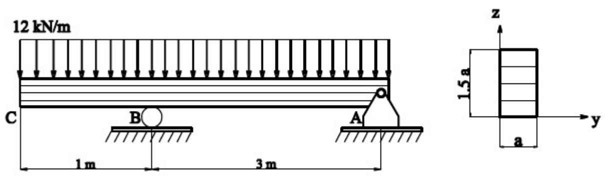
ME210 STRENGTH OF MATERIALS

Chapter 05 – Recitation 4

Example 1

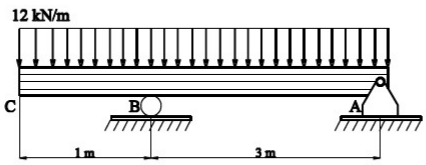


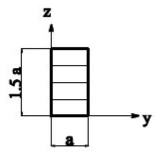
A laminated wooden beam of rectangular cross section supports a uniformly distributed load as shown. The beam is pin supported at A and simply supported at B. The beam cross section is to have a height-to-width ratio of 1.5. The allowable bending stress $\sigma_{allow} = 9\ MPa$ and the allowable shear stress is $\tau_{allow} = 0.6\ MPa$. Neglecting the weight of the beam, calculate minimum allowable a value.

$\sigma_{allow} = 9\ MPa, \quad \tau_{allow} = 0.6\ MPa.$ Find a.

- We need to find the location of **maximum shear force and moment**
- Draw shear-bending diagram

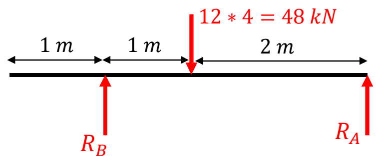
Example 1





$\sigma_{allow} = 9\ MPa,$
$\tau_{allow} = 0.6\ MPa.$
Find a.

- First find reaction forces. Draw FBD,



$\sum M_A = 0\ (CW+)$

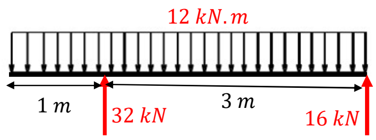
$R_B * 3 - 48 * 2 = 0 \qquad R_B = 32\ kN$

$\sum F = 0\ +\uparrow$

$R_B + R_A - 48 = 0 \qquad R_A = 16\ kN$

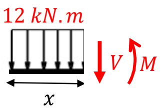
Example 1

$\frac{dV}{dx} = -w \quad V_B - V_A = -\int_A^B w dx \qquad \frac{dM}{dx} = V \quad M_B - M_A = \int_A^B V dx$



- Draw shear-moment diagram, take cuts

$\sum F = 0\ +\downarrow \qquad \sum M_A = 0\ (CCW+)$

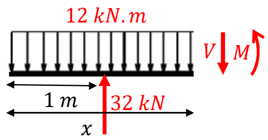


$\mathbf{0 < x < 1}$

$V + 12x = 0 \qquad M + 12x\frac{x}{2} = 0$

$\boldsymbol{V = -12x} \qquad \boldsymbol{M = -6x^2}$

$V(x=0) = 0 \quad V(x=1) = -12 \qquad M(x=0) = 0 \quad M(x=1) = -6$



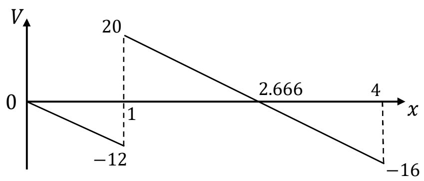
$\mathbf{1 < x < 4}$

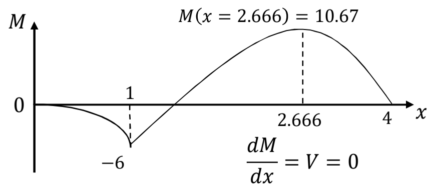
$V + 12x - 32 = 0 \qquad M + 12x\frac{x}{2} - 32(x-1) = 0$

$\boldsymbol{V = 32 - 12x} \qquad \boldsymbol{M = -6x^2 + 32x - 32}$

$V(x=1) = 20 \quad V(x=4) = -16 \qquad M(x=1) = -6 \quad M(x=4) = 0$

$M(x=2) = 8 \quad M(x=3) = 10$

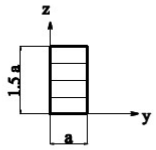




Example 1

- Check for allowable stresses

  $V_{max} = 20\ kN \qquad M_{max} = 10.67\ kN.m$



$\sigma_{allow} = 9\ MPa,$
$\tau_{allow} = 0.6\ MPa.$
Find a.

