In the 16^0 column ($A_{16} = 10$) - ($F_{16} = 15$) cannot be done, so borrow one group of 16 from the 16^1 column (leaving $B_{16} = 11$) in the 16^1 column.

We then add the borrowed 16 to the A in the 16^0 column making 16 + ($A_{16} = 10$) =26. Then 26 - ($F_{16} = 15$) =11. Now (11 = B_{16}).

Put this down in 16^0 column. In 16^1 column (B_{16}=11) - (F_{16} = 15) cannot be done, so borrow one group of 16 from 16^2 column (leaving (E_{16} = 14) – 1 = 13(D)).

We then add the borrowed 16 to the B in the 16^1 column making 16 + (B_{16} = 11) = 27.

$(372)_8$
$\times\ (66)_8$
4

Then 27 - (F_{16}= 15) =12 which is C_{16}.

Put C_{16} down in 16^1 column.

Finally, in the 16^2 column (D_{16}= 13) -2 = 11 (B_{16}).

Put B_{16} down in the 16^2 column.

Binary Coded Decimal (BCD)

As the name suggests BCD uses a binary code to represent the decimal digits. It is a 4-bit code and is only used for the representation of numeric values.

Each of the ten digits used in the decimal system is coded with its binary equivalent as follows:-

Dec. Digit	0	1	2	3	4	5	. . .	9
BCD Code	0000	0001	0010	0011	0100	0101	. . .	1001

Any number can be represented by coding each digit separately.

Example 1

A decimal value of 624 would be coded as follows:

Decimal	6	2	4
BCD Code	0110	0010	0100

0000 0
0001 1
0010 2
0011 3
0100 4
0101 5
0110 6
0111 7
1000 8
1001 9

There are ***two points*** that need to be made regarding this particular coding method:

- Only ten of the possible sixteen combinations of 4-bits are used.
- BCD uses more bits generally speaking and thus more storage than a pure binary representation of the number.

Example 2

Consider the decimal number 1265_{10}.

0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9

2^9 2^8 2^7 2^6 2^5 16 8 4 2 1

In BCD it is:- $\underbrace{0001 \quad 0010 \quad 0110 \quad 0101}_{\text{16 bits}}$

In pure Binary it is:-

1024	512	256	128	64	32	16	8	4	2	1
1	0	0	1	1	1	1	0	0	0	1

11 bits

1024+128+64+32+16+1=1265

It is therefore more economical to store in pure binary. There is a fairly simple relationship between the code for a positive number and the corresponding negative number. It is called ***twos complement***.

Twos Complement

In two complement coding, the bits have the same place values as binary numbers except that the most significant bit (the leftmost bit) represents a negative quantity.

The place values for a six bit, twos complement numbers are:

-32	16	8	4	2	1
1	0	0	1	1	0

This equals -32 + 4 + 2 = -26

So if we consider +11 in 6 bits we would have: 0 01011_2. The corresponding negative number -11 would be 110101.

1+0+4+0+16+(-32) = -11

1+2+0+8+0+0 = 11

+11 0 0 1 0 1 1

1 1 0 1 0 0

+ 1

1 1 0 1 0 1 → −11

	-32	16	8	4	2	1	
+11	0	0	1	0	1	1	
-11	1	1	0	1	0	1	(-32+21)

It can be seen that except for the units column the bits for the negative number are the opposite of those for the positive number. So to change from a positive number to a negative number in twos complement form:

Change the 0's to 1's and the 1's to 0's and then add 1.

	32	16	8	4	2	1	
+8	0	0	1	0	0	0	
-8	1	1	0	1	1	1	(bits changed)
						1	(add)
	1	1	1	0	0	0	

Subtraction Using Twos Complement

Since subtraction can be considered as the addition of a negative number (16 - 7 ≡ 16 +(-7). we can use twos complement to do subtraction by making the ***second number negative*** as described previously and then ***adding this to the first number***.

Example 25 - 16 using twos complement.

	-32	16	8	4	2	1	
16	0	1	0	0	0	0	
Reverse bits	1	0	1	1	1	1	
Add 1						1	
-16	1	1	0	0	0	0	
Add 25	0	1	1	0	0	1	
	0	0	1	0	0	1	= 9

(1) This extra bit carried over from the -32 column is called the overflow bit and in this case is ignored

Ones Complement

The twos complement of a binary number is, as we have seen, found by reversing the bits and adding 1.

Ones complement is found however by merely reversing the bits.

Example

	-31	16	8	4	2	1	
+7	0	0	0	1	1	1	
-7	1	1	1	0	0	0	(-31+24=-7)

Note: In this type of coding, the most significant bit, the leftmost bit, now represents -31 rather than -32 as in twos complement.

