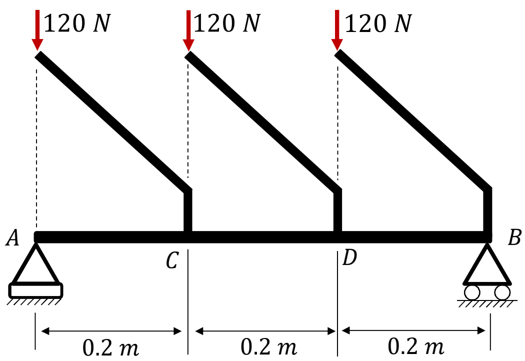
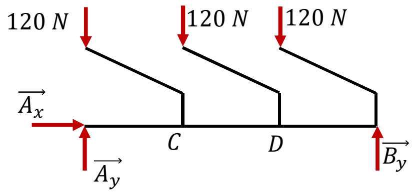
ME201 – Statics

Chapter 7&8 – Recitation 06

Example 1



Draw the shear force and bending moment diagrams for the beam AB.



$$(\circlearrowleft +) \sum \overrightarrow{M_A} = 0$$

$$-120 \times 0.2 - 120 \times 0.4 - B_y \times 0.6 = 0$$

$$B_y = 120\ N\ (\uparrow)$$

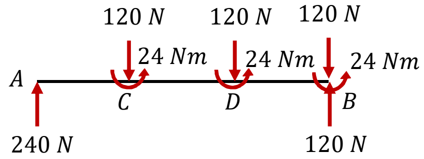
$$(\uparrow +) \quad \sum \overrightarrow{F_y} = 0$$

$$-360 + A_y + 120 = 0$$

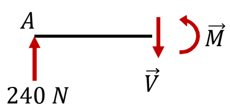
$$A_y = 240\ N\ (\uparrow)$$

$$(\rightarrow +) \quad \sum \overrightarrow{F_x} = 0$$

$$A_x = 0$$



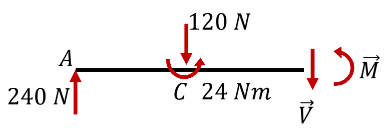
Between $A - C$



$$\vec{V} = 240\ N$$

$$\vec{M} = 240x$$

Between $C - D$

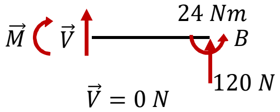


$$\vec{V} = 120\ N$$

$$\vec{M} - 240x + 120(x - 0.2) + 24 = 0$$

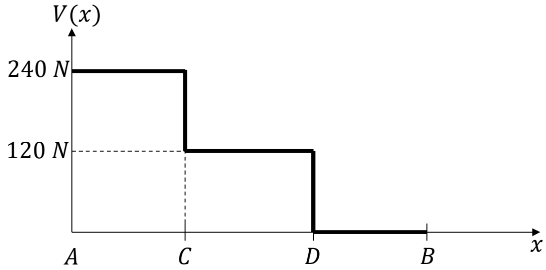
$$\vec{M} = 120x$$

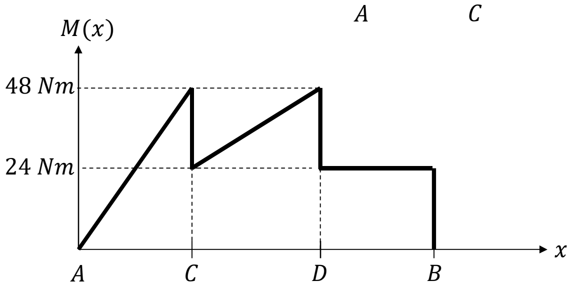
Between D – B



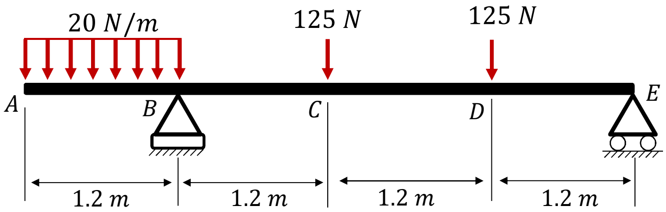
$\vec{V} = 0\ N$

$\vec{M} = 24\ Nm$

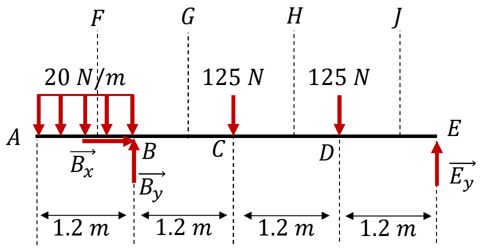




Example 2



Draw the shear force and bending moment diagrams for the beam AE.