- Bending stress,

$$\sigma = \frac{Mc}{I} = \frac{(10.67x10^3)(0.75a)}{\frac{1}{12}(a)(1.5a)^3} = \frac{28.45x10^3}{a^3} = 9x10^6 = \sigma_{allow} \qquad \boldsymbol{a = 0.1468\ m}$$

- Shear stress,

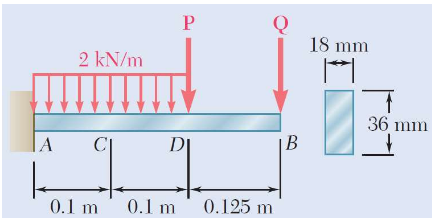
$$\tau = \frac{VQ}{It} \quad \text{or} \quad \tau_{max} = \frac{3V}{2A} = \frac{3(20x10^3)}{2(1.5a^2)} = \frac{20x10^3}{a^2} = 0.6x10^6 = \tau_{allow}$$

$$\boldsymbol{a = 0.1826\ m}$$

- Higher $\boldsymbol{a}$ will satisfy both allowable stresses

  $\boldsymbol{a = 0.1826\ m}$

Example 2



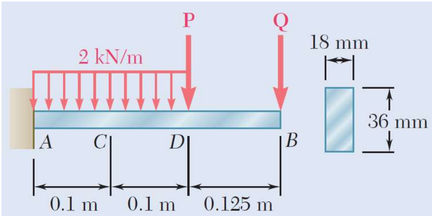
Beam AB supports a uniformly distributed load of 2 kN/m and two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is -56.9 MPa at A and -29.9 MPa at C.

a) Draw the shear and bending-moment diagrams for the beam.
b) Determine the magnitudes of the loads P and Q.
c) Find the max shearing stress on a transverse section just right of A.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.018)(0.036)^3 = 70x10^{-9}\ m^4$$

$\sigma_A = -56.9\ MPa$
$\sigma_C = -29.9\ MPa$
in the bottom edge of the beam given

Example 2



$I = 70x10^{-9}\ m^4$

$c = 0.018\ m$

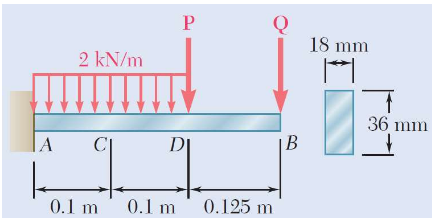
$\sigma_A = -56.9\ MPa$
$\sigma_C = -29.9\ MPa$ in the bottom edge of the beam given

$$\sigma_A = \frac{M_A c}{I} = \frac{M_A(0.018)}{70x10^{-9}} = -56.9x10^6 \qquad M_A = -221.3\ N.m$$

$$\sigma_C = \frac{M_C c}{I} = \frac{M_C(0.018)}{70x10^{-9}} = -29.9x10^6 \qquad M_C = -116.3\ N.m$$

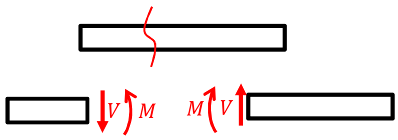
- We can use M_A and M_C to find P and Q

