INDUSTRIAL AUTOMATION & ROBOTICS TECHNOLOGY

Kinematic Singularities and Jacobian
Dynamic Analysis and Forces

M Umer khan

Department of Mechatronics Engineering

ATILIM University

UMER.KHAN@ATILIM.EDU.TR

Materials used

- Chapter 4, Introduction to Robotics, Saeed B. Niku
- Kinematic Singularities and Jacobian, PDF document

INTRODUCTION

- The dynamics, related with accelerations, loads, masses and inertias.

$$\sum \overline{F} = m \cdot \overline{a} \qquad \sum \overline{T} = I \cdot \overline{\alpha}$$

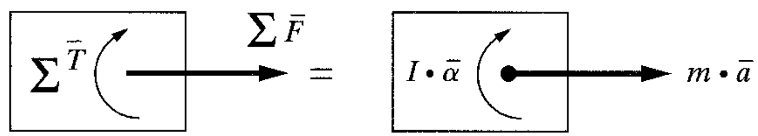

Fig. 4.1 Force-mass-acceleration and torque-inertia-angular acceleration relationships for a rigid body.

In Actuators…….

- The actuator can be accelerate a robot's links for exerting enough forces and torques at a desired acceleration and velocity.
- By the dynamic relationships that govern the motions of the robot, considering the external loads, the designer can calculate the necessary forces and torques.

DYNAMIC ANALYSIS

- Mathematical equations describing the dynamic behavior of the manipulator
  - For computer simulation
  - Design of suitable controller
  - Evaluation of robot structure

  - Joint torques $\Leftrightarrow$ Robot motion, i.e. acceleration, velocity, position

LAGRANGIAN MECHANICS: A SHORT OVERVIEW

- Lagrangian mechanics is based on the differentiation energy terms only, with respect to the system's variables and time.
- Definition: L = Lagrangian, K = Kinetic Energy of the system, P = Potential Energy, F = the summation of all external forces for a linear motion, T = the summation of all torques in a rotational motion, x = System variables

$$L = K - P$$

$$F_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i}$$

$$T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i}$$

Example 4.1

Derive the force-acceleration relationship for the one-degree of freedom system.

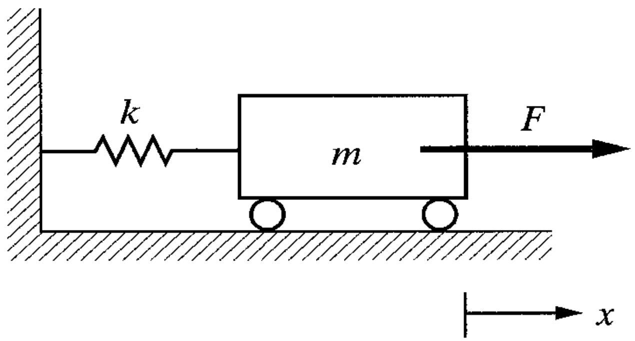

Fig. 4.2 Schematic of a simple cart-spring system.

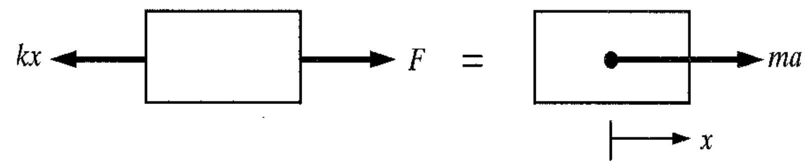

Fig. 4.3 Free-body diagram for the sprint-cart system.

Solution

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2, P = \frac{1}{2}kx^2 \quad \Rightarrow \quad L = K - P = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

- Lagrangian mechanics

$$\frac{\partial L}{\partial \dot{x}_i} = m\dot{x}, \frac{d}{dt}(m\dot{x}) = m\ddot{x}, \frac{\partial L}{\partial x} = -kx$$

$$F = m\ddot{x} + kx$$

- Newtonian mechanics

$$\sum \overline{F} = m \cdot \overline{a}$$

$$F - kx = ma \rightarrow F = ma + kx$$

- The complexity of the terms increases as the number of degrees of freedom and variables.

