Summary

Rule 1 $a^m \times a^n = a^{m+n}$
Rule 2 $\frac{a^m}{a^n} = a^{m-n}$
Rule 3 $a^{-m} = \frac{1}{a^m}$
Rule 4 $a^0 = 1$ except that a is different from 0
Rule 5 $(a^m)^n = a^{m \times n}$
Rule 6 $a^{m/n} = \sqrt[n]{a^m}$

Denary System

We are used to calculating in Base 10 (***Denary***). In this base, we use the 10 digits 0, 1, 2, 3, 4, 5,6, 7, 8 , 9. When adding or multiplying, once we get above 9, we have to write down the right hand digit and carry the left hand digit to the next column.

For example take the number 13, write down 3 and carry 1. This is because we have 3 units and 1 ten.

Since early school years, we have been familiar with the column heads Thousands, Hundreds, Tens and Units, under which we arrange the digits of large numbers. If we convert these headings into powers of 10 we get:

	Thousands		Hundreds		Tens		Units
	10^3		10^2		10^1		10^0
So 2368 =	2		3		6		8
ie.	2×10^3	+	3×10^2	+	6×10^1	+	8×10^0

In the Base 10 (denary) system, the highest value digit in any column is always one less than the base number. i.e. 10 - 1 = 9

So for other number bases, we would expect the same ruling.

Base 2 (binary) highest value digit is 2 - 1 =1. So in the binary system we use the two digits 0 and 1.

Base 8 (octal) highest value digit is 8 - 1 = 7. So in the octal system we use the digits 0, 1, 2, 3, 4, 5, 6, 7.

Base 16 (hexadecimal) highest value digit is 16 - 1 = 15.
So in the hexadecimal system we use the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15.

Some ambiguity arises here and 10–15 are represented by A–F. This is explained in more detail in the Base 16 section.

So summarising the columns for the various bases we have:

Base 10	10^5	10^4	10^3	10^2	10^1	10^0
Base 2	2^5	2^4	2^3	2^2	2^1	2^0
Base 8	8^5	8^4	8^3	8^2	8^1	8^0
Base 16	16^5	16^4	16^3	16^2	16^1	16^0

Binary System

As already discussed, since the Base is 2, the highest value digit is 1. So in Base 2 only the digits 0 and 1 are used.

Conversion of Base 10 (Denary) to Binary

To do this we repetitively divide the Base 10 number by 2, noting the remainders and reading them from the ***bottom up***.

Example Convert 45_{10} into Binary.

2) 45
2) 22 r 1
2) 11 r 0 ↑
2) 5 r 1
2) 2 r 1
2) 1 r 0 ie. 101101_2
0 r 1

Example! Convert 87_{10} to Binary

2) 87
2) 43 r1
2) 21 r1
2) 10 r1
2) 5 r0
2) 2 r1
2) 1 r0
0 r1

1010111_2

Conversion of Binary to Base 10

$\times 2^3$ $\times 2^2$ $\times 2^1$ $\times 2^0$

Example Convert 1111_2 to Base 10.

$1111_2 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 1 \times 8 + 1 \times 4 + 1 \times 2 + 1 \times 1$

$= 8 + 4 + 2 + 1$

$= 15_{10}$

Example: Convert 1010111_2 to Base 10

$1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$64 + 0 + 16 + 0 + 4 + 2 + 1 = 87_{10}$

Example: Convert 110110110011_2 to Base10

$1 \times 2^{11} + 1 \times 2^{10} + 0 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$2048 + 1024 + 0 + 256 + 128 + 0 + 32 + 16 + 0 + 0 + 2 + 1$

3507_{10}

Octal System

Since the base is 8, the highest value digit in any column is 7. So in Base 8 only the digits 0, 1, 2, 3, 4, 5, 6, 7 are used.

Conversion of Base 10 to Octal

In this case we repetitively divide the Base 10 number by 8, noting the remainder and reading them from the ***bottom up***.

Example Convert 158_{10} into octal.