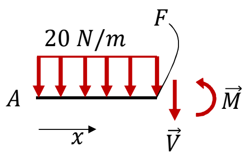


$$(\circlearrowleft +) \sum \overrightarrow{M_B} = 0$$
$$-125 \times 1.2 - 125 \times 2.4 - E_y \times 3.6 + (20 \times 1.2) \times 0.6 = 0$$
$$E_y = 121\ N\ (\uparrow)$$

$$(\uparrow +) \quad \sum \overrightarrow{F_y} = 0$$
$$-24 + B_y - 125 - 125 + 121 = 0$$
$$B_y = 153\ N\ (\uparrow)$$

$$B_x = 0$$

<u>$A - B$ @F</u>



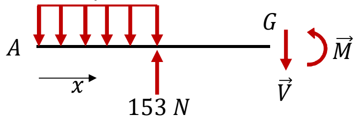
$$(\uparrow +) \quad \sum \overrightarrow{F_y} = 0$$
$$V + 20x = 0$$
$$\boxed{V = -20x}$$

$V(x = 0) = 0$
$V(x = 1.2) = -24\ N$

$$(\circlearrowleft +) \sum \overrightarrow{M_F} = 0$$
$$M + 20x\ \frac{x}{2} = 0$$
$$\boxed{M = -10\,x^2}$$

$M(x = 0) = 0$
$M(x = 1.2) = -14.4\ Nm$

<u>$B - C$ @G</u>

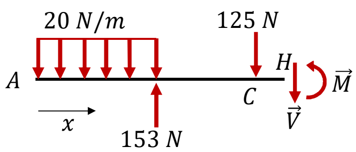


$$(\uparrow +) \quad \sum \overrightarrow{F_y} = 0$$
$$V + 20 \times 1.2 - 153 = 0$$
$$\boxed{V = 129\ N}$$

$$(\circlearrowleft +) \sum \overrightarrow{M_G} = 0$$
$$M + (20 \times 1.2)(x - 0.6) - 153 \times (x - 1.2) = 0$$
$$\boxed{M = 129x - 169.2}$$

$M(x = 1.2) = -14.4\ Nm$
$M(x = 2.4) = 140.4\ Nm$

$\underline{C - D\ @H}$



$(\uparrow +)\quad \sum \overrightarrow{F_y} = 0$

$V + 20 \times 1.2 - 153 + 125 = 0$

$\boxed{V = 4\ N}$

$(\circlearrowright +) \sum \overrightarrow{M_H} = 0$

$M + (20 \times 1.2)(x - 0.6) - 153 \times (x - 1.2)$

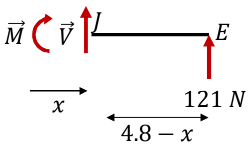
$+125 \times (x - 2.4) = 0$

$\boxed{M = 4x - 130.8}$

$M(x = 2.4) = 140.4\ Nm$

$M(x = 3.6) = 145.2\ Nm$

$\underline{D - E\ @J}$



$(\uparrow +)\quad \sum \overrightarrow{F_y} = 0$

$V + 121 = 0$

$\boxed{V = -121\ N}$

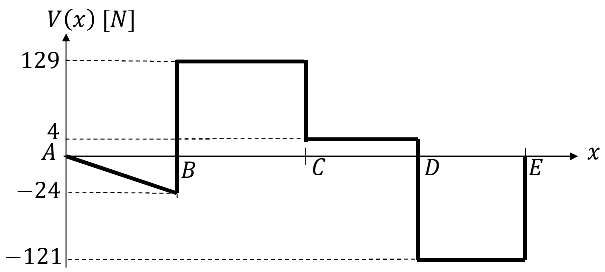
$(\circlearrowright +) \sum \overrightarrow{M_J} = 0$

