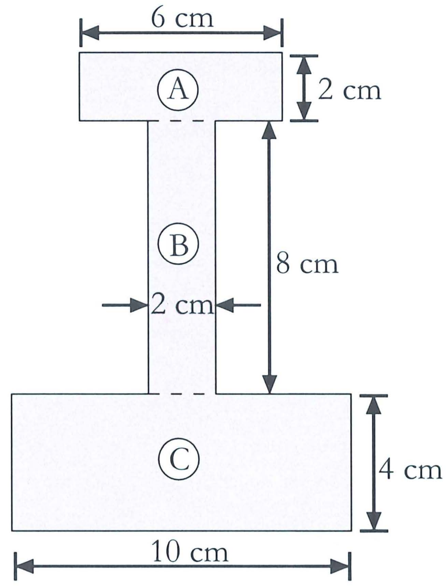
To go in the opposite direction, we divide:

To change cm^2 to m^2 we divide by 10,000

To change mm^2 to m^2 we divide by 1000,000

To change mm^2 to cm^2 we divide by 100.

Not all shapes are straightforward rectangles, as shown in the following example but they can be split up into rectangles. The area of the shape is then found by finding the area of each rectangle and then adding them together.



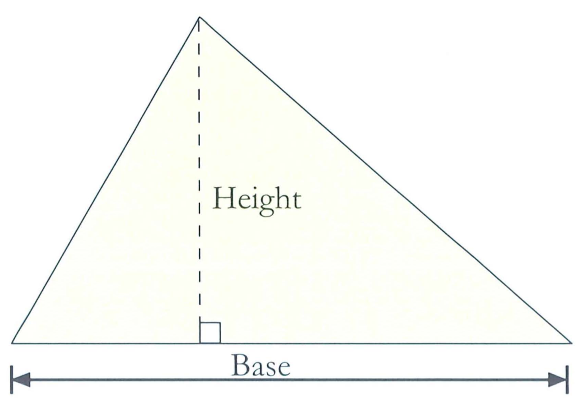
Area of A = 6cm × 2cm = $12cm^2$

Area of B = 8cm × 2cm = $16cm^2$

Area of C = 10cm × 4xm = $40cm^2$

Area of shape = $12cm^2$ + $16cm^2$ + $40cm^2$ = $68cm^2$

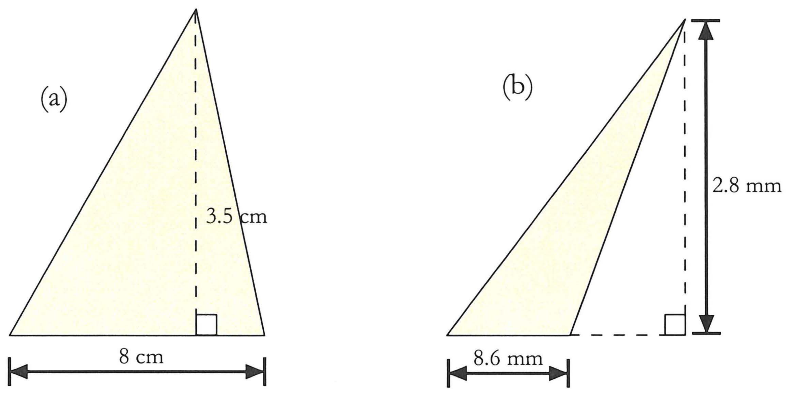
The Triangle



Area of a triangle = one half the base x perpendicular height

Example 1

Find the areas of the following triangles.

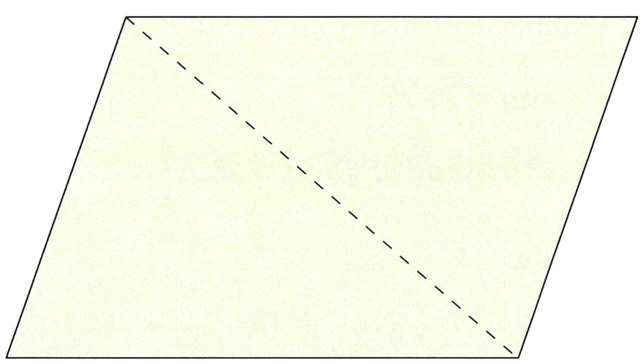


a) Area $= \frac{1}{2} \times 8 \times 3.5$
$= 14\text{cm}^2$

b) Area $= \frac{1}{2} \times 8.6 \times 2.8$
$= 12.04\text{mm}^2$

The Parallelogram

This is a four sided figure which has both pairs of its opposite sides equal and parallel.



Because of this, a parallelogram can be divided into two identical triangles by drawing in a diagonal (dotted line).

Area of a parallelogram = 2 × area of one of the triangles.

If area of a triangle = one half the base × perpendicular height

Then area of parallelogram = 2 × one half the base × perpendicular height

∴ Area of a parallelogram = base x perpendicular height