Convert Basic Units between Imperial and SI Units

Since 1971 when the British monetary system was decimalised, more and more units have changed from the Imperial system to the Metric system, but not all. There is still a necessity therefore to have a means of changing from one system to another so that comparisons can be made.

The most common equivalents are:

Imperial	Metric
1 Gallon	4.55 Litres
1 Inch	2.54 centimetres
1 Foot	30.48 centimetres
1 Yard	0.914 metres
1 Mile	1.609 Kilometres
1.76 Pints	1 Litre
2.205 lbs pound	1 Kilogramme
0.6215 miles	1 Kilometre

If a rough conversion only is requested then the above becomes:

Imperial	Metric
1 Gallon	4.6 Litres
1 Inch	2.5 centimetres
1 Foot	30 centimetres
1 Yard	1 metres
1 Mile	1.6 Kilometres
1.75 Pints	1 Litre
2.2 lbs	1 Kilogramme
0.62 miles	1 Kilometre

Example 1

Given that 1kg = 2.2lb, change 40 kg into pounds.

$$\text{If } 1 \text{ kg} = 2.2 \text{ lb}$$
$$\text{Then } 40 \text{ kg} = 40 \times 2.2 \text{ lb}$$
$$= 88 \text{ lb}$$

Example 2

Find the number of kilometres in 50 miles.

$$\begin{aligned}\text{Since } 1 \text{ mile} &= 1.6 \text{ km (approx.)}\\ \text{Then } 50 \text{ miles} &= 50 \times 1.6 \text{ km}\\ &= 80 \text{ km}\end{aligned}$$

Example 3

Convert 8 litres into pints.

$$\begin{aligned}\text{The } 1 \text{ litre} &= 1.75 \text{ pints (approx)}\\ 8 \text{ litres} &= 8 \times 1.75 \text{ pints}\\ &= 14 \text{ pints}\end{aligned}$$

Example 4

Convert 50 litres into gallons.

$$\begin{aligned}\text{Since } 4.55 \text{ litres} &= 1 \text{ gallon}\\ \text{Then } 1 \text{ litre} &= \frac{1}{4.55} \text{ gallon}\\ \text{and } 50 \text{ litres} &= \frac{1}{4.55} \times 50\\ &= 10.99 \text{ gallons}\end{aligned}$$

Roughly 11 gallons

Example 5

Convert 9lb into kilograms.

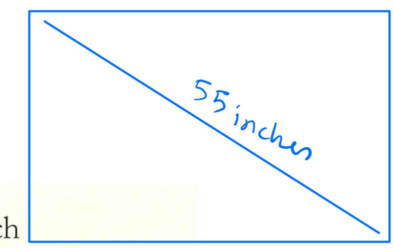
Since 2.2 lb = 1 kilogram

$$\text{Then } 1 \text{ lb} = \frac{1}{2.2} \text{ kg}$$

$$\text{and } 9 \text{ lb} = \frac{1}{2.2} \times 9 \text{ kg}$$

$$= 4.1 \text{ kg (1dp)}$$

Example 6



55×2.54
$\cong 140$ cm

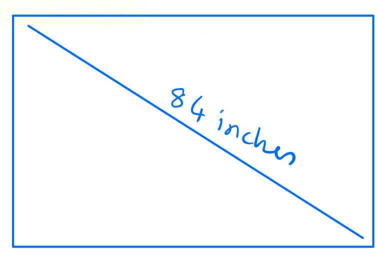
Convert 60 cm to inches

Since 2.54 cm = 1 Inch

$$\text{Then } 1 \text{ cm} = \frac{1}{2.54} \text{ inches}$$

$$\text{and } 60 \text{ cm} = \frac{1}{2.54} \times 60 \text{ inches}$$

$$= 23.6 \text{ inches}$$



84×2.54
$\cong 213$ cm

Example 7

Convert 120 km into miles.

Since 1 km = 0.625 miles

Then 120 km = 0.625 x 120 miles

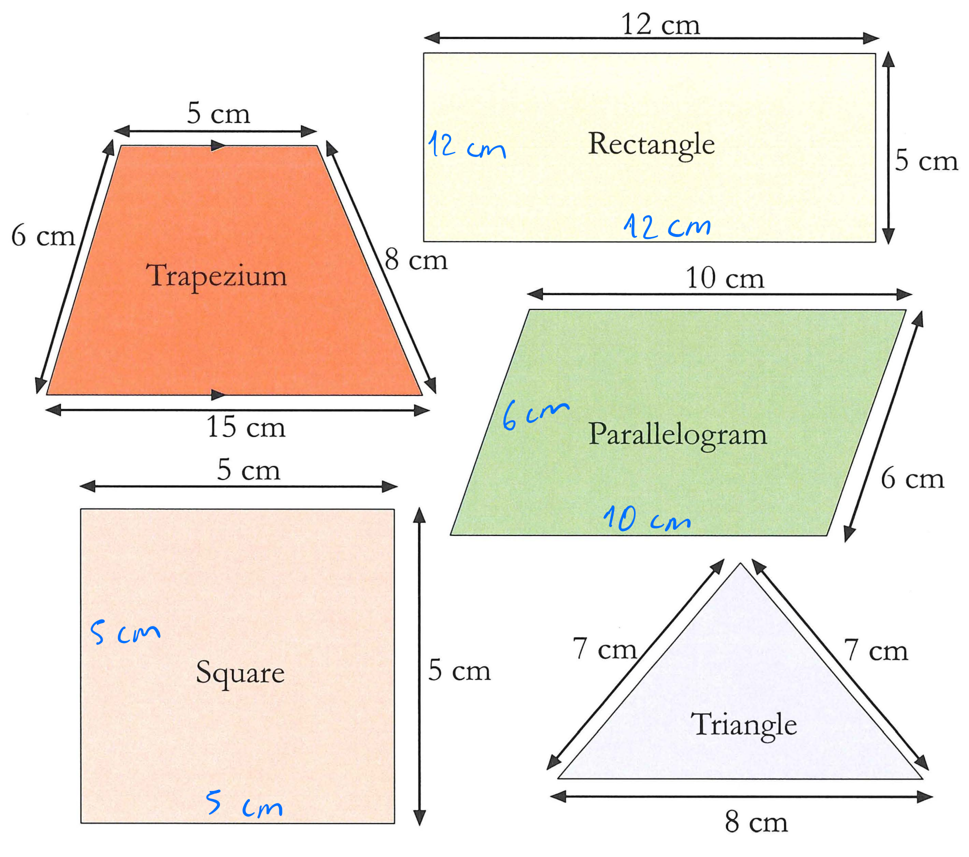
= 75 miles

$\frac{1}{1.605} \cong 0.625$ miles

$120 \text{ km} = \frac{120}{1.609} = 74.766 \cong 75$ miles

Perimeters

A plane figure is a two-dimensional shape bounded by straight lines.



The perimeter of such shapes is the distance around the outside of the figure (i.e. the total length of its sides).

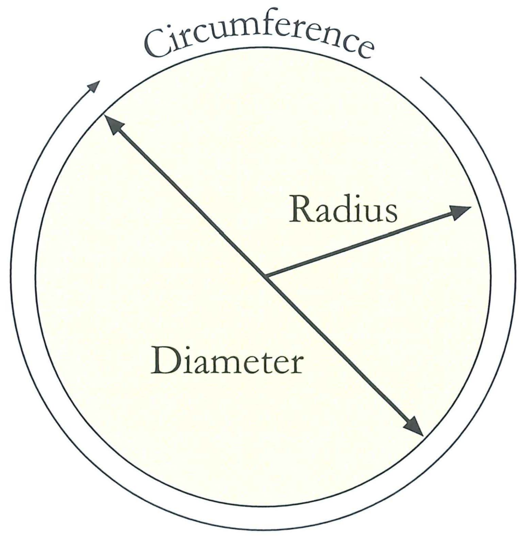
The units of perimeter are linear units - millimetres (mm), centimetres (cm) or metres (m).

So, from the diagrams:

a)	Perimeter of square	= (5 + 5 + 5 + 5) cm	= 20cm
b)	Perimeter of rectangle	= (12 + 5 + 12 +5) cm	= 34cm
c)	Perimeter of parallelogram	= (10 + 6 + 10 + 6) cm	= 32cm
d)	Perimeter of trapezium	= (5 + 8 + 15 + 6) cm	= 34cm
e)	Perimeter of triangle	= (7 + 7 + 8) cm	= 22cm

Circumference

This is the distance around the outside of a circle.



$$\text{Circumference (C)} = 2 \times \pi \times r \quad \left[\pi = \frac{22}{7} \text{ or } 3.142\right]$$

$$\text{i.e.} \quad C = 2\pi r \text{ or } \pi d \text{ (since } 2r = d)$$

Example 1

Find the circumference of a circle that has a radius of 14 cm.

$$\begin{aligned} C &= 2 \times \pi \times r \\ &= 2 \times \frac{22}{7} \times 14 \\ &= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{14}_2 \\ &= 2 \times 22 \times 2 \\ &= 88 \text{ cm} \end{aligned}$$

Example 2

Find the circumference of a circle whose diameter is 7 cm.

Circumference $= \pi d = \frac{22}{7} \times 7 = 22$ cm

$$C = \pi \times d$$
$$= \frac{22}{\cancel{7}_1} \times \cancel{7}_1$$
$$= 22 \text{ cm}$$

Example 3

Find the circumference of a circle that has a radius of 3.8cm.

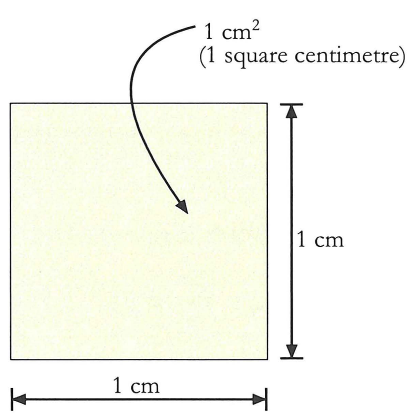
$C = 2 \times \pi \times r$

$C = 2 \times 3.142 \times 3.8$

$C = 23.88\text{cm}$ (2 dp)

Area

1 square centimetre (cm^2) is the **area** contained within a square that has a side of 1 centimetre (cm).