Example 4.2 Derive the equations of motion for the two-degree of freedom system.

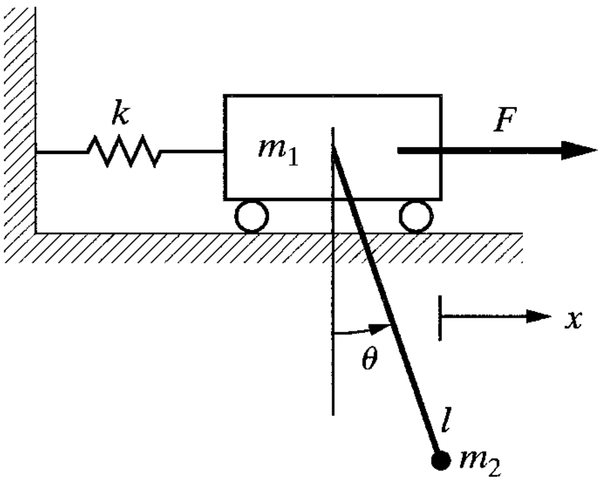

Fig. 4.4 Schematic of a cart-pendulum system.

In this system…….

- **It requires two coordinates, x and θ.**
- **It requires two equations of motion:**

  1. The linear motion of the system.

  2. The rotation of the pendulum.

Solution

$$\begin{bmatrix} F \\ T \end{bmatrix} = \begin{bmatrix} m_1 + m_2 & m_2 l \cos\theta \\ m_2 l \cos\theta & m_2 l^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & m_2 l \sin\theta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}^2 \\ \dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} kx \\ m_2 g l \sin\theta \end{bmatrix}$$

Example 4.3

Example 4.4

Using the Lagrangian method, derive the equations of motion for the two-degree of freedom robot arm.

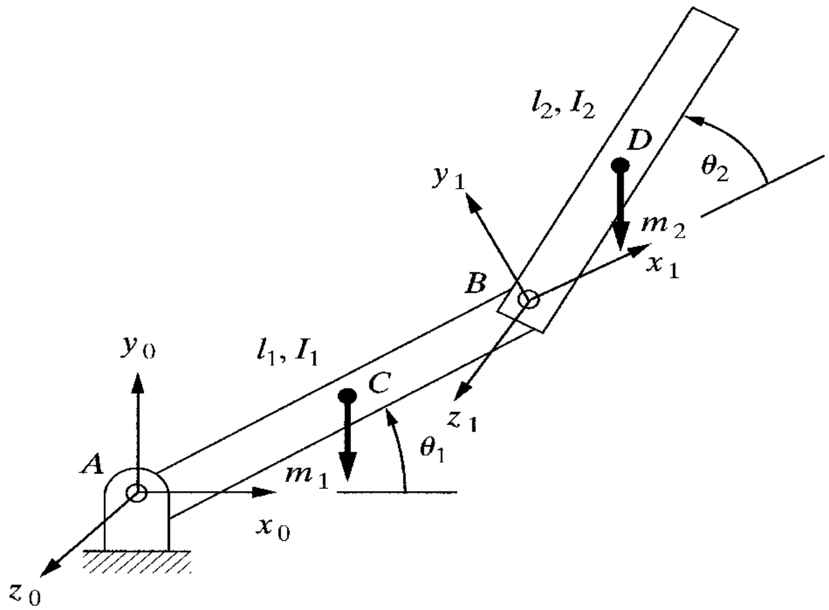

Fig. 4.6 A two-degree-of-freedom robot arm.

Solution

Follow the same steps as before…….

- **Calculates the velocity of the center of mass of link 2 by differentiating its position:**
- **The kinetic energy of the total system is the sum of the kinetic energies of links 1 and 2.**
- **The potential energy of the system is the sum of the potential energies of the two links:**

Example 4.5

Thank you!