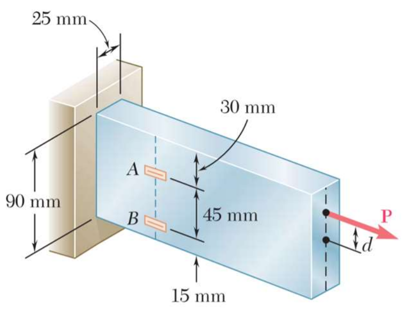
ME210 STRENGTH OF MATERIALS

Chapter 04 – Recitation 3

Example 1

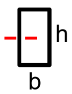


An eccentric force P is applied as shown to a steel bar of 25 90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\ \mu \qquad \varepsilon_B = -70\ \mu$$

Knowing that E=200GPa, determine
(a) The distance d
(b) The magnitude of the force P

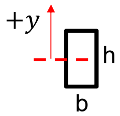
- We need to find moment of inertia I of the cross section

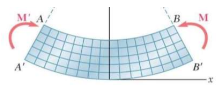


$$b = 25mm \quad h = 15 + 45 + 30 = 90\ mm \quad c = \frac{1}{2}h = 45\ mm = 0.045\ m$$

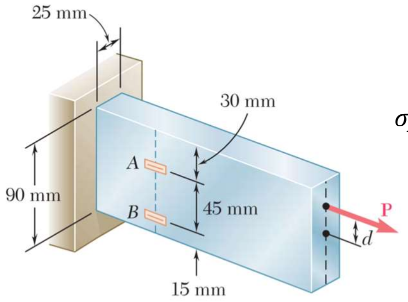
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\ x\ 10^6\ mm^4$$
$$= 1.51875\ x\ 10^{-6}\ m^4$$

Example 1





+M convention



$$\sigma_A = \frac{P}{A} - \frac{M y_A}{I} \quad (1) \qquad \sigma_B = \frac{P}{A} - \frac{M y_B}{I} \quad (2)$$

- The distances of strain gages to the center of cross section:

$$y_A = 60 - 45 = 15\ mm = 0.015\ m$$

$$y_B = 15 - 45 = -30\ mm = -0.030\ m$$

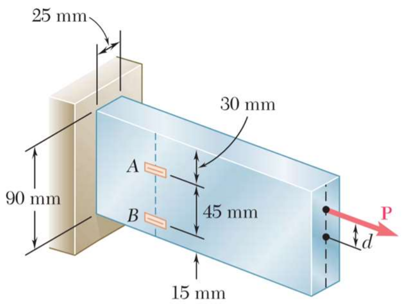
- Stresses from strain gages at *A* and *B*:

$$\sigma_A = E\varepsilon_A = (200\ x\ 10^9)(350\ x\ 10^{-6}) = 70\ x\ 10^6\ Pa$$

$$\sigma_B = E\varepsilon_B = (200\ x\ 10^9)(-70\ x\ 10^{-6}) = -14\ x\ 10^6\ Pa$$

$$A = bh = (25)(90) = 2.25\ x\ 10^3\ mm^2 = 2.25\ x\ 10^{-3} m^2$$

Example 1



$$\sigma_A = \frac{P}{A} - \frac{M y_A}{I} \quad (1) \qquad \sigma_B = \frac{P}{A} - \frac{M y_B}{I} \quad (2)$$

$\sigma_A = 70\ x\ 10^6\ Pa$ $\qquad A = 2.25\ x\ 10^{-3} m^2$

$\sigma_B = -14\ x\ 10^6\ Pa$ $\qquad y_A = 0.015\ m$

$I = 1.51875\ x\ 10^{-6}\ m^4$ $\qquad y_B = -0.030\ m$

- Substracting

$$\sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$$

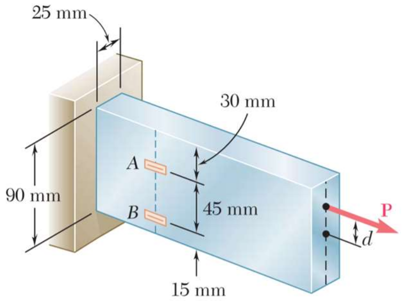
$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B}$$

$$M = -\frac{1.51875\ x\ 10^{-6}(84\ x\ 10^6)}{0.045}$$

$$M = -2835\ Nm$$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B}$$ +M convention

Example 1



$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \quad (1) \qquad \sigma_B = \frac{P}{A} - \frac{My_B}{I} \quad (2)$$