Example 2

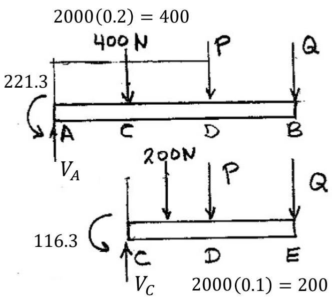


$M_A = -221.3\ N.m \qquad M_C = -116.3\ N.m$

- Reminder: Positive convention for M and V when taking cut



- Take cut at A and C



$\sum M_A = 0\ (CCW+)$

$221.3 - (400)(0.1) - P(0.2) - Q(0.325) = 0$

$P(0.2) + Q(0.325) = 181.3$

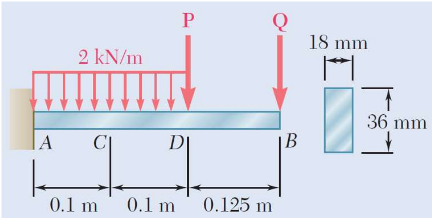
$\sum M_C = 0\ (CCW+)$

$116.3 - (200)(0.05) - P(0.1) - Q(0.225) = 0$

$P(0.1) + Q(0.225) = 106.3$

- Solving 2 eqns: **P=500 N, Q=250 N**

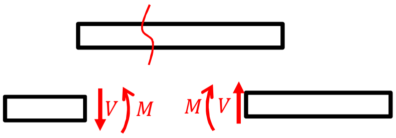
Example 2



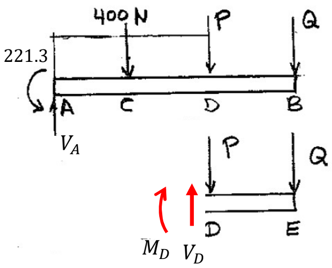
$M_A = -221.3\ N.m \qquad M_C = -116.3\ N.m$

P=500 N, Q=250 N

- Reminder: Positive convention for M and V when taking cut



- Then take cuts at A and D



$\Sigma F = 0 \ +\uparrow$

$V_A - 400 - P - Q = 0$

$V_A = 1150\ N$

$M_A = -221.3\ N.m$

$\Sigma F = 0 \ +\uparrow$

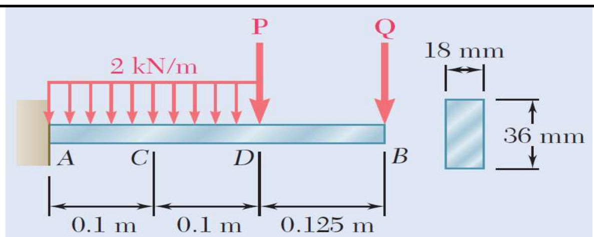
$V_D - P - Q = 0$

$V_D = 750\ N$

$\Sigma M_C = 0\ (CCW+)$

$M_D + Q(0.125) = 0$

$M_D = -31.25\ Nm$



P=500 N, Q=250 N

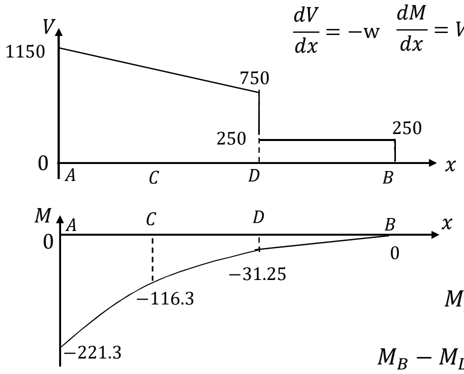
$M_C = -116.3\ N.m$

$M_A = -221.3\ N.m$

$M_D = -31.25\ Nm$

$V_A = 1150\ N \quad V_D = 750\ N$

$\frac{dV}{dx} = -w \qquad \frac{dM}{dx} = V$



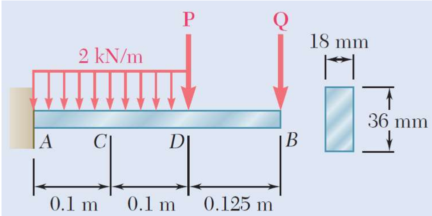
- Draw shear moment diagram

-Since there is uniformly distributed load on ACD portion, V line is linear, M line is quadratic.
-At D, there will be a sharp drop in V diagram because of concentrated force P.
-There is no force between D and B, so V is constant and M is linear.
-$M_B = 0$ because it is a free-end. You can check this using:

