ME210 STRENGTH OF MATERIALS

CHAPTER 6
Shearing Stresses in Beams and Thin-Walled Members

Introduction

Transverse loading applied to a beam results in normal and shearing stresses in transverse sections.

Distribution of normal and shearing stresses satisfies

$$F_x = \int \sigma_x dA = 0 \qquad M_x = \int (y\tau_{xz} - z\tau_{xy})dA = 0$$
$$F_y = \int \tau_{xy} dA = -V \qquad M_y = \int z\sigma_x dA = 0$$
$$F_z = \int \tau_{xz} dA = 0 \qquad M_z = \int (-y\sigma_x) = M$$

When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces

Longitudinal shearing stresses must exist in any member subjected to transverse loading.

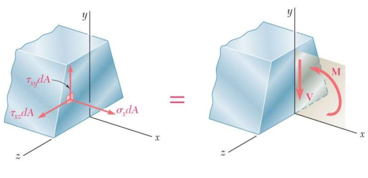

Fig. 6.1 All the stresses on elemental areas (left) sum to give the resultant shear *V* and bending moment *M*.

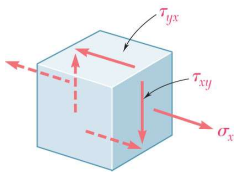

Fig. 6.2 Stress element from section of a transversely loaded beam.

Shear on the Horizontal Face of a Beam Element

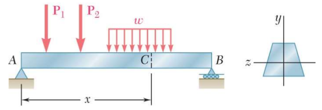

Fig. 6.4 Transversely loaded beam with vertical plane symmetric cross section.

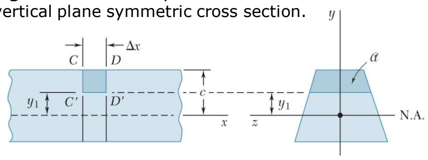

Fig. 6.5 Short segment of beam with stress element CDD'C' defined.

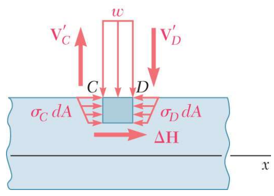

Fig. 6.6 Forces exerted on element CCD'C'.

Consider prismatic beam *AB*

For equilibrium of beam element

$$\sum F_x = 0 = \Delta H + \int_A (\sigma_D - \sigma_C) dA$$

$$\Delta H = \frac{M_D - M_C}{I} \int_A y\, dA$$

Note,

$$Q = \int_A y\, dA$$

$$M_D - M_C = \frac{dM}{dx} \Delta x = V\, \Delta x$$

Substituting,

$$\Delta H = \frac{VQ}{I} \Delta x$$

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = shear\ flow$$

Shear on the Horizontal Face of a Beam Element

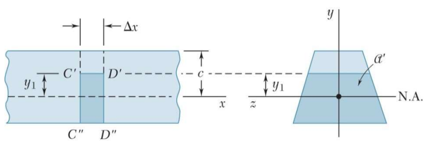

Fig. 6.7 Short segment of beam with stress element C'D'D"C" defined.

Shear flow,

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = shear\ \ flow$$

where

$$Q = \int_A y\, dA$$

$= \text{first moment of area above } y_1$

$$I = \int_{A+A'} y^2 dA$$

$= \text{second moment of full cross section}$

Same result found for lower area

$$q' = \frac{\Delta H'}{\Delta x} = \frac{VQ'}{I} = -q'$$

$$Q + Q' = 0$$

= first moment with respect to neutral axis

$$\Delta H' = -\Delta H$$

Concept Application 6.1

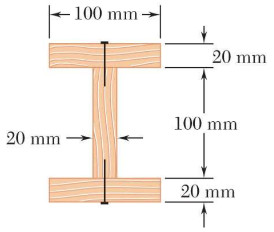

Fig. 6.8a Composite beam made of three boards nailed together.

A beam is made of three planks, nailed together. Knowing that the spacing between nails is 25 mm and that the vertical shear in the beam is $V = 500$ N, determine the shear force in each nail.

SOLUTION:

Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.

Calculate the corresponding shear force in each nail.