Subtraction Using Ones Complement

This is similar to subtraction using twos complement except that in this case, the overflow bit must be added back to the units column.

Example 25 - 16 using ones complement.

	-31	16	8	4	2	1	
16	0	1	0	0	0	0	
-16	1	0	1	1	1	1	
add 25	0	1	1	0	0	1	
	0	0	1	0	0	0	
						1	
			1	0	0	1	= 9

overflow
(1)

Simultaneous Linear Equations

If we consider the two linear equations: $3x - 2y = 4$ and $2x + 3y = 7$

Then each equation contains x and y which are unknown quantities.

The solutions are the values of x and y which satisfy both equations simultaneously. Equations such as the above are called simultaneous linear equations.

There are two methods used to solve the type of equations and we will look at each in turn.

Substitution Method

Consider the equation: $x + y = 6 \qquad (1)$

$$x - y = 2 \qquad (2)$$

The substitution method involves taking one of the equations, usually the simpler of the two, rearranging it so that either x or y is the subject and then substituting for this subject in the other equation.

Example 1 From (1) above, $x + y = 6$

Rearranging $x = -y + 6$

Substitute for x in (2)

$$x - y = 2$$

$$(-y + 6) - y = 2$$

$$-y + 6 - y = 2$$

$$-2y + 6 = 2$$

$$-2y = 2 - 6$$

$$-2y = -4$$

$$y = 2$$

Substitute for y in either of the above:-

$x + y = 6$	**or**	$x - y = 2$
$x + 2 = 6$		$x - (2) = 2$
$x = 4$		$x = 4$

Once we have values for x and y, we substitute back into one of the two original equations to see if the sides balance. This serves as a check to see if our solutions are correct.

So we have $x = 4, \quad y = 2$

From (1) $x + y = 6$

And $4 + 2 = 6$

Our solutions are correct!

Example 2 $3x + 4y = 26$ (1)

$x + y = 7$ (2)

$3(7-y)+4y=26 \rightarrow 21-3y+4y=26$

$y=26-21$

$\boxed{y=5}$

$\rightarrow x=7-y$

$x=7-5=2$

$\boxed{x=2}$

From (2) $x = 7 - y$

Substitute for x in equation (1).

$$3(7 - y) + 4y = 26$$

$$21 - 3y + 4y = 26$$

$$-3y + 4y = 26 - 21$$

$$y = 5$$

Substitute for y in either of the above.

$$3x + 4(5) = 26$$

$$3x + 20 = 26$$

$$3x = 26 - 20$$

$$3x = 6$$

$$x = 2$$

So we have $x = 2, y = 5$

From (1) $3x + 4y = 26$

$$3(2) + 4(5) = 26$$

$$6 + 20 = 26$$

So our solutions are correct!

Elimination Method

In this method, we eliminate either x or y from the two equations by adding them together or subtracting one from the other.

Before we can do this however, the coefficients (i.e. the numbers in front of the x's or the y's) must be the same. If they are not, then we must multiply one or both equations by a number/numbers to make them so.

Example 1

3 4

$2x + 4y = 22$ (1)

6 + 16 = 22 ✓

+

$6x - 4y = 2$ (2)

18 - 16 = 2 ✓

$2x + 6x + 4y + (-4y) = 22 + 2$

$8x = 24 \rightarrow \boxed{x = 3}$

$6 \times 3 - 4y = 2 \rightarrow 18 - 4y = 2 \rightarrow -4y = -16 \rightarrow \boxed{y = 4}$

In this case the coefficients (i.e. the numbers in front of the y's) are the same.

If the signs in front of these terms are the ***same*** i.e. both (+) or both (–) then we ***subtract.*** If they are ***different*** one (+) and one (–) then we ***add.***

In this case they are different so we ***add*** the equations together:

$2x + 4y = 22$ (1)

$6x - 4y = 2$ (2)

$8x = 24$

$\therefore x = 3$

Substitute for x in either of the above and we get:

$2(3) + 4y = 22$	OR	$6(3) - 4y = 2$
$6 + 4y = 22$		$18 - 4y = 2$
$4y = 22 - 6$		$-4y = 2 - 18$
$4y = 16$		$-4y = -16$
$y = 4$		$y = 4$

We have $x = 3$, $y = 4$

Check $2x + 4y = 22$

$2(3) + 4(4) = 22$

So our solutions are correct!