$$M_B - M_D = \int_D^B V dx = Area\ DB = (250)(0.125)$$

$$M_B - M_D = 31.25 \qquad M_B = 31.25 + M_D = 0$$

Example 2



P=500 N, Q=250 N

$M_C = -116.3\ N.m$

$M_A = -221.3\ N.m$

$M_D = -31.25\ Nm$

$V_A = 1150\ N \quad V_D = 750\ N$

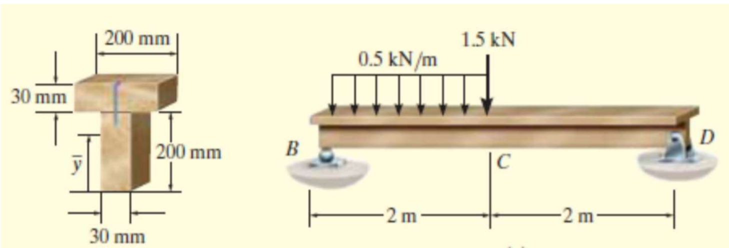
c) Find the max shearing stress on a transverse section just right of A.

$\tau = \frac{VQ}{It}$ For rectengular cross section $\tau_{max} = \frac{3V}{2A}$

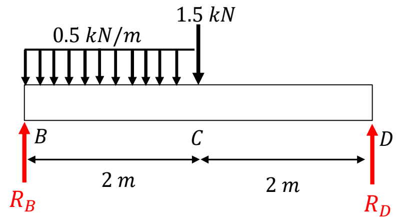
$$\frac{3V_A}{2A} = \frac{3(1150)}{2(18)(36)} = \mathbf{2.66\ MPa}$$

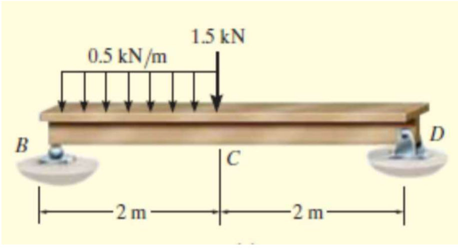
Example 3

The wooden T-beam shown is made of two 200 mm X 30 mm boards. If the allowable bending stress is $\sigma_{allow} = 12\ MPa$ and the allowable shear stress $\tau_{allow} = 0.8\ MPa$. Determine if the beam can safely support the loading shown.