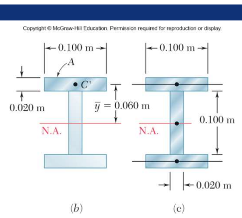

Fig. 6.8b-c Cross section with flange area for computing shear on nail highlighted. Cross section compound areas for finding entire section moment of inertia.

$$Q = A\bar{y}$$
$$= (0.020\,\text{m} \times 0.100\,\text{m})(0.060\,\text{m})$$
$$= 120 \times 10^{-6}\,\text{m}^3$$
$$I = \tfrac{1}{12}(0.020\,\text{m})(0.100\,\text{m})^3$$
$$+ 2[\tfrac{1}{12}(0.100\,\text{m})(0.020\,\text{m})^3$$
$$+ (0.020\,\text{m} \times 0.100\,\text{m})(0.060\,\text{m})^2]$$
$$= 16.20 \times 10^{-6}\,\text{m}^4$$

SOLUTION:

Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.

$$q = \frac{VQ}{I} = \frac{(500\text{N})(120 \times 10^{-6}\,\text{m}^3)}{16.20 \times 10^{-6}\,\text{m}^4}$$
$$= 3704\,\text{N}/\text{m}$$

Calculate the corresponding shear force in each nail for a nail spacing of 25 mm.

$$F = (0.025\,\text{m})q = (0.025\,\text{m})(3704\,N/m$$

$$\boxed{F = 92.6\,\text{N}}$$

Shearing Stresses in a Beam

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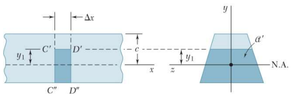


Fig. 6.7 Short segment of beam with smaller stress element C′D′D″C″ defined.

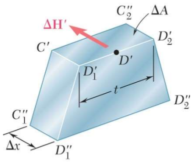


Fig. 6.9 Stress element C′D′D″C″ showing the shear force on a horizontal plane.

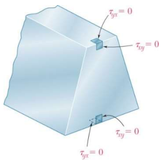


Fig. 6.11 Beam cross section showing that the shearing stress is zero at the top and bottom of the beam.

The *average* shearing stress on the horizontal face of the element is obtained by dividing the shearing force ΔH on the element by the area ΔA of the face.

$$\tau_{ave} = \frac{\Delta H}{\Delta A} = \frac{q\,\Delta x}{\Delta A} = \frac{VQ}{I}\frac{\Delta x}{t\,\Delta x}$$

$$= \frac{VQ}{It}$$

On the upper and lower surfaces of the beam, $\tau_{yx}= 0$. It follows that $\tau_{xy}= 0$ on the upper and lower edges of the transverse sections.

As long as the width of the beam cross section remains small compared to its depth, the shearing stress varies slightly along the line $D'_1D'_2$.

Shearing Stresses τ_{xy} in Common Types of Beams

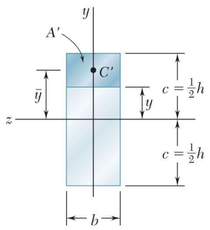


Fig. 6.13 Geometric terms for rectangular section used to calculate shearing stress.

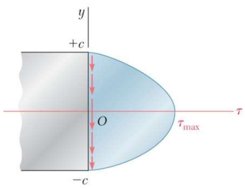


Fig. 6.14 Shearing stress distribution on transverse section of rectangular beam.

For a narrow rectangular beam,

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right)$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

For *American Standard* (S-beam) and *wide-flange* (W-beam) beams

$$\tau_{ave} = \frac{VQ}{It}$$

$$\tau_{max} = \frac{V}{A_{web}}$$

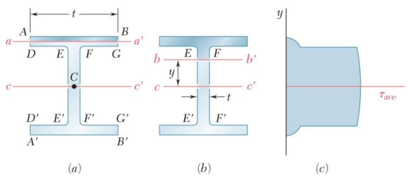


Fig. 6.15 Wide-flange beam. (a) Area for finding first moment of area in flange. (b) Area for finding first moment of area in web. (c) Shearing stress distribution.

Further Discussion on Stress Distribution

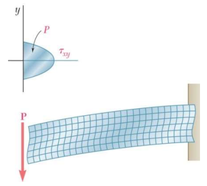


Fig. 6.18 Deformation of cantilever beam with concentrated load, with a parabolic shearing stress distribution.

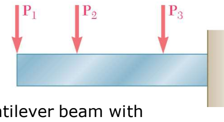


Fig. 6.19 Cantilever beam with multiple loads.

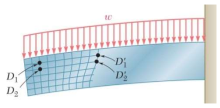


Fig. 6.20 Deformation of cantilever beam with distributed load.