Similarly 1 square metre (m^2) is the area contained within a square that has a side of 1 metre (m) and similarly 1 square millimetre (mm^2) is the area contained within a square of side 1 millimetre (mm).

So, the area of a plane figure is found by measuring the number of mm^2 or cm^2 or m^2 within it.

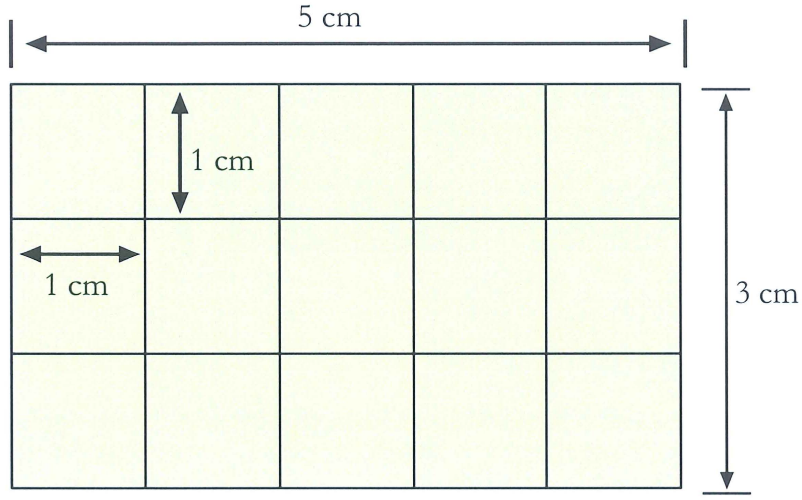
Now 10 mm = 1 cm

100 cm = 1 metre

(from which we see 1000mm = 1 metre)

The Rectangle

Consider a rectangle that has a length of 5cms and a width (breadth) of 3cm.



Now each small square within the rectangle has an area of $1cm^2$. Since there are 15 of these, the area within this rectangle must be $15 \times 1cm^2 = 15cm^2$

The same result could have been achieved by multiplying $5cm \times 3cm = 15cm^2$

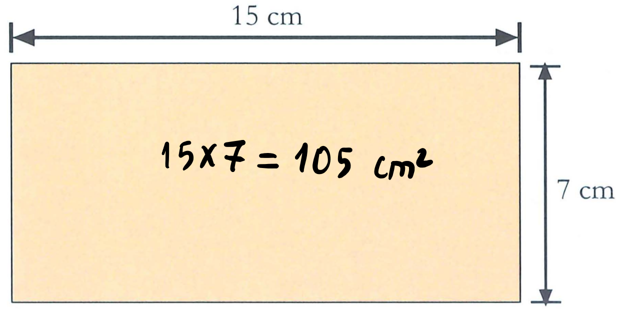
So

Area of a rectangle = length x width (breadth)

Note: The units of length and width must be the same. They must both be in, mms, cm, or m.

Example 1

Find the area of the following rectangle.

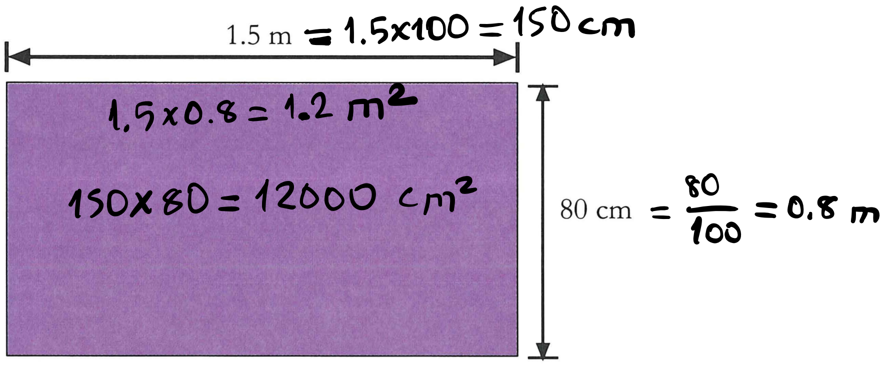


Area $= \text{length} \times \text{width}$

$= 15cm \times 7cm$

$= 105\ cm^2$

Example 2 Find the area of the rectangle.

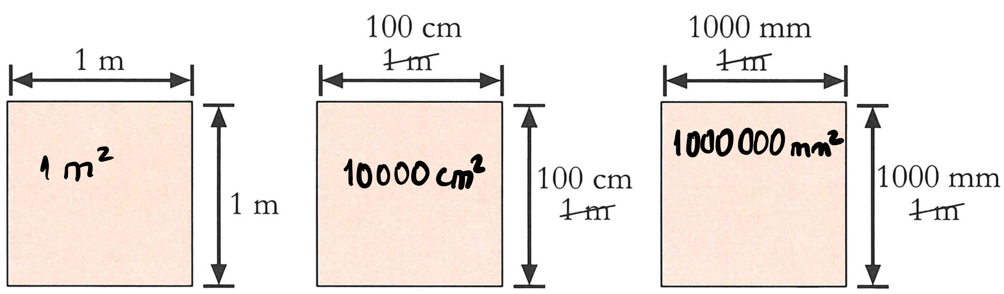


Since the units are different, we must change the metres to centimetres or the centimetres to metres before proceeding.

i.e. Area = 1.5m × 0.8m or Area =150cm × 80cm

$= 1.2\text{m}^2$ $= 12000\text{cm}^2$

Students still make the mistake of assuming that since there are 100cm in a metre there must be 100cm^2 in 1m^2. This is incorrect. To explain this, consider three identical squares of side 1 metre.



(a) (b) (c)

As shown, square (a) has its sides in metres, but square (b) could have its sides in centimetres (since 1m = 100cm) and square (c) could have its sides in millimetres (since 1m=1000mm).

When we find the area of each:-

a) 1m × 1m b) 100cm × 100cm c) 1000mm × 1000mm

$= 1\text{m}^2$ $=10000\text{cm}^2$ $=1000000\text{mm}^2$

These must be equal so:

To change 1m^2 to cm^2 we multiply by 10,000 (100 × 100)

To change 1m^2 to mm^2 we multiply by 1000,000 (1000 × 1000)

To change 1cm^2 to mm^2 we multiply by 100. (10 × 10)

As you can see these conversion factors are the squares of the linear ones (see brackets).

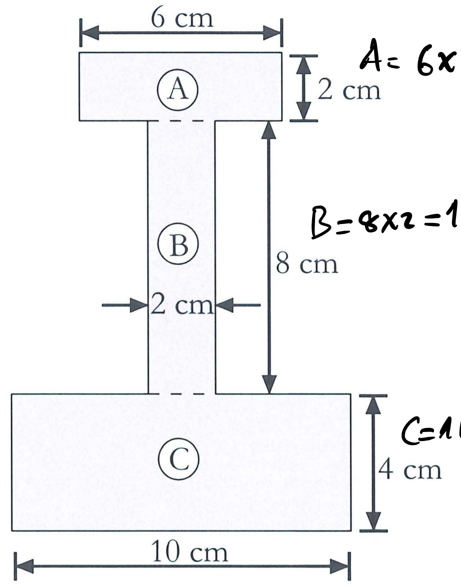
To go in the opposite direction, we divide:

To change cm^2 to m^2 we divide by 10,000

To change mm^2 to m^2 we divide by 1000,000

To change mm^2 to cm^2 we divide by 100.

Not all shapes are straightforward rectangles, as shown in the following example but they can be split up into rectangles. The area of the shape is then found by finding the area of each rectangle and then adding them together.



A= 6x2 =12 cm²

B=8x2=16cm²

The whole area
12+16+40=68cm²

C=10x4=40 cm²

Area of A = 6cm × 2cm = 12cm²

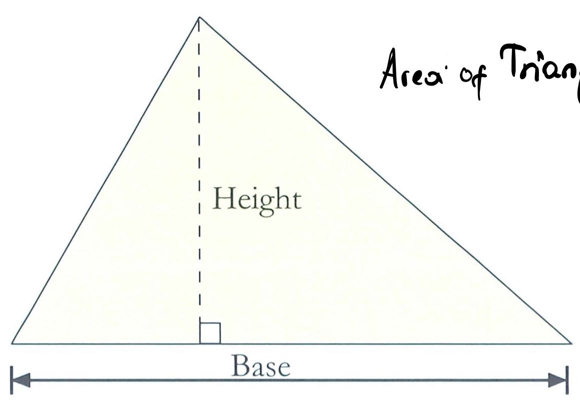
Area of B = 8cm × 2cm = 16cm²

Area of C = 10cm × 4xm = 40cm²

Area of shape = 12cm² + 16cm² + 40cm² = 68cm²

The Triangle

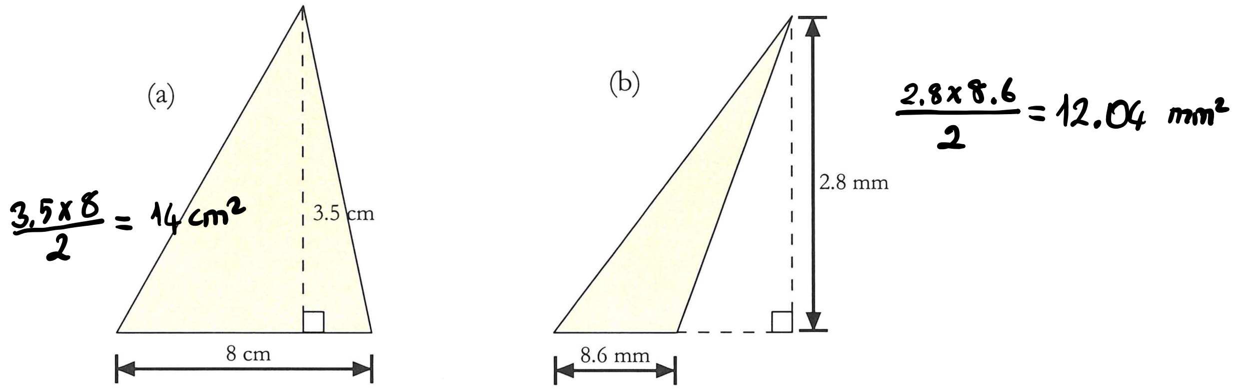
Area of Triangle = $\frac{\text{Height} \times \text{Base}}{2}$



∴ Area of a triangle = one half the base x perpendicular height

Example 1

Find the areas of the following triangles.