$8\overline{)158}$

$8\overline{)19}$ r 6 ie. 236_8

$8\overline{)2}$ r 3 ↑

0 r 2

Example: Convert 535_{10} into Base8

$8\overline{)535}$

$8\overline{)66}$ r7 ↑ 1027_8

$8\overline{)8}$ r2

$8\overline{)1}$ r0

0 r1

Conversion of Octal to Base 10

Example

Convert 642_8 to Denary (Base 10).

$$642_8 = 6 \times 8^2 + 4 \times 8^1 + 2 \times 8^0$$

$$= 6 \times 64 + 4 \times 8 + 2 \times 1$$

$$= 384 + 32 + 2$$

$$= 418_{10}$$

Example! Convert 1027_8 into Base 10

$1 \times 8^3 + 0 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 = 512 + 0 + 16 + 7$

$= 535_{10}$

0 1 2

Since each octal digit 0–7 can be encoded in 3 binary bits: i.e. 000, 001, 010, 011, 100, 101, 110, 111 there is a simple correspondence between octal and binary. In other words, any octal number can be converted to binary and vice-versa.

3 4 5 6 7

4 3 1

So 431_8 becomes $100\ 011\ 001_2$

2 5 7 1

And 2571_8 becomes $010\ 101\ 111\ 001_2$

<u>Example</u>: 1027_8 into Binary system

$001\ 000\ 010\ 111_2$

Conversely if we want to convert a binary number into octal we split the binary numbers into groups of ***three***, reading from the right hand side and treat each group as a separate binary string.

$$111|010|110|001_2$$
$$7 \quad 2 \quad 6 \quad 1_8$$

This method also affords us another way of converting a Base 8 number into Base 10, but this time via Base 2.

For example,

$$\begin{array}{ccc} & & 4 \quad 3 \quad 1 \\ 431_8 & = & 100 \ 011 \ 001 \end{array}$$

We then read the binary part as a complete binary string:

100011001_2

$=1 \times 2^8 + 0 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 256 + 0 + 0 + 0 + 16 + 8 + 0 + 0 + 1$

$= 281_{10}$

Hexadecimal System

Conversion of Base 16 to Base 10

Since the base is 16, the highest digit in any column is 15. So in Base 16 only the digits 0–15 are used.

Example 1 $46_{16} = 4 \times 16^1 + 6 \times 16^0$

$= 64 + 6$

$= 70_{10}$

Example: Convert 479_{16} to Base 10

$479_{16} = 4 \times 16^2 + 7 \times 16^1 + 9 \times 16^0 = 1024 + 112 + 9$

$= 1145_{10}$

A=10 D=13
B=11 E=14
C=12 F=15

Example: $2AF6_{16}$ to Base 10

$2AF6_{16} = 2 \times 16^3 + 10 \times 16^2 + 15 \times 16^1 + 6 \times 16^0$

$= 8192 + 2560 + 80 + 6$

$= 10998_{10}$

Example 2 $279_{16} = 2 \times 16^2 + 7 \times 16^1 + 9 \times 16^0$

$= 2 \times 256 + 112 + 9$

$= 512 + 112 + 9$

$= 633_{10}$

However if we consider 114_{16} there is a clear-cut case of ambiguity implied in the values 10-15.

114_{16} could be $11 \times 16^1 + 4 \times 16^0$ **or** $1 \times 16^1 + 14 \times 16^0$

In order to avoid confusion number 10 is replaced with letter A, number 11 with letter B, 12 with C, 13 with D, 14 with E and 15 with F

So the digits used in Base 16 are:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
10 11 12 13 14 15

Conversion of Base 10 to Hexadecimal

This is a similar method to the other bases, we repetitively divide by 16, taking stock of the remainder and reading from the bottom up.

Example 1 Convert 342_{10} into hexadecimal.

16)342
16)21 r 6 156_{16} ↑
16)1 r 5
0 r 1

Example: 10998_{10} to Hexadecimal

16)10998
16)687 r6 ↑ $2(10)(15)6_{16}$
16)42 r15 $2AF6_{16}$
16)2 r10
0 r2

Example 2 Convert 175_{10} into hexadecimal.

16)175
16)10 r ~~15~~ F ie. AF_{16}
0 r ~~10~~ A

However since each hexadecimal digit can be encoded in 4 binary bits, i.e. (0–15 can be represented as 0000–1111) there is also a simple correspondence between hexadecimal and binary: i.e. any hexadecimal number can be converted to binary and vice versa.

