

Arithmetic

Introduction

Arithmetic is the art of reckoning by figures and put simply, literally the art of counting. The word comes from the Greek *arithmetike*, which combines the ideas of two words; *arithmos*, meaning ‘number’, and *techné*, which refers to an art or skill. Numbers used for counting are called the *positive integers* and are generated by adding ‘1’ to each number in an endless series.

Numbers

Before we go into the detail of arithmetic, it is important that we define some of the components we are going to use.

A *positive number* is one that has a value greater than zero. For example, 7, 15, 356. In reality they have a (+) sign in front of them (+7, +15, +356) but this is not normally shown.

A *negative number* is one that has a value less than zero:

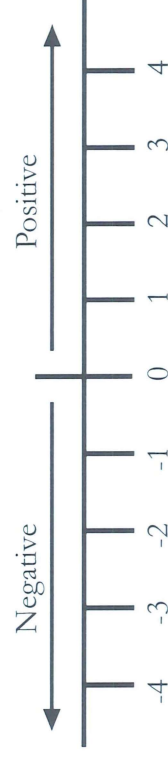
For example, -3, -15, -276. They always have a (-) sign in front of them.

We meet negative numbers during winter, when the temperature falls below freezing point (0°C). For example, -5°C.

Directed Numbers are numbers which have a (+) or (-) sign attached to them. They can be represented on a number line (similar to the x-axis of a graph)

The numbers to the left of 0 are negative and those to the right of 0 are positive.

From the number line it can be seen that -4 is lower in value than -1.



Negative numbers decrease in value the further to the left they are.

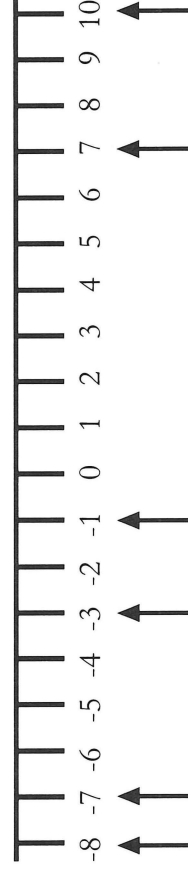
Positive numbers however increase in value, the further to the right they are.

Example

Rearrange the numbers 7, -3, 10, -8, -1, -7 in order of size starting with the smallest.

If we place these on a number line we have:

As Given: 7, -3, 10, -8, -1, -7



In order: -8, -7, -3, -1, 7, 10

The order is: -8, -7, -3, -1, 7, 10.

This is too time consuming however. An easier approach is to realise that the bigger the number that has a (-) sign in front of it, the *least value* it has.

For example, -15 is less than -7

The reverse of course is true for those numbers with (+) signs in front of them.

For example, +15 is greater than +7.

Addition and Subtraction of Directed Numbers

The arithmetic needed to be able to carry out these operations can be summarised by the following rules.

- To add together two negative numbers then *add* the numbers and put a negative sign in front of the answer.

For example, $-5 + (-2) = -7$

$$-2 + (-5) = -7$$



Example: Rearrange the integers in size

35, -2, 0, 5, -21, -72, 252, 5

Solution: -72, -21, -2, 0, 5, 5, 35, 252

b) To add a positive number and a negative number subtract the smaller number from the bigger one and put the sign of the bigger number in front of the answer.

For example, $3 + (-6) = -(6 - 3) = -3$ or $-7 + 2 = -(7 - 2) = -5$

$$-6 + 3 = -3$$

$$-6 + 3 = 3 - 6$$

$$2 + (-7) = 2 - 7$$

$$2 + (-7) = -5$$

$$\underline{2 - 7}$$



$$4 - (-5) = 4 + 5 = 9$$

Example. Find the solution of the following multiplication product

$$12 \times (-1) \times (3) \times (-2) \times 7 \times (-6) \times (-2) = 6048$$

b) To add a positive number and a negative number subtract the smaller number from the bigger one and put the sign of the bigger number in front of the answer.

For example, $3 + (-6) = -(6 - 3) = -3$ or $-7 + 2 = -(7 - 2) = -5$

c) To subtract a negative number, then say to yourself -- gives +

$$\begin{array}{r} 4(-)5 \\ = 4(-)5 \\ + \\ = 9 \end{array}$$

If you find in practice that these rules are confusing then another useful method is to think of a (+) sign as 'having' and a (-) as 'owing'.