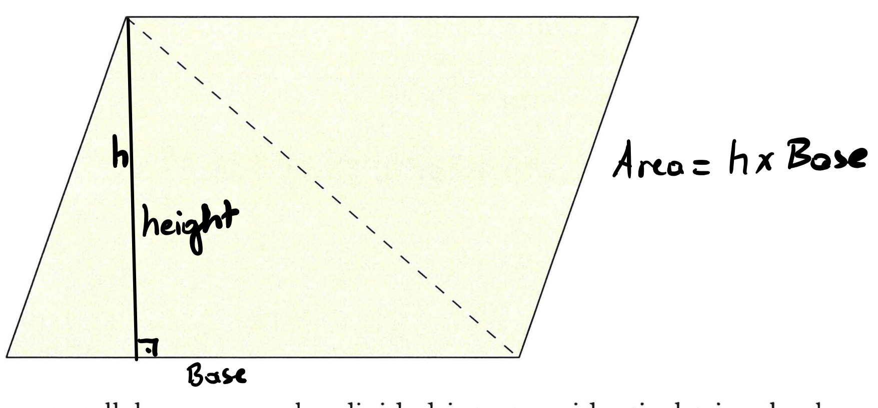


a) Area $= \frac{1}{2} \times 8 \times 3.5$
$= 14cm^2$

b) Area $= \frac{1}{2} \times 8.6 \times 2.8$
$= 12.04mm^2$

The Parallelogram

This is a four sided figure which has both pairs of its opposite sides equal and parallel.



Because of this, a parallelogram can be divided into two identical triangles by drawing in a diagonal (dotted line).

Area of a parallelogram = 2 × area of one of the triangles.

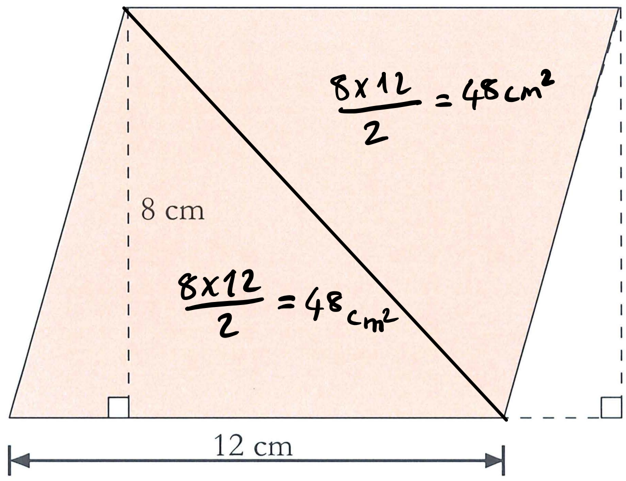
If area of a triangle = one half the base × perpendicular height

Then area of parallelogram = 2 × one half the base × perpendicular height

∴ Area of a parallelogram = base x perpendicular height

Example 1

Find the area of a parallelogram that has a base of 12cms and a height of 8cm



The whole Area $= 2 \times 48$
$= 96\text{cm}^2$

Area $= 8 \times 12 = 96\text{ cm}^2$

Area of parallelogram $= 12\text{cm} \times 8\text{cm} = 96\text{cm}^2$

Example 2

A parallelogram has an area of 160cm^2 and a base of 16cm. What is the height of the parallelogram? $160 = 16 \times \text{height} \Rightarrow \text{height} = \frac{160}{16} = 10\text{ cm}$

Since area of parallelogram $=$ base $\times$ height

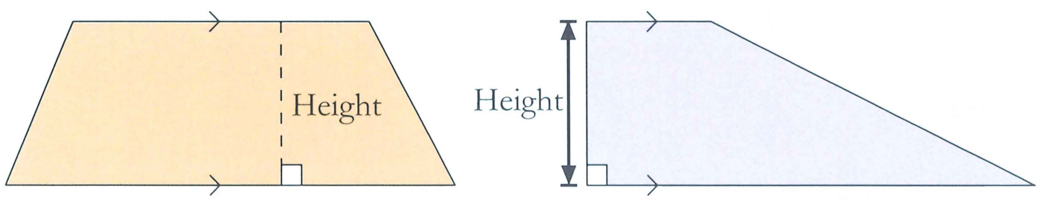
Then height $= \dfrac{\text{area}}{\text{base}}$

$= \dfrac{160}{16}$

$= 10\text{cm}$

The Trapezium

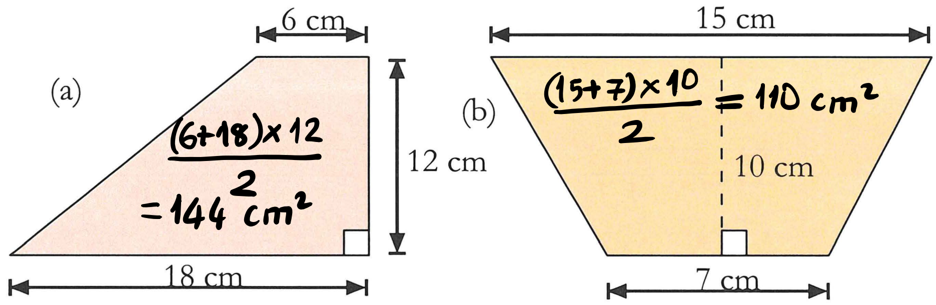
This is a four-sided figure that has one pair of opposite sides parallel.



Area of Trapezium $= \frac{1}{2} \times$ Sum of Parallel Sides $\times$ Perpendicular Distance between them

Example

Find the areas of the following trapezium:



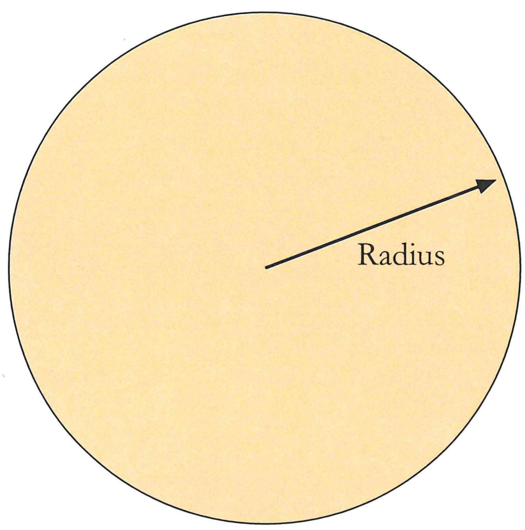
a) Area = ½ (6cm + 18cm) × 12 = ½ × 24cm × 12cm = 144cm²

b) Area = ½ (15cm + 7cm) ×10cm = ½ × 22cm × 10cm = 110cm²

The Circle

Area of a circle = πr^2

Where $\pi = \frac{22}{7}$ or 3.142 r = radius



Example 1

Find the area of a circle that has a radius of 7cm.

$$\text{Area} = \frac{22}{\cancel{7}_1} \times \cancel{7}^1 \times 7 = 154\text{cm}^2$$

Example 2

Find the area of a circle that has a diameter of 28cm.

Now since diameter = 2 × radius then radius $= \dfrac{\text{diameter}}{2} = \dfrac{28}{2} = 14\text{cm}$

$$\therefore \quad \text{Area} = \frac{22}{7} \times 14 \times 14 = 616\text{cm}^2$$

Example 3

Find the radius of a circle that has an area of 154cm^2.

$$\text{Area} = \pi r^2$$

$$\frac{\text{Area}}{\pi} = r^2$$

$$r = \sqrt{\frac{\text{Area}}{\pi}}$$

$$r = \sqrt{\frac{154}{3.142}}$$

$$r = 7\text{cm}$$

$154 = \pi r^2$

$154 = \frac{22}{7} \times r^2$

$r^2 = 49 \Rightarrow \boxed{r = 7}$

Volume

This is measured by finding how many cubic units a solid shape contains.

For example, the amount of space that is taken up by a solid shape.

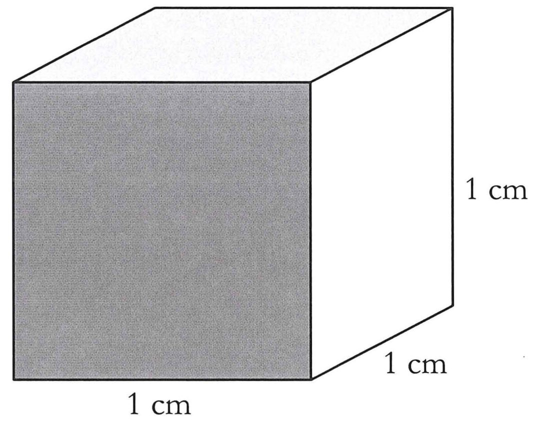
Unlike plane figures, solids have an extra dimension. (They are 3D as opposed to 2D so solids have length, breadth and height.

Example

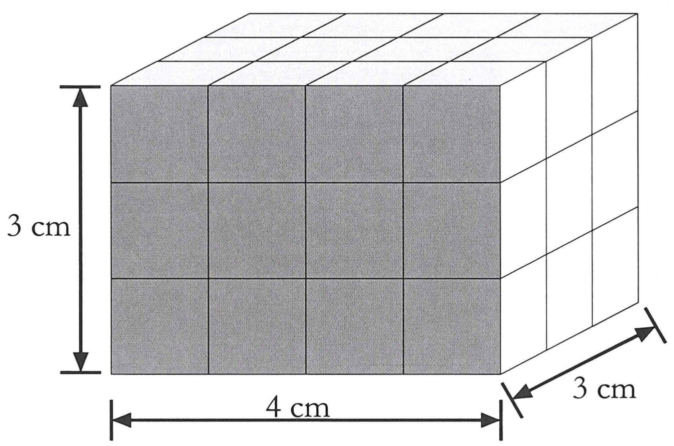
Consider a cube that is solid with six square sides. Since the sides are squares, the length, breadth and height are all equal.