0000 = 0
0001 = 1
0010 = 2
0011 = 3
0100 = 4
0101 = 5
0110 = 6
0111 = 7
1000 = 8
1001 = 9
1010 = 10 = A
1011 = 11 = B
1100 = 12 = C
1101 = 13 = D
1110 = 14 = E
1111 = 15 = F

ie. $156_{16} = \overset{1}{0001}\ \overset{5}{0101}\ \overset{6}{0110}$

And $3987_{16} = \overset{3}{0011}\ \overset{9}{1001}\ \overset{8}{1000}\ \overset{7}{0111}$

Conversely, if we want to convert a binary number into hexadecimal, we split the binary number into groups of ***four***, reading from the ***right hand side*** and treat each group as a separate binary string.

Example: Convert $2AF6_{16}$ to Binary

$\overset{2}{0010}|\overset{A}{1010}|\overset{F}{1111}|\overset{6}{0110}_2$

0111	0110	1101	1110
7	6	13	14
7	6	D	E

Once again this method affords us another way of converting a base 16 number into base 10 but this time via base 2.

For example,

$579_{16} = \overset{5}{0101}\ \overset{7}{0111}\ \overset{9}{1001}_2$

We then read the binary part as a complete binary string:

010101111001_2

$=1 \times 2^{10}+0 \times 2^9+ 1 \times 2^8+ 0 \times 2^7+ 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3+0 \times 2^2+0\times 2^1+1\times 2^0$

$=1024 + 0 + 256 + 0 + 64 + 32 + 16 + 8 + 0 +0 +1$

$=1401_{10}$

Decimal to Binary (Bicimal) Conversion

The conversion requires separating the integer (i.e. whole number part), and fractional part. The fractional part is found by successive multiplication by 2 and accumulating the digits obtained. The integer part is found by successive divisions by 2 and accumulating the remainders.

Example 1 Convert 43.5625 into bicimal.

Integers (÷ by (2))	Fractions (x by (2))
$2\overline{)\,43}$	$0.5625 \times 2 = 1.1250$
$2\overline{)\,21}$ r 1	$0.1250 \times 2 = 0.2500$
$2\overline{)\,10}$ r 1	$0.2500 \times 2 = 0.5000$
$2\overline{)\,5}$ r 0	$0.5000 \times 2 = 1.0000$
$2\overline{)\,2}$ r 1	
$2\overline{)\,1}$ r 0	
0 r 1	

Most Significant Bit

Most Significant Bit

$$43{\cdot}5625_{10} \equiv 101011{\cdot}1001_2$$

Example 2 Convert 15.875_{10} into bicimal.

Integers (÷ by (2))	Fractions (x by (2))
$2\overline{)\,15}$	$0.8750 \times 2 = 1.7500$
$2\overline{)\,7}$ r 1	$0.7500 \times 2 = 1.5000$
$2\overline{)\,3}$ r 1	$0.5000 \times 2 = 1.0000$
$2\overline{)\,1}$ r 1	
0 r 1	

Most Significant Bit

Most Significant Bit

$$15{\cdot}875_{10} \equiv 1111{\cdot}111_2$$

Note: For integer part ***read up*** (from most significant bit)

For fraction part ***read down*** (from most significant bit)

Binary (Bicimal) Conversion to Decimal

In the binary system figures to the ***right*** of the bicimal point are called ***bicimals***.

Example 1

The binary number : 1011.011 represents

2^3	2^2	2^1	2^0	·	2^{-1}	2^{-2}	2^{-3}
1	0	1	1	·	0	1	1

$=1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$

$= (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1) + (0 \times ½) + (1 \times ¼) + (1 \times ⅛)$

$= 8 + 0 + 2 + 1 + 0 + 0.25 + 0.125$

$=11.375$

Example: 110110.0010111 into decimal?