Look at these examples:

-5 - 2 I owe 5 and I owe 2, so I owe 7 altogether. So, write down -7.

-7 + 3 I owe 7 but I have 3, therefore I still owe 4. So, write down -4.

-6 - (-2) I owe 6, but I have 2 (because of the - -) so I still owe 4.

So, write down -4.

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Multiplication and Division of Directed Numbers

Multiplication

The rules for multiplication of **two** numbers are:

like signs give (+) **unlike signs give (-)**

However these rules **do not apply** if you have **three numbers**.

For example, $(-3) \times (-3) \times (-3) = -27$ i.e.: the three like signs do not give (+)

So a better method to find the sign when multiplying two or more numbers is to **count the number of negative signs**.

If the answer is **odd** the overall answer is (-).

If the answer is **even** the overall answer is (+).

Example 1 Find the product of $(-4) \times (-5) \times 2$

Here we have an **even** number of $(-)$, so the overall answer is $(+)$.

$$(-4) \times (-5) \times 2 = 40$$

Example 2 Find the product of $(-2) \times (-3) \times (-4)$

Here we have an **odd** number of $(-)$, so the overall answer is $(-)$

$$(-2) \times (-3) \times (-4) = -24$$

Example 3 Find the product of $3 \times (-4) \times (-2) \times 6$

Here we have an **even** number of $(-)$, so the overall answer is $(+)$

$$3 \times (-4) \times (-2) \times 6 = 144$$

Division

When dividing **two** numbers, **like signs** give $(+)$, **unlike signs** give $(-)$

$$\begin{array}{lcl} \frac{(+)}{(+)} = (+) & \text{e.g.} & \frac{15}{5} = 3 \\ \frac{(+)}{(-)} = (-) & \text{e.g.} & \frac{6}{-2} = -3 \\ \frac{(-)}{(+)} = (-) & \text{e.g.} & \frac{-8}{2} = -4 \\ \frac{(-)}{(-)} = (+) & \text{e.g.} & \frac{-10}{-2} = 5 \end{array}$$

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But, be careful when you have mixed multiplication and division:

$$\begin{array}{lcl} \frac{(-3) \times (-4) \times (-2)}{(-2) \times 3} = \frac{-24}{-6} & \text{(From } \times \text{ rule)} & \\ & = 4 & \text{(From } \div \text{ rule)} \\ \frac{4 \times (-6) \times (-8)}{(-2) \times (-8) \times (-2)} = \frac{+192}{-32} & \text{(From } \times \text{ rule)} & = -6 \\ & = -6 & \text{(From } \div \text{ rule)} \\ \frac{(-16)}{(-4) \times (-2)} = \frac{-16}{+8} & \text{(From } \times \text{ rule)} & = -2 \\ & = -2 & \text{(From } \div \text{ rule)} \end{array}$$

$$\frac{-24}{-6} = \frac{24}{6} = 4$$

$$= -6$$

$$= -2$$

Useful Definitions with Examples

Even Numbers

Any number that ends in 0, 2, 4, 6, or 8 is called an *even number*.

e.g. 20, 22, 24, 26, 28, 30 and so on **356, 256 355 604**

Odd Numbers

Any number that ends in 1, 3, 5, 7 or 9 is called an *odd number*.

e.g. 21, 23, 25, 27, 29 31 and so on

If we divide any number by 2 and there is no remainder, then the number is *even*. If the remainder is 1, then the number is *odd*.

Squares of Numbers

When *any* number is multiplied by itself, take for example, $3 \times 3 = 9$, then this can be shortened to $3^2 = 9$.

The 'little' 2, which tells us the number of 3's that have been multiplied together is called the *power of 2* (or *index*).

Another name for the *power of 2* is the *square*.

For example, the value of 4 squared is $4^2 = 4 \times 4 = 16$

the value of 5 squared is $5^2 = 5 \times 5 = 25$

Cubes of Numbers

If *three* identical numbers are multiplied together, for example, $3 \times 3 \times 3 = 27$ then this can be shortened to $3^3 = 27$

The 'little' 3, in this case, is called *the power of 3*. Another name for the power of 3 is the *cube*.