So 1 cubic centimetre (cm^3) is the volume of a cube that has a side of 1cm.

i.e. Volume = 1cm × 1cm × 1cm × = $1cm^3$



Consider a cuboid rectangular solid that has a length of 4cm, a width of 3cm and a height of 3cm. Each small cube has a volume of $1cm^3$.



Now each layer contains 4 × 3 (12 cubes).

Since there are 3 layers, the shape contains 3 ×12 = 36 cubes.

Volume of the shape = 36 cubic units = $36cm^3$.

The same result could have been achieved by multiplying :

4cm × 3cm × 3cm = $36cm^3$.

As for area, the units of length, width and height must be the same:

They must all be in mm, cm or m.

So volume of a cuboid = length × breadth × height

Now since the area of the end of a rectangular solid is given by length × breadth, we can write:-

Volume of Rectangular solid (cuboid) = Area of End × Height

This is true for any regular solid (i.e. one that has a constant cross section throughout its height.)

For regular solids, we may write:

$$\text{Volume of solid} = \begin{matrix}\text{Area of Constant}\\ \text{Cross Section}\end{matrix} \times \text{length or height}$$

i.e. Length is synonymous with height in these cases, because if you consider the following cuboid:-

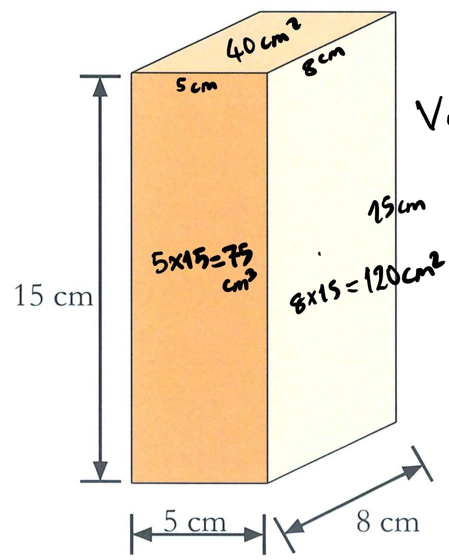
Area of End =15cm × 15cm = 75cm²

Volume = area of end × length

= 75cm² × 8cm

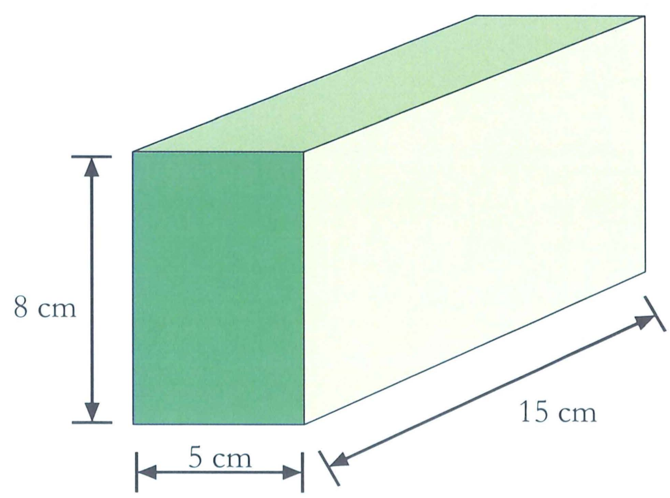
= 600cm³

Volume = 15×40 = $\boxed{600\ cm^3}$



Volume = 75×8 = $\boxed{600\ cm^3}$

Volume = 120×5 = $\boxed{600\ cm^3}$



Area of End = 8cm × 5cm = $40cm^2$

Volume = area of end × length

= $40cm^2$ × 15cm

= $600cm^2$

In other words, the same result is obtained.

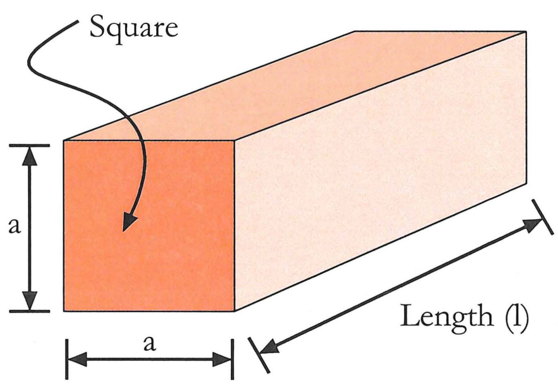
If we now consider the shapes whose areas we found earlier and add the third dimension to them we get:-

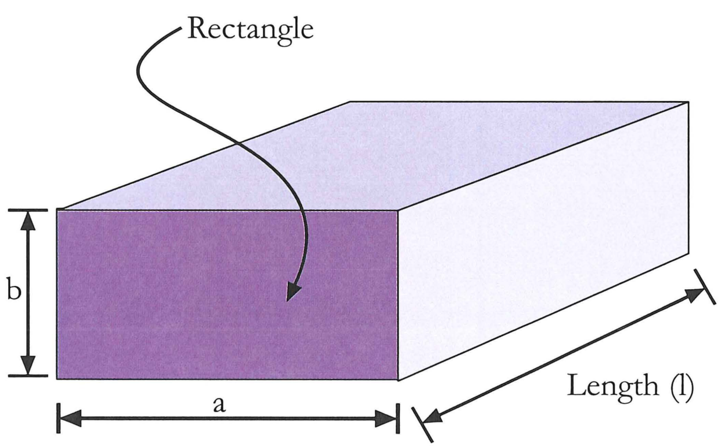
Volume of Shape

= area of square × length

= a × a × length

= a^2l $units^3$





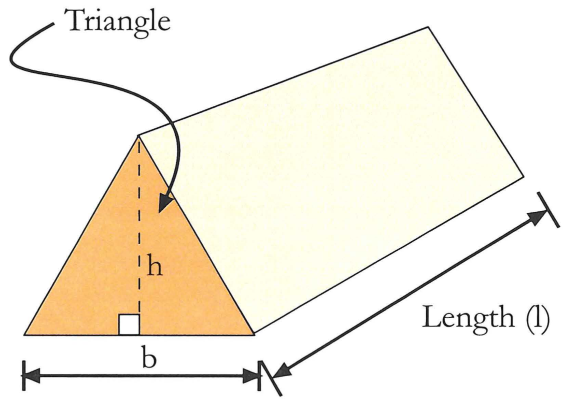
Volume of Shape = area of rectangle × length = a × b × length = abl $units^3$

Volume of Shape

= area of triangle × length

= ½ × base × height × length

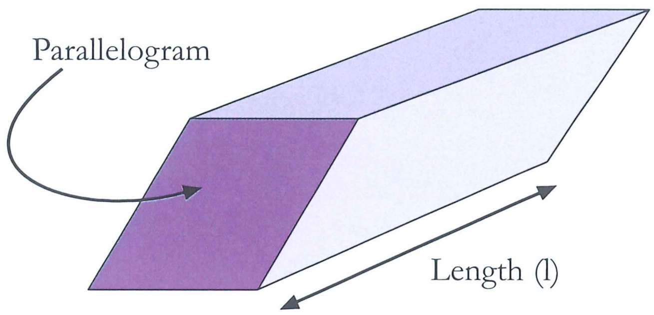
= ½ bhl $units^3$



Volume of Shape = area of parallelogram × length

= (base × height) × length

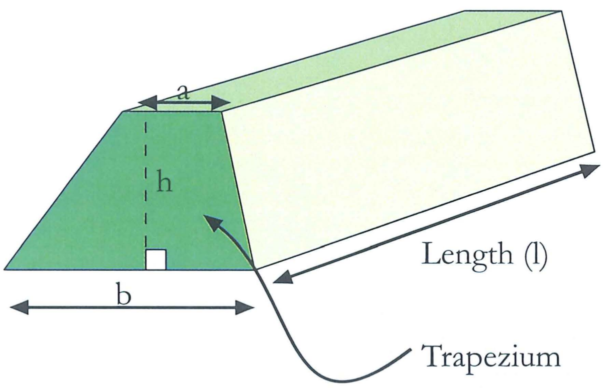
= bhl units^3



Volume of Shape = area of trapezium × length

$= (\frac{1}{2}(a+b) \times h) \times \text{length}$

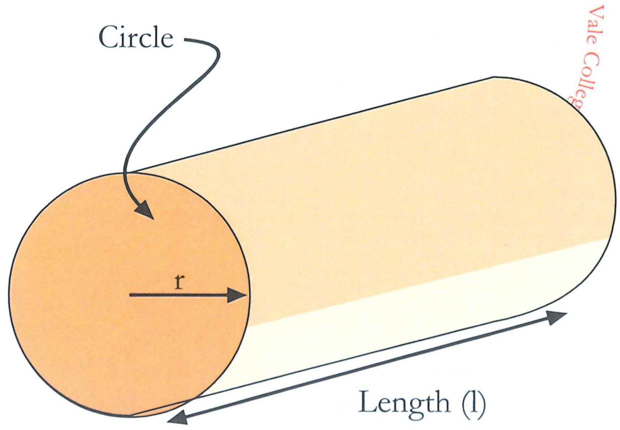
$= \frac{1}{2}(a+b) \times h \times l$ units^3