Example 2 $-1/\ 3x + 5y = 17 \quad (1) \rightarrow -3x - 5y = -17$

$4x + 5y = 21 \quad (2) \rightarrow + \ 4x + 5y = 21$

$4x - 3x + 5y - 5y = 21 - 17$

In this case, the numbers in front of the y's are the same, but in this case the signs are the same, so we ***subtract***. $\boxed{x = 4}$

$12 + 5 = 17$

$3x + 5y = 17$ ✓

$3{\times}4 + 5{\times}1$

$16 + 5 = 21$

$4x + 5y = 21$ ✓

$4{\times}4 + 5{\times}1$

$-x = -4$

$\therefore x = 4$

$3{\times}4 + 5y = 17 \rightarrow 12 + 5y = 17 \rightarrow 5y = 5 \rightarrow \boxed{y = 1}$

$4{\times}4 + 5y = 21 \rightarrow 16 + 5y = 21 \rightarrow 5y = 21 - 16$

$5y = 5 \rightarrow \boxed{y = 1}$

Substitute for x in either of the above and we get:

$3(4) + 5y = 17$

$12 + 5y = 17$

$5y = 17 - 12$

$5y = 5$

$\therefore y = 1$

We have $x = 4, y = 1$

Check $4x + 5y = 21$

$4(4) + 5(1) = 21$

$16 + 5 = 21$

So once again our solutions are correct!

Example 3 $2/\ 3x + 2y = 13 \quad (1)\ {-1}/\ 6x + 4y = 26 \ \big|\ -6x - 4y = -26$

$3/\ 2x + 3y = 12 \quad (2) \quad 6x + 9y = 36 \ \big|\ +6x + 9y = 36$

$5y = 10 \rightarrow \boxed{y = 2}$

In this case, the numbers in front of the x's and the y's are different. If we multiply (1) by 2 and (2) by 3, we get:

$2x + 3{\times}2 = 12 \rightarrow 2x + 6 = 12 \rightarrow 2x = 6$

$\boxed{x = 3}$

$6x + 4y = 26$

$6x + 9y = 36$

The numbers in front of the x's are now the same and the signs are the same (two (+)) so we ***subtract***.

$6x + 4y = 26$

$6x + 9y = 36$

$-5y = -10$

$\therefore \quad y = 2$

Substitute for y in either of the original :

$3x + 2y = 13$

$3x + 2(2) = 13$

$3x + 4 = 13$

$3x = 13 - 4$

$3x = 9$

$\therefore \quad x = 3$

We have $x = 3, \; y = 2$

Check $\quad 2x + 3y = 12$

$2(3) + 3(2) = 12$

$6 + 6 = 12$

Again, our solutions are correct.

Example:

$-3(7x - 5y) = 14$

$7(3x - 7y) = 12$

$-21x + 15y = -42$

$+ \; 21x - 49y = 84$

$-24y = 42$

$y = \frac{-42}{24} = \frac{-14}{8} = \frac{-7}{4}$

$7x - 5\left(\frac{-7}{4}\right) = 14$

$7x + \frac{35}{4} = 14 \rightarrow 7x = 14 - \frac{35}{4}$

$7x = \frac{56-35}{4}$

$7x = \frac{21}{4}$

$\boxed{x = \frac{3}{4}}$

Example 4 $\quad 2(2x - 3y) = 1 \quad (1) \qquad 4x - 6y = 2$

$\qquad 3(3x + 2y) = 8 \quad (2) \qquad 9x + 6y = 24$

$+ \; 13x = 26$

$\boxed{x = 2}$

When balancing, we always have a choice (e.g. if we multiply equation (1) by 3 and (2) by 2 the x's are balanced) and since the signs are both (+) we subtract ***or*** we can multiply (1) by 2 and (2) by 3 to balance the y's and since the signs are different we add.

$3 \times 2 + 2y = 8$

$6 + 2y = 8$

$2y = 8 - 6$

$2y = 2$

$\boxed{y = 1}$

$2 \times 2 - 3 \times 1 = 4 - 1 = 1$

$2x - 3y = 1$ ✓

$3x + 2y = 8$ ✓

$3 \times 2 + 2 \times 1 = 6 + 2 = 8$

Lets look at both cases:

	$2x - 3y = 1$ (1)	or	$2x - 3y = 1$ (1)
	$3x + 2y = 8$ (2)		$3x + 2y = 8$ (2)
$\times$ by 3	$6x - 9y = 3$	$\times$ by 2	$4x - 6y = 2$
$\times$ by 2	$6x + 4y = 16$	$\times$ by 3	$9x + 6y = 24$
Subtract		Add	
	$-13y = -13$		$13x = 26$
	$y = 1$		$x = 2$
Substitute for y			Substitute for x
$3x + 2(1) = 8$			$3(2) + 2y = 8$
$3x + 2 = 8$			$6 + 2y = 8$
$3x = 6$			$2y = 2$
$x = 2$			$y = 1$

As you can see, either result gives the same answer!