For example: The value of 4 cubed is $4^3 = 4 \times 4 \times 4 = 64$

The value of 5 cubed is $5^3 = 5 \times 5 \times 5 = 125$

Squaring a number and cubing a number is not the end of the story, as *any number can be raised to any power*, but powers above 3, do not have special names.

For example, 2 to the power of 5 is $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

And 3 to the power of 4 is $3^4 = 3 \times 3 \times 3 \times 3 = 81$

Square Root of a Number

If the square of 7, i.e. $(7 \times 7) = 49$, then 7 is called the **square root** of 49. This is written as $\sqrt{49} = 7$. More often than not, the 'little' 2 is omitted, so be aware that $\sqrt{49}$ is the same as $\sqrt[2]{49}$.

So the square root of a number is another number that when multiplied by itself gives the first number. When finding the square root of a number it should be noted that we write: $\sqrt{36} = \pm 6$

Since $6 \times 6 = 36$ and also $(-6) \times (-6) = 36$

Cube Roots of Numbers

If the cube of 6 i.e.: $(6 \times 6 \times 6) = 216$, then 6 is called the **cube root** of 216

This is written as $\sqrt[3]{216} = 6$.

So the cube root of a number is another number which when cubed gives the first number.

Arithmetical Operations

The numbering system we use in everyday life for calculations is called **Base 10** or **Denary**. This relates also to our decimal system of money and the metric system of measure, which also use multiples of 10.

In Base 10, we use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. When adding or multiplying, once we get above 9 we write down the right hand digit and 'carry' the left hand digit to the next column, which is a factor of 10 more.

Hopefully you are familiar with the columns headed Thousands, Hundreds, Tens and Units under which we arrange the digits of large numbers.

For example, 16983 can be written as:

Tens of Thousands	Thousands	Hundreds	Tens	Units
1	6	9	8	3

Addition

When adding numbers together, the figures are placed in columns making sure that units are under units, tens under tens, hundreds under hundreds and so on. Have a look at the following examples.

Example 1

Add together 5418, 613, 29, 156

$$\begin{array}{r} 5418 \\ 613 \\ 29 \\ + 156 \\ \hline 6216 \end{array}$$

Adding the units together $8 + 3 + 9 + 6$ we get 26, so we put down 6 and carry 2 to the tens column, where it is added to

$$5 + 2 + 1 + 1 + (2) = 11$$

Here we put 1 down in the tens column and carry the 1 to the hundreds column where it is added to

$$1 + 6 + 4 + (1) = 12$$

Here we put down 2 in the hundreds column and carry the 1 to the thousands column where it is added to. $5 + (1) = 6$.

Put down the 6.

Answer: 6216.

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Th	H	T	U
5	4	1	8
	6	1	3
		2	9
	+	1	5
			6
6	2	1	6
1	1	2	

Note: These column headings will be omitted in further examples so make sure you know them!

Example 2

Add together 54683, 201846, 3298 and 163219

As before, place units under units, tens under tens, hundreds under hundreds, thousands under thousands, ten thousands under ten thousands etc.

$$\begin{array}{r} 163219 \\ 201846 \\ 54683 \\ + 3298 \\ \hline 423046 \end{array}$$

Adding the units we get 26, so we put down 6 and carry the 2 to the next column where it is added to.

$$1 + 9 + 4 + 8 + (2) = 24$$

We put down the 4 and carry the 2 to the next column where it is added to:

$$6 + 8 + 2 + 2 + (2) = 20$$

There we put down 0 and carry the 2 to the next column, and so on.

i.e. we continue with this method until we complete the operation and arrive at the answer, which in this case is 423046.

$$\begin{array}{r} 5\ 4\ 6\ 8\ 3 \\ 2\ 0\ 1\ 8\ 4\ 6 \\ 3\ 2\ 9\ 8 \\ +\ 1\ 6\ 3\ 2\ 1\ 9 \\ \hline 4\ 2\ 3\ 0\ 4\ 6 \\ \hline 1\ 1\ 2\ 2\ 2 \end{array}$$

Subtraction

Example 1

Subtract 3241 from 5685

- Taking 1 from 5 in the unit's column we get 4, which is put down.
- In the tens column we take 4 from 8 to get 4 and put this down.
- In the hundreds column we take 2 from 6 to get 4 and put this down.
- Finally we take 3 from 5 to get 2 and put this down to give the answer 2444.