Consider a narrow rectangular cantilever beam subjected to load **P** at its free end:

$$\tau_{xy} = \frac{3}{2}\frac{P}{A}\left(1 - \frac{y^2}{c^2}\right) \qquad \sigma_x = +\frac{Pxy}{I}$$

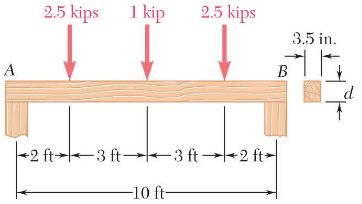
Shearing V is constant and equal in magnitude to the load **P**.

Normal strains and normal stresses are unaffected by the shearing stresses.

From Saint-Venant's principle, effects of the load application mode are negligible except in immediate vicinity of load application points.

Stress/strain deviations for distributed loads are negligible for typical beam sections of interest.

Sample Problem 6.2



A timber beam AB of span 10 ft is to support the three concentrated loads shown. Knowing that for the grade of timber used,

$$\sigma_{all} = 1800\,\text{psi} \qquad \tau_{all} = 120\,\text{psi}$$

determine the minimum required depth d of the beam.

SOLUTION:

Develop shear and bending moment diagrams. Identify the maximums.

Design the beam based on allowable normal stress.

Check shearing stress.

Redesign beam based on allowable shearing stress, if needed.

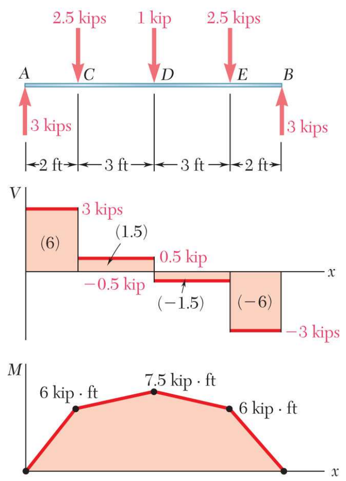


Fig. 1 Free-body diagram of beam with shear and bending-moment diagrams.

SOLUTION:

Develop shear and bending moment diagrams. Identify the maximums.

$$V_{max} = 3\,\text{kips}$$
$$M_{max} = 7.5\,\text{kip}\cdot\text{ft} = 90\,\text{kip}\cdot\text{in}$$

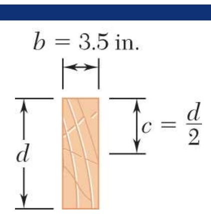


Fig. 2 Section of beam having depth *d*.

$$I = \tfrac{1}{12} b d^3$$
$$S = \frac{I}{c} = \tfrac{1}{6} b d^2$$
$$= \tfrac{1}{6}(3.5\,\text{in.})d^2$$
$$= (0.5833\,\text{in.})d^2$$

Design beam based on allowable normal stress.

$$\sigma_{all} = \frac{M_{max}}{S}$$
$$1800\,\text{psi} = \frac{90\times10^3\,\text{lb}\cdot\text{in.}}{(0.5833\,\text{in.})d^2}$$
$$d = 9.26\,\text{in.}$$

Check shearing stress.

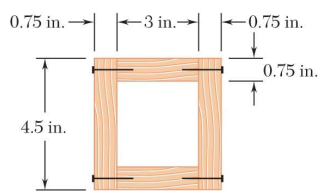
$$\tau_{all} = \frac{3}{2}\frac{V_{max}}{A} = \frac{3}{2}\frac{3000\,\text{lb}}{(3.5\,\text{in.})(9.26\,in.)} = 138.8\,psi.$$

Since τ_{all} = 120 psi, the depth d = 9.26 in. is *not* acceptable and we must redesign the beam on the basis of the requirement that $\tau_m \le$ 120 psi..

Allowable shear stress controls.

$$\tau_m = 120\ psi. = \tau_{all} = \frac{3}{2}\frac{V_{max}}{A} = \frac{3}{2}\frac{3000\ \text{lb}}{(3.5\ \text{in.})(d)} \qquad \boxed{d = 10.71\,\text{in.}}$$

Concept Application 6.4



A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.5 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.

SOLUTION:

Determine the shear force per unit length along each edge of the upper plank.

Based on the spacing between nails, determine the shear force in each nail.