$\sigma_A = 70\ x\ 10^6\ Pa$ $\quad A = 2.25\ x\ 10^{-3} m^2$

$\sigma_B = -14\ x\ 10^6\ Pa$ $\quad y_A = 0.015\ m$

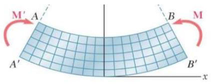
$I = 1.51875\ x\ 10^{-6}\ m^4$ $\quad y_B = -0.030\ m$

- Multiplying (2) by y_A and (1) by y_B and subtracting:

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\ x\ 10^{-3})[(0.015)(-14\ x\ 10^6) - (-0.030)(70\ x\ 10^6)]}{0.045}$$

$$\boxed{P = 94.5\ x\ 10^3 N}$$

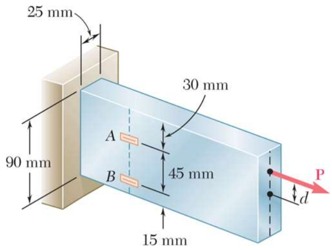


+M convention

$$M = -Pd$$

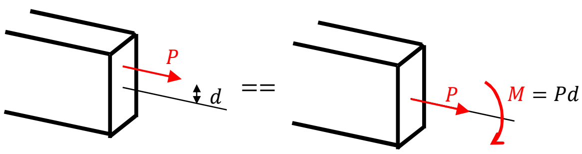
$$\boxed{d = -\frac{M}{P} = -\frac{-2835}{94.5\ x\ 10^3} = 0.030\ m}$$

Example 1



$\varepsilon_A = +350\ \mu$ $\quad \varepsilon_B = -70\ \mu$

- Alternative way using absolute values without signs
- Assume directions of P and M



- Think about each term separately, and decide if they are tension (T) or compression (C)
- Assume tension is '+', compression is '-'

$$|\sigma_A| = \frac{P}{A} \quad \frac{M|y_A|}{I} \quad (1)$$

(T) (T) (T)

$$+|\sigma_A| = +\frac{P}{A} + \frac{M|y_A|}{I}$$

$$+70x10^6 = +\frac{P}{2.25x10^{-3}} + \frac{M(0.015)}{1.51875x10^{-6}}$$

$$|\sigma_B| = \frac{P}{A} \quad \frac{M|y_B|}{I} \quad (2)$$

(C) (T) (C)

$$-|\sigma_B| = +\frac{P}{A} - \frac{M|y_B|}{I}$$

$$-14x10^6 = +\frac{P}{2.25x10^{-3}} - \frac{M(0.030)}{1.51875x10^{-6}}$$

- Solve

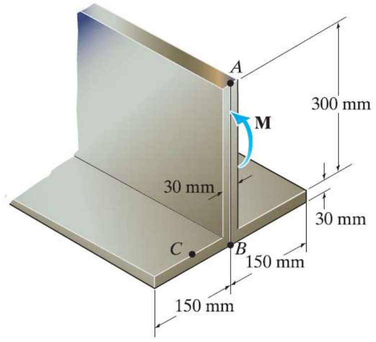
$P = 94500\ N$ (T)

$M = 2835\ Nm$ (+)

$M = Pd$

$$\boxed{d = 0.030\ m}$$

Example 2

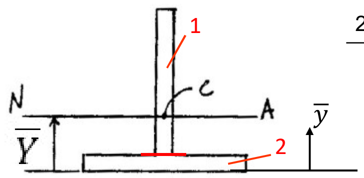


If the beam is subjected to an internal moment of M=100 kN.m, determine the bending stress developed at points A, B and C. Sketch the bending stress distribution on the cross section.)

$$\sigma = -\frac{My}{I}$$

- First, we need to find centroid and neutral axis (n.a.). $y \; and \; I$ are measured using centroid

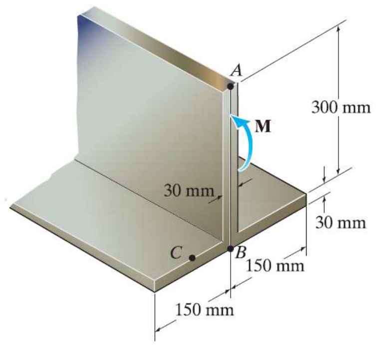
	A $[mm^2]$	$\bar{y}$ [mm]	A$\bar{y}$ $[mm^3]$
1	$(30)(300) = 9000$	30+300/2=180	1620000
2	$(300)(30) = 9000$	30/2=15	135000
	18000		1755000