$$\begin{array}{r} 5\ 6\ 8\ 5 \\ -\ 3\ 2\ 4\ 1 \\ \hline 2\ 4\ 4\ 4 \end{array}$$

Subtract 5685 from 3241

$$\begin{array}{r} 3241 \\ -\ 5685 \\ \hline \end{array}$$

Example 2

Subtract 1268 from 4517

$$\begin{array}{r} 4517 \\ -\ 1268 \\ \hline 3249 \end{array}$$

- In the units column $7 - 8$ cannot be done, so we change one of the tens in the tens column of the top number into ten units, i.e. we 'decompose' it just like changing a ten pence piece into ten pennies. This leaves 0 tens in the tens column and we add the ten units to the units column:-

i.e. $10 + 7 = 17$ in the units column and then $17 - 8 = 9$. This is put down as the answer in the units column.

$$\begin{array}{r} 4 \swarrow 10 \searrow 17 \\ 4 \cancel{8} \\ - 1 2 6 8 \\ \hline 3 2 4 9 \end{array}$$

- In the tens column, we now have $0 - 6$ which cannot be done, so we decompose one of the hundreds in the hundreds column into ten tens. This leaves 4 hundreds in the hundreds column and we add the ten tens to the ten column. i.e.: $10 + 0 = 10$ in the tens column and then $10 - 6 = 4$. This is put down as the answer in the tens column.
- In the hundreds column, we now have $4 - 2 = 2$ which is put down as the answer in the hundreds column.
- In the thousands column, we now have $4 - 1 = 3$ and this is put down in the thousands column giving a final overall answer of 3249.

Example 3

Subtract 1724 from 4012

- In the units column, $2 - 4$ cannot be done so we decompose the ten in the tens column and add it to the units column. i.e.: $10 + 2 = 12$ and then $12 - 4 = 8$, which is put down as the answer.
- In the tens column we now have $0 - 2$ which cannot be done, so we decompose one of the hundreds. The problem is we do not have any hundreds, so we decompose one of the thousands. This leaves 3 thousands, and the 10 hundreds go into the hundreds column. This becomes 9 when we transfer one to the tens column. In the tens column we now have:
 $10 + 0 = 10$ and then $10 - 2 = 8$, which is put down as the answer.

$$\begin{array}{r} 3 \swarrow 9 \searrow 10 \searrow 2 \\ 4 \cancel{0} \\ - 1 7 2 4 \\ \hline 2 2 8 8 \end{array}$$

- We now take the 7 from the 9 hundreds in the hundreds column to give $9 - 7 = 2$, which is put down as the answer.
- Finally in the thousands column $3 - 1 = 2$, which is put down giving a final answer of 2288.

Multiplication

Example 1

Multiply 327 by 8

Step 1

8 times 7 is equal to 56. So we put the 6 down in the units column and carry the 5 to the tens column

Step 2

8 times 2 is equal to 16, plus the 5 carried from the units column is equal to 21. So we put down the 1 in the tens column and carry the 2 to the hundreds column.

$$\begin{array}{r} 327 \\ \times 8 \\ \hline 2616 \\ \hline 25 \end{array}$$

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Step 3

8 times 3 is equal to 24, plus the 2 carried from the tens column is equal to 26.

Step 4

Since the multiplication is now complete we put the 26 down as shown in the figure to give the answer 2616.

Example 2

Multiply 456 by 25

Step 1 5 times 6 is equal to 30. So we put the 0 down in the units column and carry the 3 to the tens column.

Step 2 5 times 5 is equal to 25, plus the 3 carried from the units column is equal to 28. So we put down the 8 in the tens column and carry the 2 to the hundreds column.

$$\begin{array}{r}
 456 \\
 \times 25 \\
 \hline
 2280 \\
 23 \\
 9120 \\
 11 \\
 11400 \\
 11
 \end{array}$$

Line 1
This is Obtained by
multiplying 456 by 5.

Line 2
For this line put down
0 and multiply by 2.
(You are really
multiplying by 20.)

Add the two lines
together to give the
answer of 11400.

Step 3 5 times 4 is equal to 20, plus the 2 carried from the tens column is equal to 22.

Step 4 Since the first part of the multiplication is now complete we put down the 22 as shown to give 2280 on line 1.

Step 5 Next we are going to multiply by the 2 which is really 20. So we put down a 0 to compensate and multiply by 2. 2 times 6 is equal to 12, so we put down the 2 in the tens column and carry the 1 to the hundreds column as shown.