Concept Application 6.4

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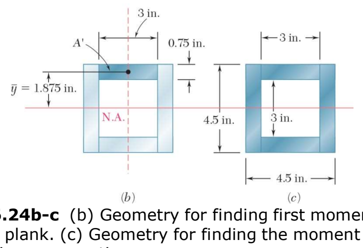

Fig. 6.24b-c (b) Geometry for finding first moment of area of top plank. (c) Geometry for finding the moment of inertia of entire cross section.

For the upper plank,

$$Q = A'y = (0.75\text{in.})(3\text{in.})(1.875\text{in.})$$

$$= 4.22\,\text{in}^3$$

For the overall beam cross-section,

$$I = \tfrac{1}{12}(4.5\,\text{in})^4 - \tfrac{1}{12}(3\,\text{in})^4$$

$$= 27.42\,\text{in}^4$$

SOLUTION:

Determine the shear force per unit length along each edge of the upper plank.

$$q = \frac{VQ}{I} = \frac{(600\,\text{lb})(4.22\,\text{in}^3)}{27.42\,\text{in}^4} = 92.3\frac{\text{lb}}{\text{in}}$$

$$f = \frac{q}{2} = 46.15\frac{\text{lb}}{\text{in}}$$

$$= \text{edge force per unit length}$$

Based on the spacing between nails, determine the shear force in each nail.

$$F = f\,\ell = \left(46.15\frac{\text{lb}}{\text{in}}\right)(1.75\,\text{in})$$

$$\boxed{F = 80.8\,\text{lb}}$$

Shearing Stresses in Thin-Walled Members

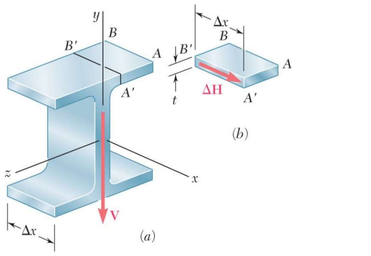

Fig. 6.25 (a) Wide-flange beam section with vertical shear V. (b) Segment of flange with longitudinal shear ΔH.

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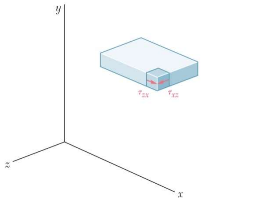


Fig. 6.26 Stress element from flange segment.

Consider a segment of a wide-flange beam subjected to the vertical shear V.

The longitudinal shear force on the element is

$$\Delta H = \frac{VQ}{I}\Delta x$$

The corresponding shear stress is

$$\tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t\,\Delta x} = \frac{VQ}{It}$$

Previously found a similar expression for the shearing stress in the web

$$\tau_{xy} = \frac{VQ}{It}$$

NOTE: $\tau_{xy} \approx 0$ in the flanges

$\tau_{xz} \approx 0$ in the web

Shearing Stresses in Thin-Walled Members

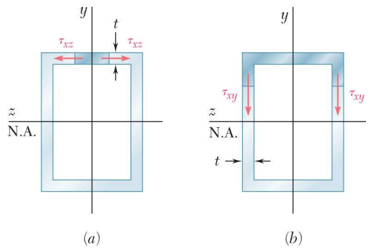


Fig. 6.28 Box beam showing shearing stress (a) in flange, (b) in web. Shaded area is used for calculating the first moment of area.

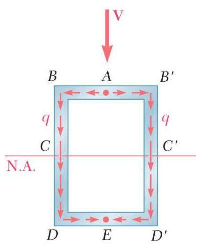


Fig. 6.30 Shear flow, q, in a box beam section.

The variation of shear flow across the section depends only on the variation of the first moment.

$$q = \tau t = \frac{VQ}{I}$$

For a box beam, q grows smoothly from zero at A to a maximum at C and C' and then decreases back to zero at E.

The sense of q in the horizontal portions of the section may be deduced from the sense in the vertical portions or the sense of the shear V.

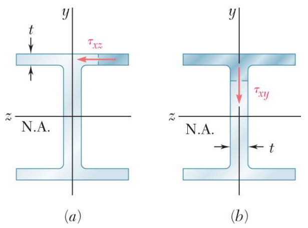


Fig. 6.27 Wide-flange beam sections showing shearing stress (a) in flange and (b) in web. The shaded area is that used for calculating the first moment of area.

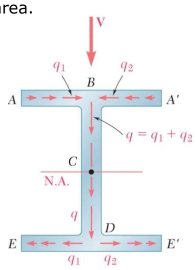


Fig. 6.31 Shear flow, q, in a wide-flange beam section.