$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{1755000}{18000} = 97.5\;mm = 0.0975\;m$$

Location of centroid from bottom edge

Example 2

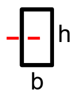


If the beam is subjected to an internal moment of M=100 kN.m, determine the bending stress developed at points A, B and C. Sketch the bending stress distribution on the cross section.)

$$\sigma = -\frac{My}{I}$$

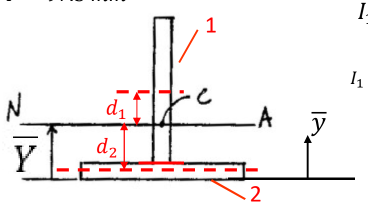
- Second, we need to find moment of inertia I for this cross section about centroid.

$$I = I_1 + I_2$$



$$I = \frac{1}{12}bh^3$$

$\bar{Y} = 97.5\;mm$



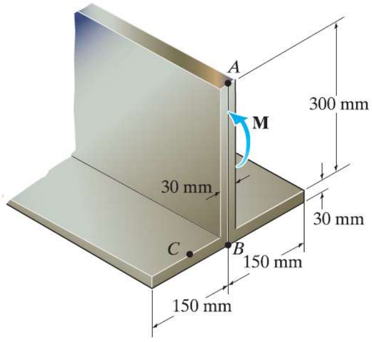
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 \qquad I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2$$

$$I_1 = \frac{1}{12}(30)(300)^3 + (9000)(30 + 150 - 97.5)^2 = 128756250\;mm^4$$

$$I_2 = \frac{1}{12}(300)(30)^3 + (9000)(97.5 - 15)^2 = 61931250\;mm^4$$

$$I = I_1 + I_2 = 190687500\;mm^4 = 0.1907x10^{-3}m^4$$

Example 2



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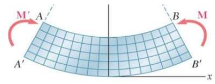
$$\sigma = -\frac{My}{I}$$



+M convention

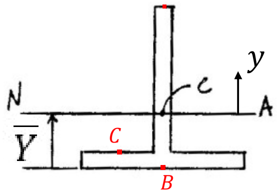
y: Distance between centoid and given point

$$y_A = 300 + 30 - 97.5 = 232.5\ mm$$

$$y_B = -97.5\ mm \qquad y_C = -(97.5 - 30) = -67.5\ mm$$

$$M = +100x10^3 N.m$$

$\overline{Y} = 97.5\ mm$

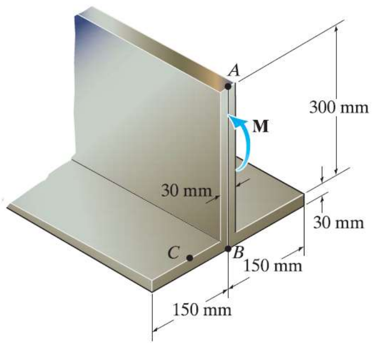


$$\sigma_A = -\frac{My_A}{I} = -\frac{100(10^3)(0.2325)}{0.1907(10^{-3})} = -122 = \mathbf{122\ MPa\ (C)}$$

$$\sigma_B = -\frac{My_B}{I} = -\frac{100(10^3)(-0.0975)}{0.1907(10^{-3})} = +51.1 = \mathbf{51.1\ MPa\ (T)}$$

$$\sigma_C = -\frac{My_C}{I} = -\frac{100(10^3)(-0.0675)}{0.1907(10^{-3})} = +35.4 = \mathbf{35.4\ MPa\ (T)}$$

Example 2



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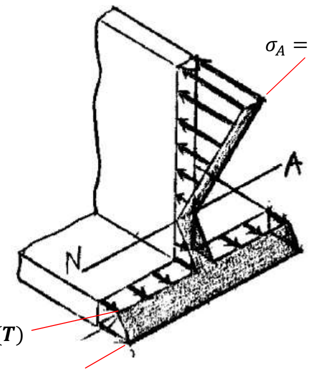

$\sigma_A = \mathbf{122\ MPa\ (C)}$

$\sigma_C = \mathbf{35.4\ MPa\ (T)}$

$\sigma_B = \mathbf{51.1\ MPa\ (T)}$

Zero at neutral axis

Changes linearly above and below n.a.

$$\sigma = -\frac{My}{I}$$