Summary

- Check to see whether the coefficients (numbers) in front of the x's or y's are the same.
- If they are, then ***add*** if the signs are ***different***, ***subtract*** if the signs are the ***same***.
- If the number in front of the x's or y's are not the same, then we must make them so by multiplying one or both equations by a number/numbers. Once we have done this we either add or subtract the equations.
- Once we have found the value of x or y, we substitute this value into one of the original equations to find the value of the other.
- Substitute both values into one of the original equations to see if the sides balance. If they do we know our solutions are correct. If they do not then backtrack to see where we have made a mistake.

Solving Of 2nd Degree Equations via Factorisation

Factors of Quadratic Expressions

An expression of the type $ax^2 + bx + c$ is called a ***quadratic*** expression. The constants a, b and c can take any numerical value. except a

$a \neq 0$

When the coefficient of the x^2 term is 1 then the factors will be of the form $(x \pm p)(x \pm q)$. Where p and q are two numbers whose sum will be equal to b and whose product will equal c.

$2 \times 3 = 6$ ✓ c
$1 \times 6 = 6$ ✓ c
$(-2) \times (-3) = 6$ ✓ c
$(-1) \times (-6) = 6$ ✓ c

Example 1 Factorise $x^2 + 5x + 6$

$2 + 3 = 5 \longrightarrow b$
$1 + 6 = 7 \neq b$
$(-2) + (-3) = -5 \neq b$
$(-1) + (-6) = -7 \neq b$

Step 1 We can write down $(x + 2)(x + 3)$

Step 2 If the second sign in the original expression is (+), this means the signs in the factor brackets are the same. The first sign in the original expression will tell us what they are. We can write down (x +) (x +)

Step 3 Look at the end non-x term and its sign and consider the factors of it. For example, $+6 = 6 \times 1$ or -6×-1, or 3×2 or -3×-2 The pair which give the number in front of the x-term when added are the required ones. i.e. 3×2 when added give 5. So we write down $(x + 3)(x + 2)$.

In effect what we are looking for are two numbers whose sum is 5 and whose product is 6.

$2 \times 3 = 6$
$1 \times 6 = 6$
$(-1) \times (-6) = 6$
$(-2) \times (-3) = 6$

Example 2 Factorise $x^2 - 5x + 6$

Step 1 $(x - 2)(x - 3)$

$2+3=5$
$1+6=7$
$-1-6=-7$
$-2-3=-5$ ✓

Step 2 The second sign is a (+), so the signs are going to be the ***same*** and they are going to be ***minuses***. (x –)(x –)

Step 3 $+6 = 6 \times 1$, -6×-1, 3×2, -3×-2. The pair which when added give -5 are -3×-2. So we have $(x - 3)(x - 2)$

The previous two examples are where the second sign in the original is a (+) and so the signs in the factor brackets will be two (+) or two (-) depending upon what the first sign in the original was.

Now if the second sign in the original is a (-) then the signs in the factor brackets will be different (i.e. one (+) and one (-)).

$1 \times (-6) = -6$
$-1 \times 6 = -6$
$-2 \times 3 = -6$
$2 \times (-3) = -6$
$1 - 6 = -5$ ✓
$-1 + 6 = 5$
$-2 + 3 = 1$
$2 - 3 = -1$

Example 3 Factorise $x^2 - 5x - 6$

Step 1 $(x - 6)(x + 1)$

Step 2 As the second sign in the original is a (-) then the signs in the factor brackets are going to be different. $(x + \quad)(x - \quad)$

Step 3 $-6 = \mathbf{-6 \times 1}$, 6×-1, -3×2, 3×-2. All these will give the end term -6 when added
The pair which when added that give -5 are -6×1.

So we have: $(x + 1)(x - 6)$

Note: Make sure you match the (+) with a (+) and the (-) with a (-) in these cases.

$-1 \times (-3) = 3$
$1 \times 3 = 3$
$-1 - 3 = -4$ ✓
$1 + 3 = 4$

Example 4 Factorise $x^2 - 4x + 3$

Step 1 $(x - 1)(x - 3)$

Step 2 $(x - \quad)(x - \quad)$

Step 3 $3 = 3 \times 1$, $\mathbf{-3 \times -1}$ and $(-3 + -1) = -4$

So we have: $(x - 3)(x - 1)$

$-2 \times 4 = -8$
$2 \times (-4) = -8$
$-1 \times 8 = -8$
$1 \times (-8) = 8$
$-2 + 4 = 2$
$2 - 4 = -2$ ✓
$-1 + 8 = 7$ $1 - 8 = -7$