$110110.0010111_2 = 1x2^5+1x2^4+0x2^3+1x2^2+1x2^1+0x2^0+0x2^{-1}+0x2^{-2}+1x2^{-3}+0x2^{-4}+1x2^{-5}+1x2^{-6}+1x2^{-7}$

$=32+16+0+4+2+0+0+0+1/8+0+1/32+1/64+1/128$

$= 54+0.125+0.03125+0.015625+0.0078125 = \boxed{54.1796875}$

Example 2

111.01101 can be represented as:

$1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$

$= (1 \times 4) + (1 \times 2) + (1 \times 1) + (0 \times ½) + (1 \times ¼) + (1 \times ⅛) + (0 \times 1/16) + (1 \times 1/32)$

$= 4 + 2 + 1 + 0 + 0.25 + 0.125 + 0 + 0.03125$

$= 7.40625$

Arithmetical Operations with Binary Numbers

Addition

Remember from our work in the binary section, the largest value we can have in any column of a binary calculation is 1.

The columns are:

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0

a) $1 + 0 = 1_2$ (no problem here, sum does not exceed 1).

Revision

Basic Algebra

Questions

1) **Three times a certain number x plus five is equal to five times the number plus two. The correct algebraic expression for this is:**

a. $3x + 5 = 5 + x + 2$

b. $3 + x + 5 = 5x + 2$

[✓ c.] $3x + 5 = 5x + 2$

$3 \times x + 5 = 5 \times x + 2$

$3x + 5 = 5x + 2$

2) **$(3x + 5) - (x + 3)$ is equal to:**

$3x + 5 - x - 3 = 2x + 2$

[✓ a.] $2x + 2$

b. $2x - 8$

c. $2x - 2$

3) **Which of the following is NOT equal to $\frac{1}{3}xy$:**

a. $\frac{xy}{3}$

[✓ b.] $\frac{x}{3y}$

c. $\frac{1}{3}yx$

4) **If $p = 3$ and $q = 4$, then $q^2 - p^2$ is equal to:**

$q^2 - p^2 = q \times q - p \times p = 4 \times 4 - 3 \times 3 = 16 - 9 = 7$

[✓ a.] 7

b. -7

c. 1

5) When the brackets are removed form 2(3p - 2q) the answer is:

a. 23p - 22q

[✓ b.] 6p - 4q

c. 6p + 4q

$2(3p-2q) = 2\times 3p - 2\times 2q = 6p - 4q$

6) When 5x + 3x - 4x is simplified, the answer is:

a. 12x

[✓ b.] 4x

c. -4x

$\underline{5x+3x}-4x = 8x-4x = 4x$

7) When $8a^2$ is divided by $2a^2$, the answer is:

a. $4a^2$

b. 4a

[✓ c.] 4

$\frac{8\cancel{a^2}}{2\cancel{a^2}} = 4$

8) When 4x + 8y is *completely* factorised, the correct answer is:

a. 4 (x + y)

[✓ b.] 4 (x + 2y)

c. 4(2x + 2y)

$4x+8y = 2\times 2\times x + 2\times 2\times 2\times y$

$= 4\times x + 4\times 2y$

$= 4\times(x+2y) = \boxed{4(x+2y}$

9) When the equation 3x - 8 = 19 is solved, the value of x is:

a. 8

b. 7

[✓ c.] 9

$3x-8 = 19$

$3x\cancel{-8}\cancel{+8} = 19+8$

$3x = 27$

$x = \frac{27}{3} = \boxed{9}$

10) When (x + 5)(x + 2) is expanded and simplified, the answer is:

a. $2x + 7$

b. $x^2 + 10$

[✓ c.] $x^2 + 7x + 10$

$$(x+5)(x+2) = x^2+\underline{2x+5x}+10 = \boxed{x^2+7x+10}$$

11) When c is made the subject of the formula $\frac{bc^2}{d}$ = a, the answer is:

a. $c = \frac{da}{b}$

b. $c = \sqrt{\frac{a-d}{b}}$

[✓ c.] $c = \sqrt{\frac{ad}{b}}$

$$\frac{bc^2}{d} = a \Rightarrow \frac{bc^2}{d} \times \frac{a}{1}$$

$$\boxed{\sqrt{x^2} = x}$$

$$bc^2 = ad$$

$$\frac{\cancel{b}c^2}{\cancel{b}} = \frac{ad}{b}$$

$$c^2 = \frac{ad}{b}$$

$$\sqrt{c^2} = \sqrt{\frac{ad}{b}} \Rightarrow \boxed{c = \sqrt{\frac{ad}{b}}}$$

12) When $x^2 - 9$ is factorised, the answer is:

a. $(x - 3)$

b. $(x - 3)(x - 3) = x^2 - 3x - 3x + 9 = x^2 - 6x + 9 \neq x^2 - 9$

[✓ c.] $(x - 3)(x + 3) = x^2 + \cancel{3x} - \cancel{3x} - 9 = x^2 - 9$ ✓

13) Log $\frac{a}{b}$ is exactly the same as:

a. log (a - b)

b. log a - log b

c. $\frac{\log a}{\log b}$

14) If $\log_{10} 2 = 0.3010$, then $\log_{10} 2000$ is:

- a. 2.3010
- b. 3.3010
- c. 301

15) 58_{10} expressed in Binary is:

- a. 111001
- b. 111010
- c. 101111

16) In common logarithmic form the characteristic of the number 0.002968 is:

- a. $\overline{2}$
- b. $\overline{3}$
- c. 2.968

17) The value of x in $\log_5 125 = x$ is:

- a. 5
- b. 25
- c. 3

18) The binary number 1110111_2 expressed as a Base_{10} i.e. Denary number is:

- a. 121
- b. 119
- c. 117

19) When the Octal (Base 8) numbers 37 and 42 are added together, the answer is:

a. 79_8

b. 101_8

c. 110_8

20) When the Hexadecimal (Base 16) numbers 35_{16} and 48_{16} are added, the correct answer is:

a. 83_{16}

b. 713

c. 7D

21) The decimal value 625 coded in BCD is:

a. 0110 0010 0100

b. 0110 0010 0101

c. 0110 0010 0111

22) In the simultaneous equations x + y = 6, x - y = 2. If y = 2 then x is:

a. 3

b. 5

c. 4

23) When $x^2 - 2x - 8$ is factorised the correct equation is:

a. $(x - 2)(x - 4)$ $= x^2 - 4x - 2x + 8 = x^2 - 6x + 8$

b. $(x + 2)(x + 4)$ $= x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

[✓ c.] $(x + 2)(x - 4)$ $= x^2 - 4x + 2x - 8 = x^2 - 2x - 8$

24) The logarithm of two numbers multiplied together may be found by:

a. Adding the logarithms of the two numbers together

b. Multiplying the logarithms together

c. Subtracting the logarithms of the two numbers

25) $a^5 \times a^6$ is equal to:

$a^5 x a^6 = a^{5+6} = a^{11}$

a. a^{30}

[✓ b.] a^{11}

c. a^{15}

26) $\frac{x^9}{x^2}$ is equal to:

$$\frac{x^9}{x^2} = x^{9-2} = x^7$$

a. x^{18}

b. x^{11}

[✓ c.] x^7

27) $\frac{1}{\sqrt{x}}$ is equal to:

$$\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$$

a. $x^{½}$

[✓ b.] $x^{-½}$

c. x^2

28) 125^0 is equal to:

$a^0 = 1$

a. 125^0

b. δ

[✓ c.] 1

29) $\frac{1}{9^{1/2}}$ **is equal to:**

$$\frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

or $\frac{1}{9^{1/2}} = \frac{1}{(3^2)^{1/2}} = \frac{1}{3^{2\times\frac{1}{2}}} = \frac{1}{3}$

$(x^a)^b = x^{a\times b}$

[✓ a.] $\frac{1}{3}$

b. $\frac{1}{4.5}$

c. 81

Revision

Basic Algebra

Answers

1.	**C**	16.	**B**
2.	**A**	17.	**C**
3.	**B**	18.	**B**
4.	**A**	19.	**B**
5.	**B**	20.	**C**
6.	**B**	21.	**B**
7.	**C**	22.	**C**
8.	**B**	23.	**C**
9.	**C**	24.	**A**
10.	**C**	25.	**B**
11.	**C**	26.	**C**
12.	**C**	27.	**B**
13.	**B**	28.	**C**
14.	**B**	29.	**A**
15.	**B**		