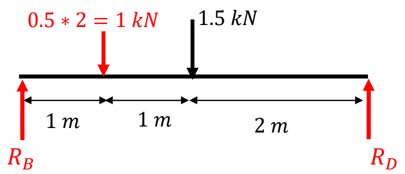


Example 3





- First we need to find reaction forces



$+\circlearrowleft\ \Sigma M_D = 0\ (CCW+)$

$-R_B * 4 + 1 * 3 + 1.5 * 2 = 0$

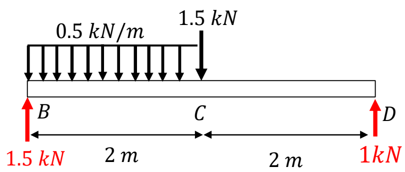
$R_B = 1.5\ kN$

$+\uparrow\ \Sigma F = 0$

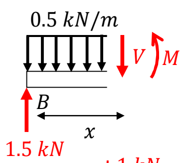
$R_B + R_D - 1 - 1.5 = 0$

$R_D = 1\ kN$

Example 3



- We can take cuts and draw shear-moment diagrams



$0 < x < 2$

$+\uparrow\ \Sigma F = 0$

$1.5 - 0.5x - V = 0$

$\boldsymbol{V = -0.5x + 1.5}$

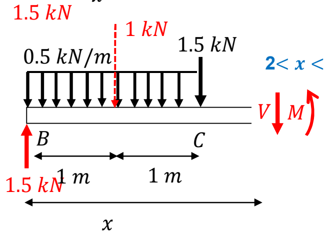
$V(x=0) = 1.5 \quad V(x=2) = 0.5$

$+\circlearrowleft\ \Sigma M_A = 0\ (CCW+)$

$-1.5x + 0.5x\frac{x}{2} + M = 0$

$\boldsymbol{M = -0.25x^2 + 1.5x}$

$M(x=0) = 0 \quad M(x=2) = 2$



$2 < x < 4$

$1.5 - 0.5(2) - 1.5 - V = 0$

$\boldsymbol{V = -1}$

$V(x=2) = -1 \quad V(x=4) = -1$

$-1.5x + 1(x-1)$
$+ 1.5(x-2) + M = 0$

$\boldsymbol{M = -x + 4}$

$M(x=2) = 2 \quad M(x=4) = 0$

Example 3

$0 < x < 2$

$V = -0.5x + 1.5$ $\quad V(x = 0) = 1.5$

$V(x = 2) = 0.5$

$M = -0.25x^2 + 1.5x$ $\quad M(x = 0) = 0$

$M(x = 2) = 2$

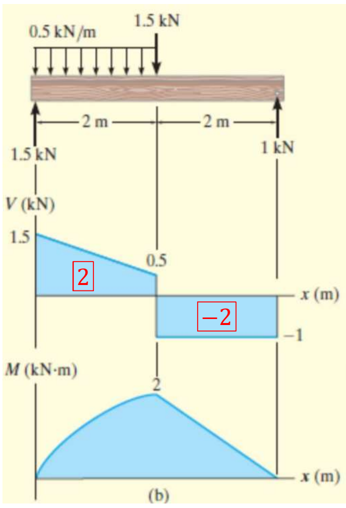
$2 < x < 4$

$V = -1$ $\quad V(x = 2) = -1$

$V(x = 4) = -1$

$M = -x + 4$ $\quad M(x = 2) = 2$

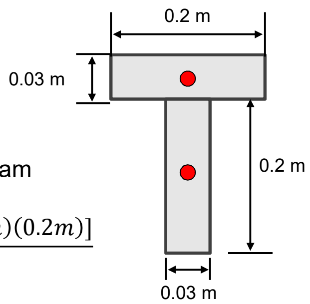
$M(x = 4) = 0$



$V_{max} = 1.5kN$

$M_{max} = 2kNm$

Example 3



- Locate the neutral axis from the bottom of the beam

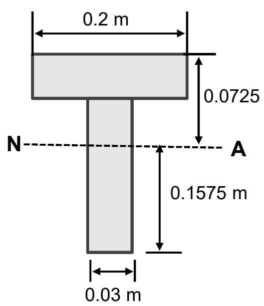
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{(0.1m)[(0.03m)(0.2m)] + (0.215m)[(0.03m)(0.2m)]}{(0.03m)(0.2m) + (0.03m)(0.2m)}$$

$$\bar{y} = 0.1575\ m$$

$$I = \frac{1}{12}(0.03m)(0.2m)^3 + (0.03m)(0.2m)(0.1575m - 0.1m)^2$$

$$+\frac{1}{12}(0.2m)(0.03m)^3 + (0.03m)(0.2m)(0.215m - 0.1575m)^2$$

$$I = 60.125\ x\ 10^{-6}\ m^4$$



Example 3

- We need to use higher value of c in order to find maximum value of stress

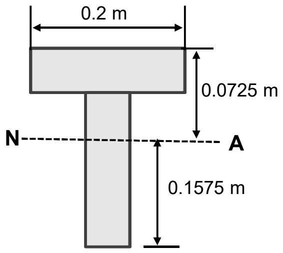
$$c = 0.1575\ m\ \ (not\ 0.23 - 0.1575 = 0.0725\ m)$$

$$\sigma_{allow} \geq \frac{M_{max}c}{I}$$

$$12\ x\ 10^6\ Pa \geq \frac{(2\ x\ 10^3\ Nm\ )(0.1575m)}{60.125\ x\ 10^{-6}\ m^4}$$

$$12\ x\ 10^6\ Pa \geq 5.24\ x\ 10^6\ Pa$$

OK ✓



Example 3

- Max shear occurs at neutral axis.
- Neutral axis is in the web where t = 0.03 m

For simplicity we can use the rectangular area below the neutral axis to calculate Q

$$Q = \bar{y}'A = \left(\frac{0.1575}{2}\right)\ x\ (0.1575\ x\ 0.03)$$

$$Q = 0.372\ x\ (10^{-3})\ m^3$$

So that

$$\tau_{allow} \geq \frac{V_{max}Q}{It}$$

$$0.8\ x10^6 Pa \geq \frac{(1.5\ x\ 10^3\ N)(0.372\ x\ 10^{-3}m^3)}{(60.125\ x\ 10^{-6}\ m^4)(0.03m)}$$

$$0.8\ x10^6 Pa \geq 0.309\ x\ 10^6\ Pa$$

OK ✓