Step 6 2 times 5 is equal to 10, plus the 1 carried from the tens column is equal to 11. So we put down the 1 in the hundreds column and carry the other 1 to the thousands column.

Step 7 2 times 4 is equal to 8, plus the 1 carried from the hundreds column is equal to 9.

Step 8 Since the second part of the multiplication is now complete we put down the 9 as shown to give 9120 on line 2.

Step 9 Finally we add lines 1 and 2 together to give the answer 11400.

Example 3

Multiply 327 by 128

Example 4:

$$\begin{array}{r}
 3251 \\
 \times 221 \\
 \hline
 3251 \\
 65020 \\
 +650200 \\
 \hline
 718471
 \end{array}$$

$$\begin{array}{r}
 327 \\
 \times 128 \\
 \hline
 2616 \\
 225 \\
 6540 \\
 1 \\
 32700 \\
 41856 \\
 11
 \end{array}$$

This is Obtained by
multiplying 327 by 8.

For this line put down
0 and multiply by 2.
(You are really
multiplying by 20.)

For this line put down
00 and multiply by 1.
(You are really
multiplying by 100.)

Add the three lines
together to give the
answer of 41856.

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Division

When we divide 76362 by 13, the answer is 5874.

76362 is called the dividend. 13 is called the divisor.

5874 is called the quotient.

$$\begin{array}{r} \text{QUOTIENT} \\ \text{DIVISOR} \overline{) \text{DIVIDEND}} \end{array}$$

Example 1

Divide 9522 by 6

Step 1 6 into 9 goes once. Put 1 in the quotient position above 9 and put the 6 under the 9.

Step 2 Subtract 6 from 9 to find the remainder 3.

Step 3 Bring down the next figure in the dividend 5 and put it alongside the 3, as shown in the figure.

$$\begin{array}{r} 1\ 5\ 8\ 7 \\ 6 \overline{) 9\ 5\ 2\ 2} \\ \underline{6} \\ 3\ 5 \\ \underline{3\ 0} \\ 5\ 2 \\ \underline{4\ 8} \\ 4\ 2 \\ \underline{4\ 2} \\ 0 \end{array}$$

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Step 4 6 into 35 goes 5 times ($5 \times 6 = 30$). Put the 5 in the quotient position alongside the 1.

Step 5 Subtract 30 from 35 to give the remainder 5. Bring down the next figure in the dividend 2 and put it alongside the 5.

Step 6 6 into 52 goes 8 times ($8 \times 6 = 48$). Put the 8 in the quotient position alongside the 5.

Step 7 Subtract 48 from 52 to give the remainder 4. Bring down the next figure in the dividend 2 and put it alongside the 4.

Step 8 6 into 42 goes 7 times exactly. ($7 \times 6 = 42$)

Put the 7 in the quotient position alongside the 8 and put 42 under 42.

Subtraction gives no remainder as expected so the division is complete

Example 2

73392 divide by 16

16 into 7 will not go. 16 into 73 goes 4 times. ($16 \times 4 = 64$).

Put the 4 in the quotient position and put the 64 under the 73 to give remainder 9.

Bring down the next figure 3 in the dividend and put it alongside the 9 to get 93.

16 into 93 will go 5 times ($16 \times 5 = 80$).

Put the 5 in the quotient position alongside the 4 and put the 80 under the 93 to give remainder 13.

Bring down the next figure 9 in the dividend and put it alongside the 13 to get 139.

16 into 139 will go 8 times ($16 \times 8 = 128$).

Put the 8 in the quotient position alongside the 5 and put 128 under 139 to get remainder 11.

Bring down the last figure 2 from the dividend and put it alongside the 11 to give 112.

16 into 112 will go 7 times exactly. ($16 \times 7 = 112$).

$$\begin{array}{r}
 4587 \\
 16 \overline{) 73392} \\
 \underline{64} \\
 93 \\
 \underline{80} \\
 139 \\
 \underline{128} \\
 112 \\
 \underline{112} \\
 0
 \end{array}$$

Put the 7 in the quotient position alongside the 8, and put the 112 under 112.

Subtraction gives no remainder as expected so the division is complete.