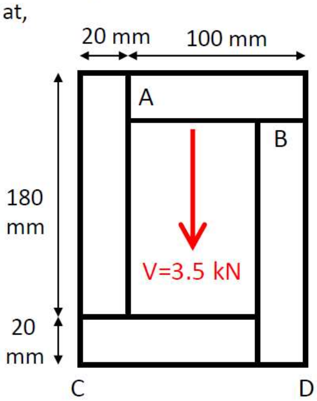
For a wide-flange beam, the shear flow increases symmetrically from zero at *A* and *A'*, reaches a maximum at *C* and then decreases to zero at *E* and *E'*.

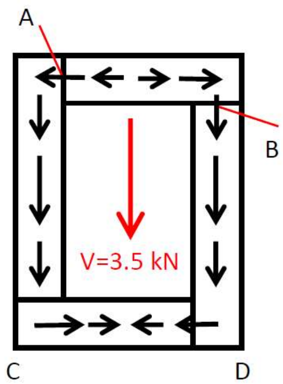
The continuity of the variation in q and the merging of q from section branches suggests an analogy to fluid flow.

Example 1

Two 100x20 mm and two 180x20 mm boards are glued together as shown to form a 120x200 mm box beam. If the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint at,

a) point A
b) point B





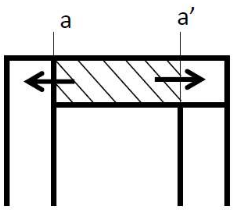
$$I = \frac{1}{12}(0.12)(0.2)^3 - \frac{1}{12}(0.08)(0.16)^3 = 5.27x10^{-5}m^4$$

Example 1

Two 100x20 mm and two 180x20 mm boards are glued together as shown to form a 120x200 mm box beam. If the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint at,

a) point A
b) point B

a)



τ is the same at a and a’

$$Q = A\bar{y} \qquad A = (0.12 - 0.04)(0.02)$$
$$\bar{y} = 0.09$$
$$Q = (0.12 - 0.04)(0.02)(0.09) = 1.44x10^{-4}m^3$$

$t = 0.02\ m$ but for both cuts $t = 2x0.02 = 0.04\ m$

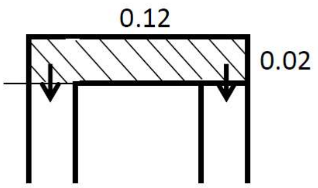
$$\tau_A = \frac{VQ}{It} = \frac{3.5x10^3x1.44x10^{-9}}{5.27x10^{-5}x0.04} = \mathbf{239.1\ kPa}$$

Example 1

Two 100x20 mm and two 180x20 mm boards are glued together as shown to form a 120x200 mm box beam. If the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint at,

a) point A
b) point B

b)



$$Q = A\bar{y} \qquad A = (0.12)(0.02)$$
$$\bar{y} = 0.09$$
$$Q = (0.12)(0.02)(0.09) = 2.16x10^{-4}m^3$$

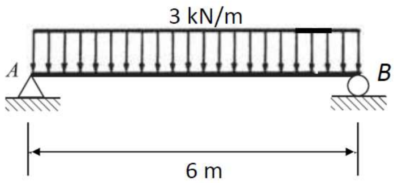
$$t = 2x0.02 = 0.04\ m$$

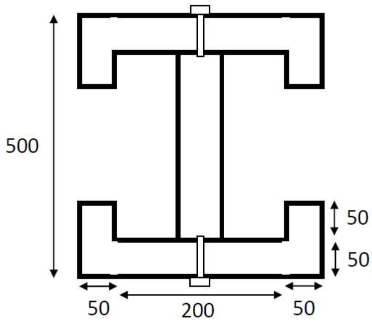
$$\tau_B = \frac{VQ}{It} = \frac{3.5x10^3x2.16x10^{-9}}{5.27x10^{-5}x0.04} = \mathbf{358.6\ kPa}$$

Example 2

A simple beam carries a uniform load of 3 kN/m. The beam's cross section is made of three pieces joint together with screws.