Example 5 Factorise $x^2 - 2x - 8$

Step 1 $(x + 2)(x - 4)$

Step 2 $(x + \quad)(x - \quad)$

Step 3 $-8 = 8 \times -1$, -8×1, 4×-2, $\mathbf{-4 \times 2}$ and $(-4 + 2) = -2$

So we have: $(x + 2)(x - 4)$

$-4 \times 5 = -20$
$4 \times (-5) = -20$ $4 - 5 = -1$ ✓
$-2 \times 10 = -20$
$2 \times (-10) = -20$
$-1 \times 20 = -20$
$1 \times (-20) = -20$

Example 6 Factorise $x^2 - x - 20$

Step 1 $(x + 4)(x - 5)$

Step 2 $(x + \quad)(x - \quad)$

Step 3 $-20 = -20 \times 1$, 20×-1, -10×2, 10×-2, 5×-4, $\mathbf{-5 \times 4}$ and $(-5 + 4) = -1$

So we have: $(x + 4)(x - 5)$

$(-1) \times (-9) = 9$
$-1 + (-9) = -10$

Example 7 Factorise $x^2 - 10x + 9$

Step 1 $(x - 1)(x - 9)$

Step 2 $(x - \quad)(x - \quad)$

Step 3 $+9 = 9 \times 1$, $\mathbf{-9 \times -1}$, 3×3, -3×-3 and $(-9 + -1) = -10$

So we have: $(x - 9)(x - 1)$

Quadratic Equations

A quadratic equation is represented by an expression of the type $ax^2 + bx + c = 0$, where the constants a, b and c can take on any value.

It is satisfied by two values of x, but these values may be equal to each other. Such an equation should be solved by factorisation if possible. If not the quadratic formula must be used.

Example 1 Solve the equation $x^2 - 4x - 12 = 0$

Factorise

Step 1 $(x + 2)(x - 6) = 0$

$x + 2 = 0$ | $x - 6 = 0$
$x = -2$ | $x = 6$

Step 2 $(x + \quad)(x - \quad) = 0$

Step 3 $-12 = -12 \times 1$, 12×-1, 6×-2, $\mathbf{-6 \times 2}$, 4×-3, -4×3 and $(-6 + 2)$

So we have: $(x + 2)(x - 6) = 0$

Now if the product of the two expressions $(x + 2)$ and $(x - 6)$ is zero, then one of them itself must be zero, or they are both equal to zero.

So either $x + 2 = 0$ i.e. $x = -2$ or $x - 6 = 0$ i.e. $x = 6$

Check:

When $x = -2$

$x^2 - 4x - 12$

$= (-2)^2 - 4(-2) - 12$

$= 4 + 8 - 12$

$= 0$

When $x = 6$

$x^2 - 4x - 12$

$= (6)^2 - 4(6) - 12$

$= 36 - 24 - 12$

$= 0$

Example 2 Solve $x^2 - 7x - 18 = 0$

$2 \times (-9) = -18$

$2 + (-9) = -7$

Step 1 $(x + 2)(x - 9) = 0$

$x + 2 = 0$ | $x - 9 = 0$

$x = -2$ | $x = 9$

Step 2 $(x + \quad)(x - \quad) = 0$

Step 3 $-18 = -18 \times 1,\ 18 \times -1,\ -6 \times 3,\ 6 \times -3,\ 9 \times -2,$ **-9×2** and $(-9 + 2) = -7$

So we have: $(x - 9)(x + 2) = 0$

Either $x - 9 = 0$ i.e. $x = 9$ or $x + 2 = 0$ i.e. $x = -2$

Check:

When x=9 $x^2 - 7x - 18$

$= 9^2 - 7(9) - 18$

$= 81 - 63 - 18$

$= 0$

When x = -2 $x^2 - 7x - 18$

$= (-2)^2 - 7(-2) - 18$

$= 4 + 14 - 18$

$= 0$

Example 3 Solve $x^2 - 8x + 15 = 0$

Step 1 $(x \quad)(x \quad) = 0$

Step 2 $(x - \quad)(x - \quad) = 0$

Step 3 $+15 = 15 \times 1,\ -15 \times -1,\ 5 \times 3,$ **-5×-3** and $(-5 + -3) = -8$

So we have: $(x - 5)(x - 3) = 0$

Either $x - 5 = 0$ i.e. $x = 5$ or $x - 3 = 0$ i.e. $x = 3$

Check

When x = 5 $5^2 - 8(5) + 15$

$= 25 - 40 + 15$

$= 0$

When x = 3 $3^2 - 8(3) + 15$

$= 9 - 24 + 15$

$= 0$

If a quadratic equation cannot be solved by factorisation, then the quadratic formula should be used.The quadratic formula is based upon the general quadratic expression $ax^2 + bx + c = 0$ and it is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x_1 = \frac{2+\sqrt{(-2)^2 - 4\times 7\times(-6)}}{2\times 7} = \frac{2+2\sqrt{43}}{2\times 7} = \frac{1+\sqrt{43}}{7}$$

$$x_2 = \frac{1-\sqrt{43}}{7}$$

So if we had the quadratic equation $7x^2 - 2x - 6 = 0$, then $a = 7$, $b = -2$ and $c = -6$.