Volume of Shape = area of circle × length

$= (\pi r^2) \times l$

$= \pi r^2 l$ units^3



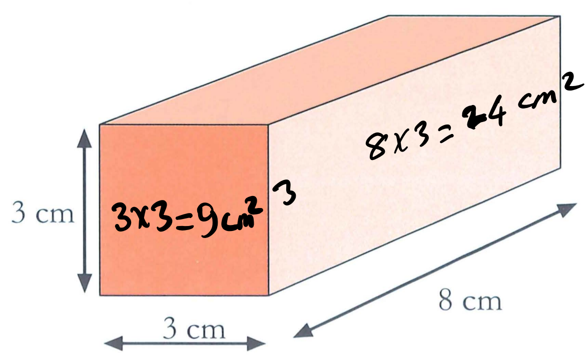
Examples

$V = (3 \times 3) \times 8$

$V = 72cm^3$

$V = 9 \times 8 = 72 cm^3$

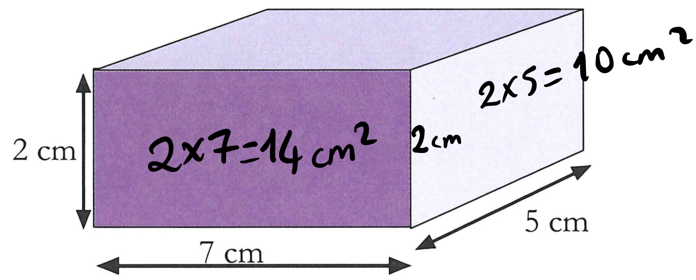
$V = 24 \times 3 = 72 cm^3$



$V = 14 \times 5 = 70\ cm^3$ $V = 10 \times 7 = 70\ cm^3$

$V = (2 \times 7) \times 5$

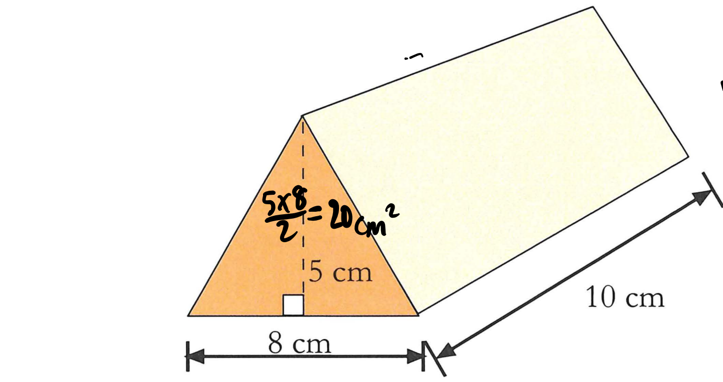
$V = 70cm^3$



$V = (\frac{1}{2} \times 8 \times 5) \times 10$

$V = 200cm^3$

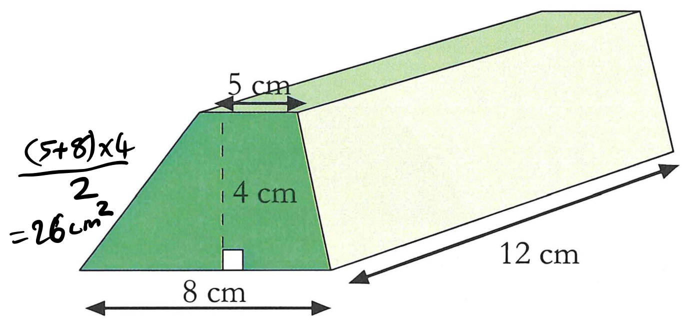
$V = 20 \times 10 = 200\ cm^3$



$V = (\frac{1}{2}\ (8+5) \times 4) \times 12$

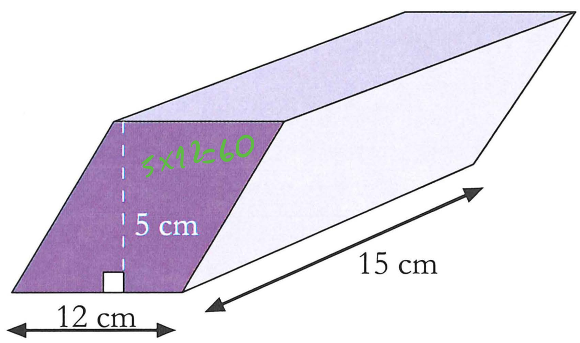
$V = 312cm^3$

$\boxed{V = 26 \times 12 = 312\ cm^3}$



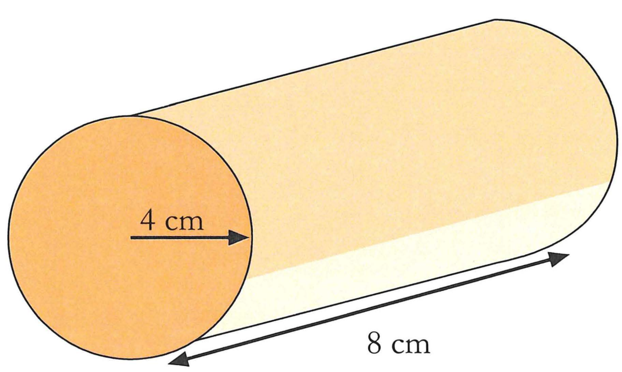
$V = (12 \times 5) \times 15$

$V = 900cm^3$

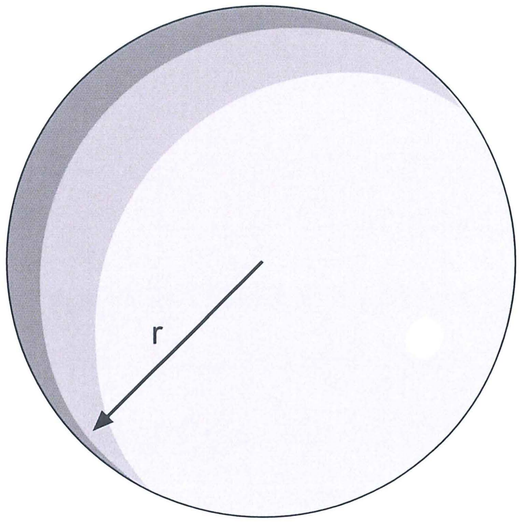


$V = (3.142 \times 4 \times 4) \times 8$

$V = 402.2cm^3$ (1dp)



Sphere



Volume of sphere $= \frac{4}{3} \times \pi r^3$ (r is the radius)

Example 1

Calculate the volume of a sphere that has a radius of 2.46cm.

$$\text{Volume} = \frac{4}{3} \times 3.142 \times 2.46^3$$

$$= 62.4\text{cm}^3 \text{ (1 dp)}$$

Example 2

Calculate the radius of a sphere that has a volume of 400cm³.

Since $\quad \text{Volume} = \frac{4}{3} \pi r^3$

then $\quad 400 = \frac{4}{3} \times 3.142 \times r^3$

i.e. $\quad 400 = 4.19 \times r^3$

$$\frac{400}{4.19} = r^3$$

$$95.48 = r^3$$

$$r = \sqrt[3]{95.48}$$

$$r = 4.57\text{cm}$$

$\sqrt[3]{27} = 3$
$\sqrt[3]{64} = 4$

Surface area of a sphere

Surface area of a sphere = $4\pi r^2$

Example 3

Calculate the surface area of a sphere, which has a diameter of 12 cm.

Surface area $= 4\pi r^2$

$= 4 \times 3.142 \times 6 \times 6$

$= 452.45 \text{ cm}^2$ (2dp)

Cone

$$\text{Volume of Cone} = \frac{1}{3}\pi r^2 h$$

Example 1

Find the volume of a cone of radius 2cm and length 5cm.

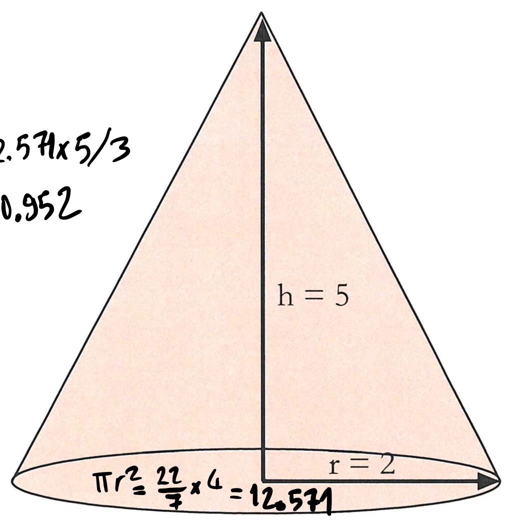
$= ⅓ \times 3.142 \times 2 \times 2 \times 5$

$= ⅓ \times 62.84$

$= 20.95\text{cm}^3$ (2 dp)

$V = 12.571 \times 5/3$

$V = 20.952$



Example 2

The volume of a cone of radius 4cm is 101cm^3. Find the height.

$V = ⅓ \pi r^2 h$

$101 = ⅓ \times 3.142 \times 4 \times 4 \times h$

$101 = 16.76 \times h$

$101 = \frac{1}{3}\left(\frac{22}{7} \times 4^2\right) \times h$

$h = \frac{101}{16.762} = 6.02$

$$h = \frac{101}{16.76}$$

h = 6.0cm (1 dp)

Curved surface area of a cone

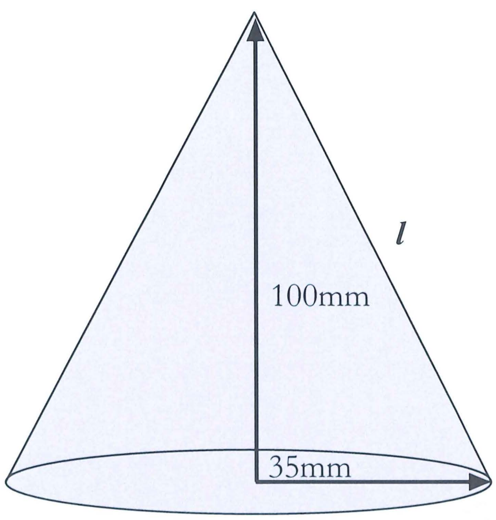
curved surface area of a cone = $\pi r l$ cm^2 (l = slant height)

Total surface area of a cone

Total surface area of a cone (including base) $= \pi r^2 + \pi r l$

$= \pi r(r + l)$ cm^2

e.g. A cone has a diameter of 70mm and a vertical height of 100mm. Calculate the total surface area.



To find the slant height l, we use Pythagoras theorem.