The Decimal System

The number 888.888 means:

$$\begin{array}{r} 800 \\ 80 \\ + 8 \\ \hline 888 \end{array}$$

$$\begin{array}{r} 0.800 \\ 0.080 \\ + 0.008 \\ \hline 0.888 \end{array}$$

$$\begin{array}{r} 888.000 \\ + 0.888 \\ \hline 888.888 \end{array}$$

$$800 + 80 + 8 + \frac{8}{10} + \frac{8}{100} + \frac{8}{1000}$$

$$0.8 \quad 0.08 \quad 0.008$$

This is an extension of the *Denary* (Base 10) system where reading from left to right each figure 8 is ten times the value of the next one. The whole numbers and the fractional parts are separated by the decimal point.

$$\frac{8}{10} \quad \text{means} \quad 8 \div 10 \quad \text{and can be written as } 0.8$$

$$\frac{8}{100} \quad \text{means} \quad 8 \div 100 \quad \text{and can be written as } 0.08$$

$$\frac{8}{1000} \quad \text{means} \quad 8 \div 1000 \quad \text{and can be written as } 0.008$$

So the number 888.888 is:

$$\begin{array}{r} 800.0 \\ 80.0 \\ 8.0 \\ 0.8 \\ 0.08 \\ + 0.008 \\ \hline 888.888 \end{array}$$

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When no whole numbers are involved we write a zero in front of the decimal point.

For example, .8 is written as 0.8 and .08 as 0.08

When we wrote a number such as 507 previously, the zero had to be included to keep the place for the missing tens.

$$\begin{array}{r} 8 \\ 10 \\ + 8 \\ \hline 18 \end{array}$$

$$\frac{8}{10} + \frac{8}{100} = 0.808$$

$$0.8 \quad 0.008$$

The zero this time keeping the place for the missing hundredths.

Example 1

Write down the place value of the figure 7 in each of the following:

$$40 + 6 + \frac{8}{10} + \frac{7}{100} \rightarrow 40 + \frac{6}{10} + \frac{2}{100} + \frac{7}{1000}$$

a) 46.87

b) 3.75

c) 0.627

$$3 + \frac{7}{10} + \frac{5}{100}$$

a) The place value is 7 hundredths

$$\text{i.e. } \frac{7}{100}$$

b) The place value is 7 tenths

$$\text{i.e. } \frac{7}{10}$$

c) The place value is 7 thousandths

$$\text{i.e. } \frac{7}{1000}$$

Example 2

Change the following fractions to decimals and then add them together.

$$\text{a) } \frac{7}{10} + \frac{8}{100} = \frac{0.70}{+0.08} = 0.78$$

$$\begin{array}{r} 0.7 \\ + 0.08 \\ \hline 0.78 \end{array}$$

$$\frac{7}{10} = 7 \text{ tenths} = 0.7$$

$$\frac{8}{100} = 8 \text{ hundredths} = 0.08$$

$$\text{b) } \frac{3}{100} + \frac{7}{1000} = \frac{0.03}{+0.007} = 0.037$$

$$\begin{array}{r} 0.03 \\ + 0.007 \\ \hline 0.037 \end{array}$$

$$\frac{3}{100} = 3 \text{ hundredths} = 0.03$$

$$\frac{7}{1000} = 7 \text{ thousandths} = 0.007$$

$$\text{c) } \frac{9}{10} + \frac{5}{1000}$$

$$\begin{array}{r} 0.900 \\ + 0.005 \\ \hline 0.905 \end{array}$$

$$\frac{9}{10} = 9 \text{ tenths} = 0.9$$

$$\frac{5}{1000} = 5 \text{ thousandths} = 0.005$$

Addition and Subtraction of Decimals

Addition of decimals is carried out in exactly the same way as for whole numbers, but care must be taken to see that the **decimal points are placed underneath each other**. This ensures that figures having the same place value are in the same column.

Example 1 Add together 17.58, 3.06 and 0.257

$$\begin{array}{r} 17.58 \\ 3.06 \\ +0.257 \\ \hline 20.897 \end{array}$$

Example 2 Add together $186.2 + 0.685 + 13.21$

$$\begin{array}{r} 186.200 \\ 0.685 \\ +13.210 \\ \hline 200.095 \end{array}$$

For subtraction of decimals again make sure once again that the decimal points are placed directly underneath each other.