Since factorisation is not possible, then the values of x will not be whole numbers and they are usually quoted to so many decimal places.

Example 1

Solve $4x^2 + x - 3 = 0$ giving your answer correct to 2 decimal places.

In this case $a = 4$, $b = 1$, $c = -3$

So:-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-1 \pm \sqrt{1^2 - 4(4)(-3)}}{8}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{8}$$

$$x = \frac{-1 \pm \sqrt{49}}{8}$$

$$x = \frac{-1 \pm 7}{8}$$

so $x = \frac{-8}{8}$ or $x = \frac{6}{8}$

$= -1\ 00$ $\qquad$ $= 0\ 75$

Example 2

Solve $3x^2 - 8x + 2 = 0$ giving your answer correct to 2 decimal points.

In this case $a = 3$, $b = -8$, $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x_1 = \frac{+8+\sqrt{(-8)^2 - 4\times 3\times 2}}{2\times 3} = \frac{8+\sqrt{64-24}}{6} = \frac{8+\sqrt{40}}{6} = \frac{8+2\sqrt{10}}{6}$$

$$x_1 = \frac{4+\sqrt{10}}{3}, \quad x_2 = \frac{4-\sqrt{10}}{3}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{6}$$

$$x = \frac{8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{8 \pm \sqrt{40}}{6}$$

$$x = \frac{8 \pm 6.325}{6}$$

so $x = \frac{14.325}{6}$ or $x = \frac{1.675}{6}$

$= 2.39$ (2 dp) $= 0.28$ (2 dp)

Logarithms

Logarithm is another word for ***index*** or ***power***. Consider $2^4 = 16$. 4 is the power to which the base 2 must be raised to give 16 or 4 is the logarithm which with a base 2 gives 16.

We can write this simply as: $4 = \log_2 16$

In a similar fashion: $3^3 = 27$, i.e. 3 is the logarithm which with a base 3 gives 27 or $3 = \log_3 27$

The base of a logarithm may be any number. Logarithms with a base of 10 are called ***common logarithms***. If we use a calculator, we see that the logarithm of 6 is 0.7782.

Therefore $10^{0.7782} = 6$ or $\log_{10} 6 = 0.7782$

Further examples

$10 = 10^1$ $\log_{10} 10 = 1$ $10000 = 10^4$ $\log_{10} 10000 = 4$

$100 = 10^2$ $\log_{10} 100 = 2$ $100000 = 10^5$ $\log_{10} 10^5 = 5$

$1000 = 10^3$ $\log_{10} 1000 = 3$ and so on

In general then we can state: **Number = Base$^{\text{Logarithm}}$** and the logarithm of a number is the power to which the base must be raised to give that number.

The Laws of Logarithms

Multiplication

The logarithm of two numbers multiplied together may be found by adding the logarithm of the numbers together.

For example, if we have two numbers P & Q

$\log_a P + \log_b Q \neq \log_{ab} PQ$

Then $\log_b PQ = \log_b P + \log_b Q$ (where b is the log base).

Division

The logarithm of two numbers divided may be found by subtracting the logarithm of the number you are dividing by from the logarithm of the number you are dividing into.

So if we have two numbers P & Q then,

$$\log_b \frac{P}{Q} = \log_b P - \log_b Q$$

(where b is the log base).

Powers

The logarithm of a number raised to any power is found by multiplying the logarithm of that number by the power.

If we have a number P and wish to raise it to a power 'n'

Then: $\log_b P^n = n\log_b P$

$\log_{10} 100000 = \log_{10} 10^5 = 5 \underbrace{\log_{10} 10}_{1} = 5$

Composition of a Logarithm

A logarithm consists of two parts. The whole number before the decimal point is called the ***characteristic***. The part after the decimal point is called the ***mantissa.***

LOGARITHM TABLE (for numbers 1 to 5.49)