$l^2 = 100^2 + 35^2$

$l^2 = 10{,}000 + 1225$

$l = \sqrt{11{,}225}$

$l = 105.95$ mm (2dp)

So the total surface area $= (3.142 \times 35 \times 35) + (3.142 \times 35 \times 105.95)$

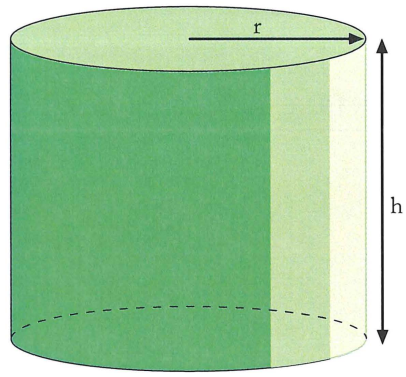
$= 15500.27mm^2$ (2dp)

Cylinder

Volume of Cylinder = $\pi r^2 h$

Example

Find the volume of a cylinder of height 8cm and radius 3cm.

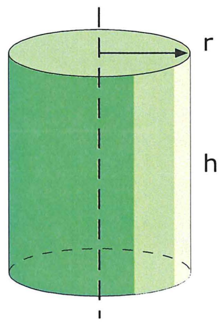


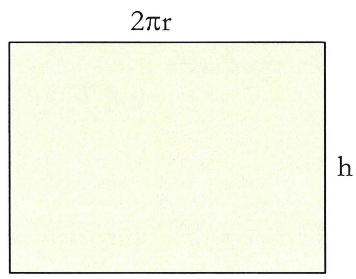
Volume = $\pi r^2 h$

$= 3.142 \times 3 \times 3 \times 8$

$= 226.2\text{cm}^3$ (1dp)

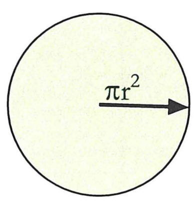
Curved Surface Area of a Cylinder



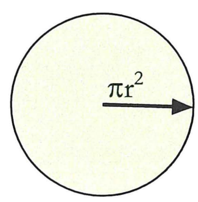


Curved Surface Area (ie circular ends excluded) $= 2\pi rh$

Total Surface Area (ie circular ends included) =



$+ 2\pi rh +$



$= 2\pi r^2 + 2\pi rh$

Example

Calculate the total surface area of a solid cylinder of radius 4cm and height 10cm.

Area of circle $= \frac{22}{7} \times 4^2 = 50.286$

$2\pi r \times h = 2 \times \frac{22}{7} \times 4 \times 10 = 251.429$

Curved surface area $= 2\,\pi\,r\,h$

Area of circular ends $= 2 \times \pi r^2$

Area of Solid Cylinder $= 2 \times 50.286 + 251.429$

$= 352.0002\ \text{cm}^2$

Total surface area $= 2\,\pi\,r\,h + 2\,\pi r^2$

$= (2 \times 3.142 \times 4 \times 10) + 2\,(3.142 \times 4^2)$

$= 251.36\text{cm}^2 + 100.54\text{cm}^2$

$= 351.9\text{cm}^2$

Revision

Arithmetic

Questions

1) Eighteen thousandths, written as a decimal is:

a. 0.0018

b. 0.018

c. 0.18

$\frac{18}{1000} = 0.018$

2) 40 divided by ⅛ is equal to:

a. 5

b. 320

c. 1/5

$\frac{40}{\frac{1}{8}} = 40 \times \frac{8}{1} = 320$

3) The next number in the sequence 1, 3, 6, 10 is:

$\underbrace{1,3}_{2}\underbrace{,6}_{3}\underbrace{,10}_{4}\underbrace{\ }_{5}$

a. 11

b. 13

c. 15

$10+5=15$

4) The value of [(-1) - (-1)] -1 is:

a. -2

b. -1

c. 0

$[-1+1]-1 = [0]-1 = -1$

5) $7 \times 6 - 12 \div 3 + 1$ is equal to:

$\underbrace{7 \times 6}_{42} - \underbrace{12 \div 3}_{4} + 1 = 38+1=39$

a. 39

b. 28

c. -44

6) If £182.50 is shared equally between 5 people, how much would each person receive:

a. £35.50

b. £37.50

c. £36.50

$5\overline{)182.5}$ = 36.5
15
32
30
25

7) The arithmetic mean of ten numbers is 36. If one of the numbers is 18, what is the mean of the other nine.

a. 32

b. 36

c. 38

$\frac{\text{Sum of ten numbers}}{10} = 36$

One of the numbers is 18

Sum of ten numbers = 360

Sum of ten numbers − 18 = Sum of nine other numbers

360 − 18 = 342

The mean of nine = $\frac{342}{9} = 38$

8) Three thousand and forty nine written in numbers is:

a. 3,490

b. 30049

c. 3049

9) If the fractions $\frac{2}{5}$ $\frac{3}{7}$ and $\frac{1}{3}$ arranged in order of size, smallest first, the order would be:

a. $\frac{2}{5}, \frac{1}{3}, \frac{3}{7}$

b. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$

c. $\frac{3}{7}, \frac{2}{5}, \frac{1}{3}$

$\frac{2}{5} \times \frac{21}{21} = \frac{42}{105} = \frac{2}{5}$

$\frac{3}{7} \times \frac{15}{15} = \frac{45}{105} = \frac{3}{7}$

$\frac{1}{3} \times \frac{35}{35} = \frac{35}{105} = \frac{1}{3}$

10) $3\frac{2}{3} + 4\frac{3}{5}$ **is equal to:**

$3+\frac{2}{3}+4+\frac{3}{5}$

$7+\frac{2}{3}+\frac{3}{5} = 7+\frac{2}{3}\times\frac{5}{5}+\frac{3}{5}\times\frac{3}{3}$

$= 7+\frac{10}{15}+\frac{9}{15}$

$= 7+\frac{19}{15} = 7+1+\frac{4}{15} = 8\frac{4}{15}$

a. $7\frac{1}{3}$

b. $6\frac{13}{15}$

c. $8\frac{4}{15}$

11) $2\frac{1}{2} \times 1\frac{1}{3}$ **is equal to:**

$\frac{2\times2+1}{2}\times\frac{1\times3+1}{3} = \frac{5}{2}\times\frac{4}{3} = \frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$

a. $2\frac{1}{5}$

b. $2\frac{1}{6}$

c. $3\frac{1}{3}$

12) $\frac{5}{6} \div \frac{1}{3}$ **is equal to:**

$\frac{5}{\cancel{6}_{2}}\times\frac{\cancel{3}}{1} = \frac{5}{2}$

a. $\frac{5}{2}$

b. $\frac{5}{18}$

c. $2\frac{1}{2}$

Two of the answers are correct

13) **The number 46700 when written in Standard Form is:**

a. 46.7×10^3

b. 4.67×10^4

c. 4.67×10^5

14) **When written in Standard Form 0.00075 is equal to:**

a. 75×10^{-5}

b. 7.5×10^{-4}

c. 0.75×10^{-3}

15) **$3 \times 10^2 \times 2 \times 10^4$ is equal to:**

a. 6×10^8

b. 6×10^6

c. 6×10^{-2}

16) **$4 \times 10^6 \div 2 \times 10^3$ is equal to:** $\frac{4 \times 10^6}{2 \times 10^3} = 2 \times 10^{6-3} = 2 \times 10^3$

a. 2×10^2

b. 2×10^3

c. 2×10^{-3}

17) **Which of the following is a prime number:**

a. $15/3 = 5$ $15/5 = 3$

b. $27/3 = 9$ $27/9 = 3$

c. 41

18) **2, 3 and 5 are the factors of :**

a. 6

b. 10

c. 30

19) **The HCF of 20, 30 and 60 is:**

a. 2

b. 10

c. 20

30 15 12 10

20) **The LCM of 2, 4, 5 and 6 is:**

a. 240

b. 60

c. 6

21) **If 1 km = 0.6 miles, then 66 miles is equivalent to:**

a. 110 km

$\frac{66}{0.6} = \frac{660}{6} = 110$ km

b. 1100km

c. 11km

22) **If £120 is divided in the ratio 2 : 3, then the larger share is:**

a. £48

$2+3=5$ the sum shares

b. £80

$\frac{120}{5} = 24$ for each share

c. £72

The large share is proportional to 3
so $3 \times 25 = 72$

23) **If 50% of a certain length is 500mm, the complete length is:**

a. 250mm

b. 1000mm

c. 100mm

24) **What is the square root of 16 × 64:**

a. 40

b. 32

c. 256

$\sqrt{16 \times 64} = \sqrt{16} \times \sqrt{64} = 4 \times 8 = 32$

25) **The difference between 2^3 and 3^2 is:**

a. 9

b. 1

c. 17

$2^3 = 8 = 2 \times 2 \times 2$

$3^2 = 3 \times 3 = 9$

$9 - 8 = 1$

26) **What is 0.0059 correct to 2 decimal places:**

a. 0.01

b. 0.10

c. 0.006

$0.0059 = 0.01$

27) **Correct to 2 significant figures 3.0394 is:**

a. 3.0

b. 3.04

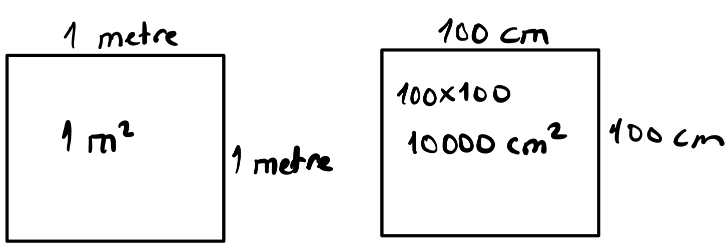
c. 3.03

28) **How many square centimetres are there in a square metre:**

a. 100

b. 1000

c. 10,000

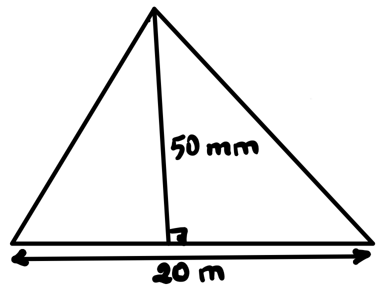


29) **A triangle has an altitude of 50mm and a base of 20mm. Its area is:**

a. 250mm²

b. 0.25cm²

c. 500mm²



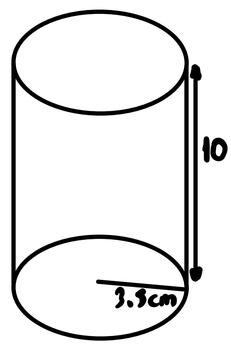
$$\frac{50 \times 20}{2} = 500\ mm^2$$

30) **Oil is sold in a closed cylindrical container whose diameter is 7cm and whose height is 10cm. How much oil does the can hold:**

a. Less than 0.4 litres

b. 0.5 litres

c. More than 1 litre



V= Area of the circle x height

$V = \pi r^2 \times h$

$V = \frac{22}{7} \times (3.5)^2 \times 10$

$V = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 10$

$V = \frac{22}{4} \times 70 = 385\ cm^3$

$1000\ cm^3$ can hold a litre

Revision

Arithmetic

Answers

1.	**B**	21.	**A**
2.	**B**	22.	**C**
3.	**C**	23.	**B**
4.	**B**	24.	**B**
5.	**A**	25.	**B**
6.	**C**	26.	**A**
7.	**C**	27.	**A**
8.	**C**	28.	**C**
9.	**B**	29.	**C**
10	**C**	30.	**A**
11.	**C**		
12.	**C**		
13.	**B**		
14.	**B**		
15.	**B**		
16.	**B**		
17.	**C**		
18.	**C**		
19.	**B**		
20.	**B**		