Example 1 Simplify $29.825 - 13.278$

$$\begin{array}{r} 29.825 \\ -13.278 \\ \hline 16.547 \end{array}$$

Example 2 Subtract 0.798 from 3.265

$$\begin{array}{r} 3.265 \\ -0.798 \\ \hline 2.467 \end{array}$$

Example 3 Calculate the difference between 37.685 and 13.293.

$$\begin{array}{r} 37.685 \\ -13.293 \\ \hline 24.392 \end{array}$$

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Multiplication of Decimals

The method is:

Step 1 Ignore the decimal points in the two numbers to be multiplied and multiply them as though they were whole numbers.

Step 2 Count the total number of figures after the decimal points in the number doing the multiplication and the number being multiplied.

Step 3 Make sure that the decimal point in the answer is placed so that the number of figures after the point is equal to the total found in (2) above.

Example 1 Multiply 13.58 by 3.5

1358 is divided by 100
35 is divided by 10

$$100 \times 10 = 1000$$

$$\begin{array}{r} 13.58 \quad \leftarrow 2 \text{ Figures} \\ \times 3.5 \quad \leftarrow 1 \text{ Figure} \\ \hline 6790 \quad \leftarrow 3 \text{ Figures} \\ 124 \quad \leftarrow \text{Total} \\ 40740 \\ 1112 \\ \hline 47.530 \end{array}$$

$$\frac{47510}{1000} = 47.510$$

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Example 2

Find the value of 18.69×8.63

100 → 100

1869
× 863
1612947

100 × 100 = 10000

$$\begin{array}{r} 18.69 \quad \leftarrow 2 \text{ Figures} \\ \times 8.63 \quad \leftarrow 2 \text{ Figures} \\ \hline 5607 \quad \leftarrow 4 \text{ Figures} \\ 222 \quad \leftarrow \text{Total} \\ 112140 \\ 1545 \\ 1495200 \\ 1657 \\ \hline 161.2947 \\ 11 \end{array}$$

Division of Decimals

Consider $150 \div 5 = 30$.

~~3~~ ~~3~~

If we multiply the divisor and the dividend by the same number say 3, we obtain $450 \div 15 = 30$, so the answer is the same.

We make use of this procedure when we divide decimals.

If the divisor is a decimal number, we change it into a whole number by moving the decimal point to the right. If we do this, the decimal point in *the dividend* must be moved the same number of places to the right. We then proceed as for ordinary division.

Example 1 Divide 31.15 by 3.5 $\rightarrow 31.15 \times 100 = 3115$
 $3.5 \times 100 = 350$

Move the decimal point in 3.5 one place right to give 35. This is the same as multiplying by 10.

Now move the decimal point in 31.15 the same number of places (one place to the right) to give 311.5.

We now have $311.5 \div 35$ and we proceed as in whole number division.

35 into 3 will not go. 35 into 31 will not go.

35 into 311 goes 8 times ($8 \times 35 = 280$). Put the 8 in the quotient position and the 280 under 311 to give a remainder 31.

Bring down the 5 alongside 31 to give 315. Since 5 lies to the right of the point in the dividend, insert a point in the quotient.

35 into 315 goes 9 exactly. Put the 9 in the quotient position.

$$\begin{array}{r} 8.9 \\ 35 \overline{) 311.5} \\ \underline{280} \\ 315 \\ \underline{315} \\ 0 \end{array}$$

$$\begin{array}{r} 31.15 \\ 3.5 \overline{) 31.15} \\ \underline{28.0} \\ 3.15 \\ \underline{3.15} \\ 0 \end{array}$$

31.15 = 8.9
3.5

315
35
9
→ remainder
→ 315 x 10

Since there is no remainder, this is the final answer:

i.e. $31.15 \div 35 = 8.9$

$$38.7296 \times 10000 = 387296$$

$$2.47 \times 10000 = 24700$$

Example 2

$$\frac{387296}{247} = \frac{387296}{24700}$$

Find the value of $38.7296 \div 2.47$

As before: $38.7296 \div 2.47$ becomes: $3872.96 \div 247$

i.e. By moving the decimal point in both numbers two places to the right.

$$\begin{array}{r} 15.68 \\ 247 \overline{) 3872.96} \\ \underline{-24700} \\ 140296 \\ \underline{123500} \\ 167960 \\ \underline{167960} \\ 0 \end{array}$$

remainder

$$\begin{array}{r} 15.68 \\ 247 \overline{) 3872.96} \\ \underline{247} \\ 1402 \\ \underline{1235} \\ 1679 \\ \underline{1482} \\ 1976 \\ \underline{1976} \\ \dots \end{array}$$

247 into 3 will not go. 247 into 38 will not go. 247 into 387 goes once.