No.	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.000	0.004	0.009	0.013	0.017	0.021	0.025	0.029	0.033	0.037
1.1	0.041	0.045	0.049	0.053	0.057	0.061	0.064	0.068	0.072	0.076
1.2	0.079	0.083	0.086	0.090	0.093	0.097	0.100	0.104	0.107	0.111
1.3	0.114	0.117	0.121	0.124	0.127	0.130	0.134	0.137	0.140	0.143
1.4	0.146	0.149	0.152	0.155	0.158	0.161	0.164	0.167	0.170	0.173
1.5	0.176	0.179	0.182	0.185	0.188	0.190	0.193	0.196	0.199	0.201
1.6	0.204	0.207	0.210	0.212	0.215	0.217	0.220	0.223	0.225	0.228
1.7	0.230	0.233	0.236	0.238	0.241	0.243	0.246	0.248	0.250	0.253
1.8	0.255	0.258	0.260	0.262	0.265	0.267	0.270	0.272	0.274	0.276
1.9	0.279	0.281	0.283	0.286	0.288	0.290	0.292	0.294	0.297	0.299
2.0	0.301	0.303	0.305	0.307	0.310	0.312	0.314	0.316	0.318	0.320
2.1	0.322	0.324	0.326	0.328	0.330	0.332	0.334	0.336	0.338	0.340
2.2	0.342	0.344	0.346	0.348	0.350	0.352	0.354	0.356	0.358	0.360
2.3	0.362	0.364	0.365	0.367	0.369	0.371	0.373	0.375	0.377	0.378
2.4	0.380	0.382	0.384	0.386	0.387	0.389	0.391	0.393	0.394	0.396
2.5	0.398	0.400	0.401	0.403	0.405	0.407	0.408	0.410	0.412	0.413
2.6	0.415	0.417	0.418	0.420	0.422	0.423	0.425	0.427	0.428	0.430
2.7	0.431	0.433	0.435	0.436	0.438	0.439	0.441	0.442	0.444	0.446
2.8	0.447	0.449	0.450	0.452	0.453	0.455	0.456	0.458	0.459	0.461
2.9	0.462	0.464	0.465	0.467	0.468	0.470	0.471	0.473	0.474	0.476
3.0	0.477	0.479	0.480	0.481	0.483	0.484	0.486	0.487	0.489	0.490
3.1	0.491	0.493	0.494	0.496	0.497	0.498	0.500	0.501	0.502	0.504
3.2	0.505	0.507	0.508	0.509	0.511	0.512	0.513	0.515	0.516	0.517
3.3	0.519	0.520	0.521	0.522	0.524	0.525	0.526	0.528	0.529	0.530
3.4	0.531	0.533	0.534	0.535	0.537	0.538	0.539	0.540	0.542	0.543
3.5	0.544	0.545	0.547	0.548	0.549	0.550	0.551	0.553	0.554	0.555
3.6	0.556	0.558	0.559	0.560	0.561	0.562	0.563	0.565	0.566	0.567
3.7	0.568	0.569	0.571	0.572	0.573	0.574	0.575	0.576	0.577	0.579
3.8	0.580	0.581	0.582	0.583	0.584	0.585	0.587	0.588	0.589	0.590
3.9	0.591	0.592	0.593	0.594	0.595	0.597	0.598	0.599	0.600	0.601
4.0	0.602	0.603	0.604	0.605	0.606	0.607	0.609	0.610	0.611	0.612
4.1	0.613	0.614	0.615	0.616	0.617	0.618	0.619	0.620	0.621	0.622
4.2	0.623	0.624	0.625	0.626	0.627	0.628	0.629	0.630	0.631	0.632
4.3	0.633	0.634	0.635	0.636	0.637	0.638	0.639	0.640	0.641	0.642
4.4	0.643	0.644	0.645	0.646	0.647	0.648	0.649	0.650	0.651	0.652
4.5	0.653	0.654	0.655	0.656	0.657	0.658	0.659	0.660	0.661	0.662
4.6	0.663	0.664	0.665	0.666	0.667	0.667	0.668	0.669	0.670	0.671
4.7	0.672	0.673	0.674	0.675	0.676	0.677	0.678	0.679	0.679	0.680
4.8	0.681	0.682	0.683	0.684	0.685	0.686	0.687	0.688	0.688	0.689
4.9	0.690	0.691	0.692	0.693	0.694	0.695	0.695	0.696	0.697	0.698
5.0	0.699	0.700	0.701	0.702	0.702	0.703	0.704	0.705	0.706	0.707
5.1	0.708	0.708	0.709	0.710	0.711	0.712	0.713	0.713	0.714	0.715
5.2	0.716	0.717	0.718	0.719	0.719	0.720	0.721	0.722	0.723	0.723
5.3	0.724	0.725	0.726	0.727	0.728	0.728	0.729	0.730	0.731	0.732
5.4	0.732	0.733	0.734	0.735	0.736	0.736	0.737	0.738	0.739	0.740

LOGARITHM TABLE (for numbers from 5.5 to 10)