Basic Algebra

Introduction

In algebra we use letters as well as numbers to represent quantities. The following examples show how phrases involving associated quantities are translated into algebraic expressions.

a) Eight times a number

Let the number be x

$\therefore$ eight times the number $= 8 \times x = 8x$

b) Five times a number minus three

Let the number be y

$\therefore$ five times the number minus three $= 5y - 3$

c) Three times the product of two numbers.

Let the numbers be x and y

$\therefore$ three times the product of x and y $= 3xy$

d) Four times a number minus three times another number.

Let the first number be a and the second b

$\therefore$ Four times a number a minus three times another number b $=4a - 3b$

e) One number divided by another.

Let one number be x and the other y

$\therefore$ one number divided by another $= \frac{x}{y}$

f) The sum of two numbers divided by a third number

Let the numbers be a, b and c.

$\therefore$ The sum of two numbers divided by the third $= \dfrac{a+b}{c}$

Algebraic Expressions, Equations And Identities

An ***algebraic expression*** is a collection of letters or symbols separated by arithmetical operators.

$+ \quad - \quad \div \quad \text{x}$

For example: $2x^3 + 4x + 6$ is an algebraic expression.

We can only find the value of it when x is given a specific value. If we give a value of 2 to x then:-

$2(2)^3 + 4(2) + 6 = 16 + 8 + 6 = 30$

An ***algebraic equation*** involves an 'equals' sign (a statement of equality).

For example: $6r - 5 = 19$

The statement of equality i.e. the equation only holds for a specific value of r.

i.e.

$$6r = 19 + 5$$

$$6r = 24$$

$$r = \frac{24}{6}$$

$$r = 4$$

An ***algebraic identity*** differs from an equation in that it is true for every value of the variable.

For example: $x^2 - 9 = (x + 3)(x - 3)$

If we substitute $x = 8$

$$8^2 - 9 = (8 + 3)(8 - 3)$$

$$64 - 9 = (11)(5)$$

$$55 = 55$$

This is the case for any value of x.

Substitution

This is the process of finding the numerical value of an algebraic expression by replacing the symbols or letters in it by given values.

Example 1

If $a = 2$ $b = 3$ and $c = 5$ find the values of:-

$5 \times 2 - 5 = 5$ $15 - 2 = 13$

a) $3a + 2b + 4c$ b) $5a - c$ c) $15 - a$

$3 \times 2 + 2 \times 3 + 4 \times 5 = 32$

a) $3a + 2b + 4c$

$= 3(2) + 2(3) + 4(5)$

$=6 + 6 + 20$

$=32$

b) $5a - c$

$= 5(2) - 5$

$=10 - 5$

$=5$

c) $15 - a$

$=15 - 2$

$=13$

Example 2

If $x = 5$ $y = -2$ and $z = 3$ find the values of:-

a) $5x - 3y$ b) $4y - z$ c) $z - 5y$ d) $x^2 - 4xy + 3yz^2$

$5^2 - 4 \times 5 \times (-2) + 3(-2) \times 3^2$

$25 + 40 - 54 = 11$

a) $5x - 3y$

$= 5(5) - 3(-2) = 31$

b) $4y - z$

$= 4(-2) - 3 = -11$

c) $z - 5y$

$= 3 - 5(-2) = 13$

Evaluation of Formulae by Substitution

The formula E = IR can be described as a formula for E in terms of I and R. The only way we can find a numerical value for E is by substituting given values of I and R into it.

Example 1

$$P = \frac{Fs}{t}$$

This formula is used in the mechanical engineering field.

Find the value of P when F = 30, S = 10 and T = 2.

$$P = \frac{Fs}{t}$$

$$P = \frac{30 \times 10}{2}$$

$$P = 150$$

Example 2

$$I = \frac{V}{R}$$

This formula is used to find the current in an electrical circuit.

Find the value of I when V = 240 and R = 20

$$I = \frac{V}{R}$$

$$I = \frac{240}{20}$$

$$I = 12$$

Example 3

When n cells are connected in series, the following formula is used to find the current:

$$I = \frac{nE}{R + nr}$$

Find the value of this current when E = 1.8, n = 6, r =1.1and R= 0.4

$$I = \frac{6 \times 1.8}{0.4 + 6(1.1)}$$

$$I = \frac{10.8}{7}$$

$$I = 1.54$$

Example 4

This is the formula for the total resistance when two electrical resistances are wired in parallel:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Find the value of R when $R_1 = 10$ and $R_2 = 15$.

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{15}$$

$$\frac{1}{R} = \frac{3 + 2}{30}$$

$$\frac{1}{R} = \frac{5}{30}$$

Cross Multiplying:

$$5R = 30$$

$$R = 6$$

Example 5

The acceleration of a train 'a' is found by using the formula:

$$a = \frac{v^2 - u^2}{2s}$$

Find 'a' when $v = 20$, $u = 10$ and $s = 5$

$$a = \frac{20^2 - 10^2}{2 \times 5}$$

$$a = \frac{400 - 100}{10}$$

$$a = \frac{300}{10}$$

$$a = 30$$

Example 6

The distance travelled 's' by an accelerating aircraft is given by:

$$s = ut + \tfrac{1}{2}at^2$$

Find 's' when u = 4, t = 150 and a = 0.2

i.e.

$$s = ut + \tfrac{1}{2}at^2$$
$$s = 4(150) + (\tfrac{1}{2}(0.2) \times 150 \times 150)$$
$$s = 600 + (0.1 \times 22500)$$
$$s = 600 + 2250$$
$$s = 2850$$

Example 7

The surface area of a cylinder is given by:

$$s = 2\pi r(r + h)$$

Find s when r = 5cm, and h = 15cm. (Take π as 3.142)

i.e.

$$s = 2 \times 3.142 \times 5(5 + 15)$$
$$s = 31.142 \times 20$$
$$s = 628.4\text{cm}^2$$

Example 8

The formula: $C = \frac{5}{9}(F - 32)$

is used to convert degrees Fahrenheit (F) to degrees Celsius (C). Convert to degrees Celsius 86°F.

$$C = \frac{5}{9}(86 - 32)$$
$$C = \frac{5}{9}(54)$$
$$C = 30°C$$

Example: Convert to degrees celcius 100°F

$$C = \frac{5}{9}(100-32) = \frac{5 \times 68}{9} = 37.778$$

$32°F = 0°C$

Transposition of Formulae

If we consider the formula: $v = u + at$

then v is said to be its subject.

$u = v - at \rightarrow u$ is the subject

$at = v - u \Rightarrow a = \frac{v-u}{t} \rightarrow a$ is the subject

By rearranging this formula, we can make u, a or t the subject. This process is called ***transposition of formulae*** or quite simply ***changing the subject***.

The algebraic terms in a formula can be connected by the four arithmetical operations (+ - × ÷) as well as by roots and powers.

There are rules for transposing a formula and they should be carried out in a specific order:

a) Get rid of square roots or other roots. This is accomplished by squaring or raising both sides of the formula to another power.

e.g.

If $a = \sqrt{bc}$

then squaring both sides $a^2 = bc$

b) If any fractions are present, then they can be removed by the process of ***cross multiplication.***

e.g.

If $\frac{x}{y} = \frac{a}{b}$

then $xb = ya$

c) Process (b) sometimes results in brackets being formed. ***These must be removed, i.e. expanded***.

e.g.

If $V = \frac{2R}{R - r}$

Cross multiplying

$V(R - r) = 2R$

ie $VR - Vr = 2R$

d) The required subject may appear in more than one place in the formula. This is not allowed so the subject terms must be ***collected together.***

e.g.

If $VR - Vr = 2R$ and R is the required subject then

$VR - 2R = Vr$ (Transferring terms to the other side and changing sign)

e) Process (d) could lead to the need to ***factorise***.

e.g.

$$\text{If } VR - 2R = Vr$$
$$\text{then } R(V - 2) = Vr$$

f) The final stage is to ***isolate*** the new subject.

e.g.

If $R(V - 2) = Vr$

then $R = \dfrac{Vr}{V - 2}$

ie Dividing both sides by the bracket $(V - 2)$.

Although there are six steps listed above, in many cases they will not all be needed. However, it is good practice to maintain the order given.

Example 1

Transpose

$$y = \frac{5}{3 + x} \quad \text{for x}$$

$y(3+x) = 5$

$3y + xy = 5 \Rightarrow xy = 5 - 3y$

$x = \dfrac{5-3y}{y} = \dfrac{5}{y} - \dfrac{3y}{y}$

$= \dfrac{5}{y} - 3$

In this case there are no roots present and so the first step would be to get rid of fractions.