Put 1 in the quotient position and the 247 under the 387 to give a remainder 140

Bring down the 2 alongside the 140 to give 1402.

247 into 1402 goes 5 times ($5 \times 247 = 1235$).

Put the 5 in the quotient position alongside the 1 and put 1235 under 1402 to give a remainder 167.

Bring down the 9 alongside the 167 to give 1679 and since the 9 lies to the right of the point in the dividend, insert a point in the quotient.

247 into 1679 goes 6 times ($6 \times 247 = 1482$).

Put 6 alongside the point in the quotient and take 1482 from 1679 to give a remainder 197.

Finally bring the 6 down alongside the 197 to give 1976.

247 into 1976 goes 8 times exactly.

Put this 8 alongside the 6 in the quotient and 1976 under 1976 to give no remainder so the division is complete.

i.e. $38.7296 \div 2.47 = 15.68$

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Factors

A factor of a number is another number that will divide into the first number without leaving a remainder.

Example

5 is a factor of 15 because $15 \div 5 = 3$ No remainder

3 is a factor of 12 because $12 \div 3 = 4$ No remainder

but

3 is not a factor of 16 because $16 \div 3 = 5$ Remainder 1

2 is not a factor of 21 because $21 \div 2 = 10$ Remainder 1

Method to find the Factors of a Number

Example 1 Find the factors of 60.

As 1 is a factor of every number then start with the obvious (1×60) and then proceed to see if 2,3,4, etc. will go into 60.

$$1 \times 60 = 60$$

$$4 \times 15 = 60$$

$$2 \times 30 = 60$$

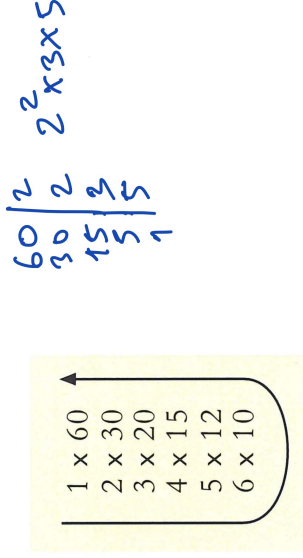
$$5 \times 12 = 60$$

$$3 \times 20 = 60$$

$$6 \times 10 = 60$$

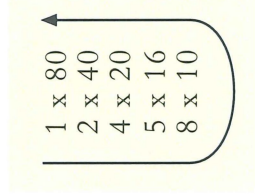
Now 7, 8 and 9 will not go into 60 exactly, but 10 does, but we already have this, so stop the proceedings.

So we now read from 1 as shown by the curved arrow.



The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60

Example 2 Find the factors of 80



The factors of 80 are: 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80.

Multiples

These are just the “*times table*” of numbers.

Multiples of 3 are: 3, 6, 9, 12, 15, 18, 21,

Multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40,

Method to find the highest common factor (HCF)

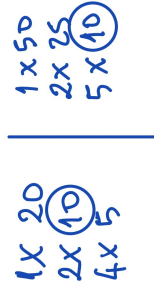
Example Find the HCF of 60 and 80

The factors that are ringed are common to both 60 and 80.

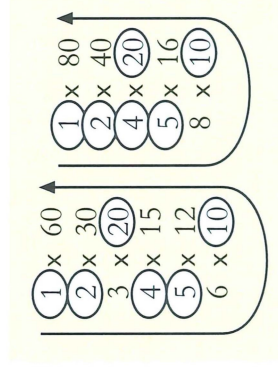
The highest of these is 20.

So the HCF of 60 and 80 is 20.

Find the HCF of 50 and 20



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Method to find the Lowest Common Multiple (LCM)

Example Find the LCM of 3 and 5

Multiples of 3: 3, 6, 9, 12, (15), 18, 21, 24, 27, 30

Multiples of 5: 5, 10, (15), 20, 25, 30, 35,

The first multiple of 3 and 5 to appear in both lists is ringed (15). This by definition must be the lowest common multiple. So the LCM of 3 and 5 is 15.