No.	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.5	0.740	0.741	0.742	0.743	0.744	0.744	0.745	0.746	0.747	0.747
5.6	0.748	0.749	0.750	0.751	0.751	0.752	0.753	0.754	0.754	0.755
5.7	0.756	0.757	0.757	0.758	0.759	0.760	0.760	0.761	0.762	0.763
5.8	0.763	0.764	0.765	0.766	0.766	0.767	0.768	0.769	0.769	0.770
5.9	0.771	0.772	0.772	0.773	0.774	0.775	0.775	0.776	0.777	0.777
6.0	0.778	0.779	0.780	0.780	0.781	0.782	0.782	0.783	0.784	0.785
6.1	0.785	0.786	0.787	0.787	0.788	0.789	0.790	0.790	0.791	0.792
6.2	0.792	0.793	0.794	0.794	0.795	0.796	0.797	0.797	0.798	0.799
6.3	0.799	0.800	0.801	0.801	0.802	0.803	0.803	0.804	0.805	0.806
6.4	0.806	0.807	0.808	0.808	0.809	0.810	0.810	0.811	0.812	0.812
6.5	0.813	0.814	0.814	0.815	0.816	0.816	0.817	0.818	0.818	0.819
6.6	0.820	0.820	0.821	0.822	0.822	0.823	0.823	0.824	0.825	0.825
6.7	0.826	0.827	0.827	0.828	0.829	0.829	0.830	0.831	0.831	0.832
6.8	0.833	0.833	0.834	0.834	0.835	0.836	0.836	0.837	0.838	0.838
6.9	0.839	0.839	0.840	0.841	0.841	0.842	0.843	0.843	0.844	0.844
7.0	0.845	0.846	0.846	0.847	0.848	0.848	0.849	0.849	0.850	0.851
7.1	0.851	0.852	0.852	0.853	0.854	0.854	0.855	0.856	0.856	0.857
7.2	0.857	0.858	0.859	0.859	0.860	0.860	0.861	0.862	0.862	0.863
7.3	0.863	0.864	0.865	0.865	0.866	0.866	0.867	0.867	0.868	0.869
7.4	0.869	0.870	0.870	0.871	0.872	0.872	0.873	0.873	0.874	0.874
7.5	0.875	0.876	0.876	0.877	0.877	0.878	0.879	0.879	0.880	0.880
7.6	0.881	0.881	0.882	0.883	0.883	0.884	0.884	0.885	0.885	0.886
7.7	0.886	0.887	0.888	0.888	0.889	0.889	0.890	0.890	0.891	0.892
7.8	0.892	0.893	0.893	0.894	0.894	0.895	0.895	0.896	0.897	0.897
7.9	0.898	0.898	0.899	0.899	0.900	0.900	0.901	0.901	0.902	0.903
8.0	0.903	0.904	0.904	0.905	0.905	0.906	0.906	0.907	0.907	0.908
8.1	0.908	0.909	0.910	0.910	0.911	0.911	0.912	0.912	0.913	0.913
8.2	0.914	0.914	0.915	0.915	0.916	0.916	0.917	0.918	0.918	0.919
8.3	0.919	0.920	0.920	0.921	0.921	0.922	0.922	0.923	0.923	0.924
8.4	0.924	0.925	0.925	0.926	0.926	0.927	0.927	0.928	0.928	0.929
8.5	0.929	0.930	0.930	0.931	0.931	0.932	0.932	0.933	0.933	0.934
8.6	0.934	0.935	0.936	0.936	0.937	0.937	0.938	0.938	0.939	0.939
8.7	0.940	0.940	0.941	0.941	0.942	0.942	0.943	0.943	0.943	0.944
8.8	0.944	0.945	0.945	0.946	0.946	0.947	0.947	0.948	0.948	0.949
8.9	0.949	0.950	0.950	0.951	0.951	0.952	0.952	0.953	0.953	0.954
9.0	0.954	0.955	0.955	0.956	0.956	0.957	0.957	0.958	0.958	0.959
9.1	0.959	0.960	0.960	0.960	0.961	0.961	0.962	0.962	0.963	0.963
9.2	0.964	0.964	0.965	0.965	0.966	0.966	0.967	0.967	0.968	0.968
9.3	0.968	0.969	0.969	0.970	0.970	0.971	0.971	0.972	0.972	0.973
9.4	0.973	0.974	0.974	0.975	0.975	0.975	0.976	0.976	0.977	0.977
9.5	0.978	0.978	0.979	0.979	0.980	0.980	0.980	0.981	0.981	0.982
9.6	0.982	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986	0.986
9.7	0.987	0.987	0.988	0.988	0.989	0.989	0.989	0.990	0.990	0.991
9.8	0.991	0.992	0.992	0.993	0.993	0.993	0.994	0.994	0.995	1.00
9.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10.0	1.00									