- Cross multiplying $y(3+x) = 5$
- A bracket has now been formed and this should be expanded:
  $3y + xy = 5$
- There is now one term which contains x, so $xy = 5 - 3y$
- Finally x is isolated by dividing both sides by y:

$$x = \frac{5 - 3y}{y} \quad \text{or} \quad x = \frac{5}{y} - 3$$

Example 2

Transpose

$$y = \frac{3x + 2}{x + 3} \quad \text{for } x$$

$y(x+3) = 3x+2$
$xy+3y = 3x+2$
$xy-3x = 2-3y$
$x(y-3) = 2-3y$
$x = \frac{2-3y}{y-3}$

There are no roots present and so the first step is to get rid of fractions:

- Cross multiplying $y(x+3) = 3x + 2$
- The next step is to expand the bracket: $xy + 3y = 3x + 2$
- The required subject x appears in two places and so these terms must be collected together $xy - 3x = 2 - 3y$
- Factorising we get $x(y - 3) = 2 - 3y$
- Finally dividing both sides by the bracket $(y - 3)$ we get:

$$x = \frac{2 - 3y}{(y - 3)}$$

Note: It is in order for y to appear in two places because it is not the subject of the formula. If you look at the original formula above there was only one y when it was the subject.

Let us now look at an example in which all six stages need to be processed:

Example 3

Transpose

$$x = \sqrt{\frac{y - 1}{y + 1}} \quad \text{for } y$$

$x^2 = \frac{y-1}{y+1} \Rightarrow x^2(y+1) = y-1$
$x^2y + x^2 = y-1$
$x^2y - y = -x^2-1$
$y(x^2-1) = -x^2-1$
$y = \frac{-x^2-1}{x^2-1} = \frac{-1}{-1} \times \frac{-x^2-1}{x^2-1}$
$= \frac{x^2+1}{1-x^2}$

a) Get rid of square root.

Squaring both sides:

$$x^2 = \frac{y - 1}{y + 1}$$

b) Get rid of fractions.

$$x^2 = \frac{y - 1}{y + 1}$$

Cross multiplying

$$x^2(y + 1) = y - 1$$

c) Expand brackets

$$x^2(y + 1) = y - 1$$
$$x^2y + x^2 = y - 1$$

d) Collect terms containing subject

$$x^2y - y = -1 - x^2$$

e) Factorise

$$y(x^2 - 1) = -1 - x^2$$

f) Isolate the subject

$$y = \frac{-1 - x^2}{x^2 - 1}$$

This is a little untidy because of the minus signs so multiply top and bottom of the right hand side by -1, to give:-

$$y = \frac{1 + x^2}{1 - x^2}$$

Addition and Subtraction of Algebraic Terms

If you had two fruit bowls, one with five oranges in it and the other with 3 oranges in it, you would have 8 oranges altogether.

However, if you had one fruit bowl with 2 apples in it and another with 3 bananas in it, you would have 2 apples and 3 bananas.

Algebra is very similar to this in that only ***like terms*** can be added or subtracted.

Like terms are numerical multiples of the ***same*** algebraic quantity.

For example,

$$5a, \ 26a, \ -1.5a, \ \frac{2}{5}a$$

are four like terms

Examples

1) $5a + 3a + 2a = 10a$

2) $8x - 5x = 3x$

3) $6p - 8p = -2p$

4) $-2q - 5q = -7q$

An expression such as $3a + 2b - c$ is an expression consisting of three ***unlike terms*** and this cannot be simplified further.

It is possible however to have an expression consisting of several sets of like terms. In this case each of the sets can be simplified.

Examples

1) $5p + 2q - 3r + 2p - 5q + 6r$

$= 5p + 2p + 2q - 5q - 3r + 6r$

$= 7p - 3q + 3r$

2) $6a + 3b + 4c - 8a + 2b - 3c$ $-2a+5b+c$

$= 6a - 8a + 3b + 2b + 4c - 3c$

$= -2a + 5b + c$

Multiplication of Algebraic Quantities

When multiplying algebraic terms together then:

- Multiply the numbers (if present)
- Multiply the letters or symbols
- Multiply the signs

A good method of sorting out the signs in an algebraic multiplication is to count the number of ***negative*** signs involved.

a) If the overall total is an ***odd number*** then the answer is minus (-)

b) If the overall total is an ***even number*** then the answer is plus (+)

Examples

1) $2 \times -p \times 5 \times r \times -q = 10prq$ (2 minus signs present – ***even***)

2) $-3 \times a \times -2 \times b \times -5 \times c = -30abc$ (3 minus signs present – ***odd***)

3) $4a \times 3b = 12\ ab$

4) $-2a \times 5b = -10\ ab$

5) $3a \times -5b \times -c = 15\ abc$

6) $-3p \times -2q = 6\ pq$

When multiplying the same letters or symbols then ***indices*** are used:-

Examples

1) $3a \times 2a = 6a^2$

2) $-5p \times 2p = -10p^2$

3) $-4x \times -3x = 12x^2$

4) $5m \times -3m = -15m^2$

5) $2a \times -3a \times 5a = -30a^3$

6) $3ab \times 5ab = 15a^2b^2$

Division of Algebraic Quantities

When algebraic expressions are divided it is often possible to cancel between numerator and denominator. This is the same as dividing top and bottom by the same number or symbol.

Examples

1) $\frac{+a}{+b} = +\frac{a}{b} = \frac{a}{b}$

2) $\frac{-4p}{3q} = -\frac{4p}{3q} = \frac{4p}{-3q}$

3) $\frac{-7x}{-8y} = +\frac{7x}{8y} = \frac{7x}{8y}$

4) $\frac{5x}{-3y} = -\frac{5x}{3y} = \frac{-5x}{3y}$

5) $\frac{4a^2b}{8ab^2} = \frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{a}} \times a \times \overset{1}{\cancel{b}}}{\underset{2}{\cancel{8}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{b}} \times b} = \frac{a}{2b}$

$\frac{4}{8} \cdot \frac{a^2}{a} \cdot \frac{b}{b^2} = \frac{1}{2} \cdot \frac{a}{1} \cdot \frac{1}{b}$
$= \frac{a}{2b}$

6) $\frac{12p^2q^2r}{4pqr} = \frac{\overset{3}{\cancel{12}} \times \overset{1}{\cancel{p}} \times p \times \overset{1}{\cancel{q}} \times q \times \overset{1}{\cancel{r}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{p}} \times \underset{1}{\cancel{q}} \times \underset{1}{\cancel{r}}} = \frac{3pq}{1} = 3pq$

Note: Division of algebraic quantities is dealt with in more detail in algebraic fractions.

Brackets

Sometimes it is useful to group terms together especially when we want to multiply an expression by a number or symbol. Brackets are used to do this. For example, if we wanted to multiply the expression $3x + 2y$ by 4, we would write $4(3x + 2y)$.

When brackets are removed (i.e. expanded), each term inside the bracket is multiplied by the quantity outside the bracket. ***Care must be taken with the signs when doing this.***

Remember:

$(+) \times (+) = (+)$

$(+) \times (-) = (-)$

$(-) \times (+) = (-)$

$(-) \times (-) = (+)$

Examples

1. $3(a + b) = 3 \times a + 3 \times b = 3a + 3b$

2. $4(x - y) = 4 \times x - 4 \times y = 4x - 4y$

3. $a(b + c) = a \times b + a \times c = ab + ac$

4. $3p(2q - 5) = 3p \times 2q - 3p \times 5 = 6pq - 15p$

5. $-3(a + b) = (-3 \times a) + (-3 \times b) = -3a - 3b$

In the case where there is just a sign outside the bracket, $-(a + b)$, it is best to imagine a 'one' there.

For example, $-(a + b) = -1(a + b) = (-1 \times a) + (-1 \times b) = -a - b$

$$-(a - b) = -1(a - b) = (-1 \times a) - (-1 \times -b) = -a + b$$

Some expressions contain more than one bracket. These can be removed i.e. (expanded) separately and the expression simplified, i.e. (like terms added or subtracted).

Example 1 $2(3x + 4y) + 3(2x + 3y)$

$= 6x + 8y + 6x + 9y$

$= 12x + 17y$

Example 2 $3(4x + 5y) - 2(2x + 3y)$

$= 12x + 15y - 4x - 6y$

$= 8x + 9y$

Example 3 $5(3a - 2b) - 2(a + 4b)$

$= 15a - 10b - 2a - 8b$

$= 13a - 18b$

Example 4 $p(q + r) - q(p - r)$

$= pq + pr - qp + qr$

$= pr + qr$

Note: pq is the same as qp and ab the same as ba and so on.

Example 5 $3(a - b) - 2(2a - 3b)$ $= 3a - 3b - 4a + 6b = -a + 3b$

$= 3a - 3b - 4a + 6b$

$= - a + 3b$

Example 6 $2x(x - 5) - x(x - 2) - 3x(x - 5)$ $= 2x^2 - 10x - x^2 + 2x - 3x^2 + 15x$

$= 2x^2 - x^2 - 3x^2 - 10x + 2x + 15x$

$= 2x^2 - 10x - x^2 + 2x - 3x^2 + 15x$

$= -2x^2 + 7x$

$= -2x^2 + 7x$

Factorisation

An expression such as $2x + 2y$ has the number 2 common to both terms.

i.e. $2x + 2y = 2(x + y)$

This is the reverse procedure of expanding brackets. The number 2 and the bracket $(x + y)$ are called ***the factors*** of $2x + 2y$. The easiest way to factorise an expression is to write out the expression in full breaking down the numbers into prime numbers (if they are not prime to start).

For example, $4 = 2 \times 2$ $6 = 2 \times 3$ $8 = 2 \times 2 \times 2$ $9 = 3 \times 3$ (and so on).

Example 1 Factorise $8p + 12q$

If we breakdown 8 and 12 into prime numbers and write out the expression in full we get:

$2 \times 2 \times 2 \times p + 2 \times 2 \times 3 \times q$

Now highlight numbers or letters which appear in ***both*** terms:

(2) $\times$ **(2)** $\times 2 \times p +$ **(2)** $\times$ **(2)** $\times 3 \times q$

So **(2)** $\times$ **(2)** $= 4$ and this is placed outside the bracket. The unhighlighted parts which are left form the bracket. (In this case $2p + 3q$). These are the factors of $8p + 12q$.

$$8p + 12q = 4(2p + 3q)$$

factors