Specify the necessary minimum spacing of screws if one screw is capable of transmitting lateral load of 2 kN





$I = 2.367x10^9 \; mm^4$

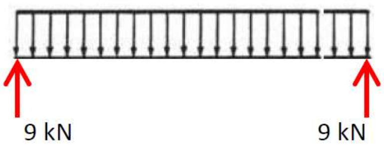
$$Q = \underbrace{2x50x100}_{A_1}x\underbrace{200}_{\bar{y}_1} + \underbrace{50x200}_{A_2}x\underbrace{225}_{\bar{y}_2} = 4.25x10^6 \; mm^3$$

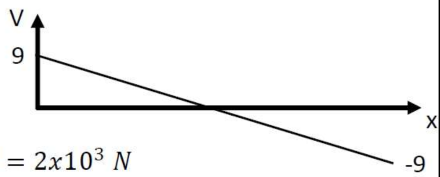
$$q = \frac{VQ}{I} = \frac{4.25x10^6}{2.367x10^9} x V \; ?$$

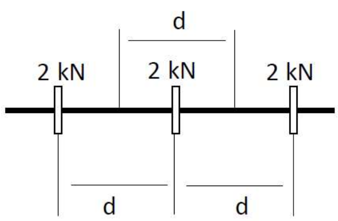
Example 2

$V = V_{max} = 9 \; kN$

$$q = \frac{VQ}{I} = \frac{4.25x10^6}{2.367x10^9} x9x10^3 = 16.2 \; N/mm$$





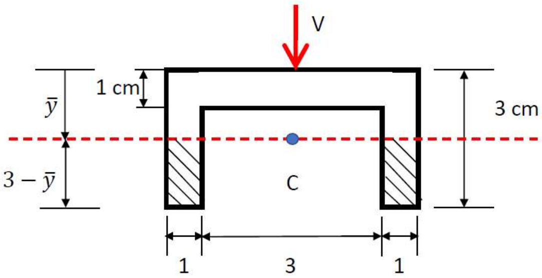


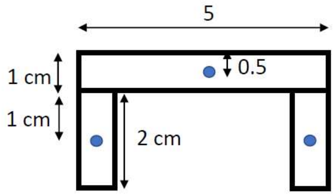
$16.2 \; N/mm \text{ x } d = 2x10^3 \; N$

$\mathbf{d = 123 \; mm}$ it can be increased for the mid span of the beam

Example 3

Determine the largest shear force V that the member can sustain if $\tau_{all} = 8\ MPa$ (allowable)



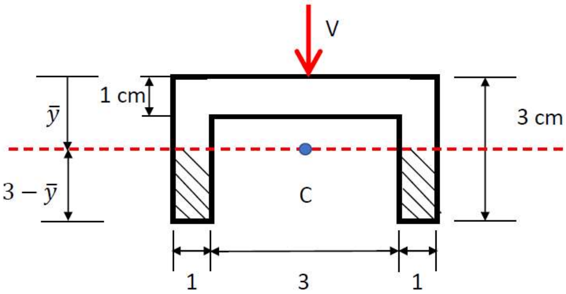


$$\bar{y} = \frac{\underbrace{(5\ x\ 1)}_{A}\ x\ \underbrace{0.5}_{y} + 2\ x\ \underbrace{(2\ x\ 1)}_{A}\ x\ \underbrace{2}_{y}}{(5\ x\ 1) + 2\ x\ (2\ x\ 1)} = 1.1667\ \text{cm}$$

$$I = \underbrace{\frac{1}{12}\ x\ 5\ x\ 1^3}_{\frac{1}{12}bh^3} + \underbrace{(5\ x\ 1)\ x\ (1.1667 - 0.5)^2}_{\text{P.A.T}} + 2\left[\frac{1}{12}\ x\ 1\ x\ 2^3 + (1\ x\ 2)\ x\ (2 - 1.1667)^2\right]$$

$$I = 2.6391 + 4.1109 = 6.75\ cm^4 = 6.75\ x\ 10^4\ mm^4$$

Example 3



$$Q_{max} = 2\ x\underbrace{[(3 - \bar{y})\ \ x\ 1]}_{A'}\ x\ \underbrace{\left(\frac{3 - \bar{y}}{2}\right)}_{\bar{y}'} = 3.3611\ cm^3$$

$$\tau_{max} = \tau_{all} = \frac{VQ}{It} = \frac{V\ x\ (3.3611\ x\ 10^3\ mm^3)}{(6.75\ x\ 10^4\ mm^4)\ x\ (2\ x\ 10\ mm)} = 8\ N/mm^2$$

$$V = 3213\ N$$