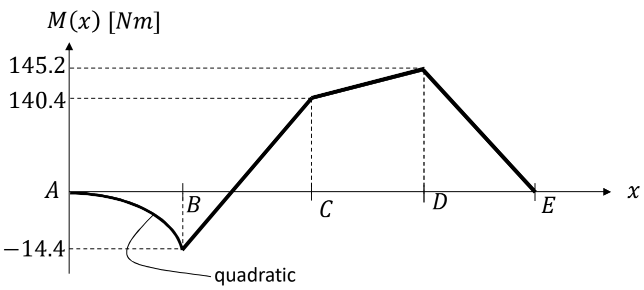
$M - 121 \times (4.8 - x) = 0$

$\boxed{M = 580.8 - 121x}$

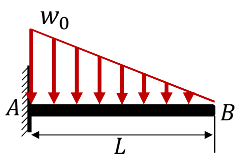
$M(x = 3.6) = 145.2\ Nm$

$M(x = 4.8) = 0\ Nm$

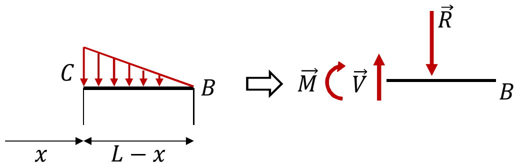




Example 3



Draw the shear force and bending moment diagrams for the beam AB.



$$R = \frac{L-x}{L} w_0 \frac{L-x}{2}$$

$$R = \frac{(L-x)^2 w_0}{2L}$$

$$(\uparrow +) \quad \sum \vec{F_y} = 0$$

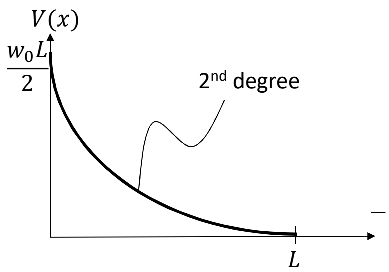
$$-\frac{(L-x)^2 w_0}{2L} + V = 0$$

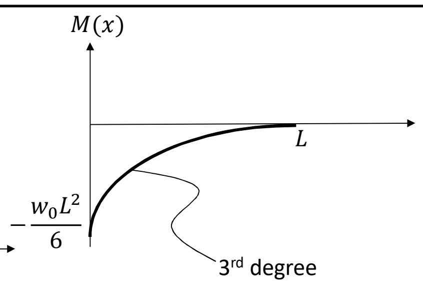
$$\boxed{V = \frac{(L-x)^2 w_0}{2L}}$$

$$(\circlearrowleft +) \sum \vec{M_c} = 0$$

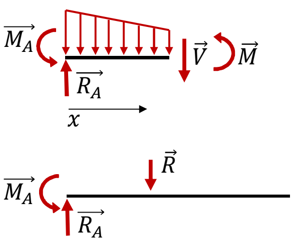
$$\frac{L-x}{3} \times \frac{(L-x)^2 w_0}{2L} + M = 0$$

$$\boxed{M = -w_0 \frac{(L-x)^3}{6L}}$$





One can solve the problem considering the left portion of the beam as well.



First determine the reactions.

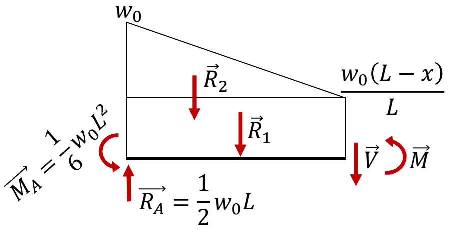
$$(\uparrow +) \quad \sum \vec{F_y} = 0$$

$$R_A - \frac{1}{2} L w_o = 0 \ (\uparrow)$$

$$(\circlearrowleft +) \sum \vec{M_A} = 0$$

$$-\frac{1}{3} L \times \frac{L}{2} L w_0 + M_A = 0$$

$$M_A = \frac{1}{6} w_0 L^2 \ (\circlearrowleft)$$



$$w(x) = \frac{w_0}{L}(L-x)$$

$$\vec{R}_1 = \frac{w_0(L-x)}{L}x \ (\downarrow)$$

$$\vec{R}_2 = \frac{1}{2}x\left(w_0 - \frac{w_0(L-x)}{L}\right)$$

$$\vec{R}_2 = \frac{1}{2}\frac{w_0x^2}{L} \ (\downarrow)$$

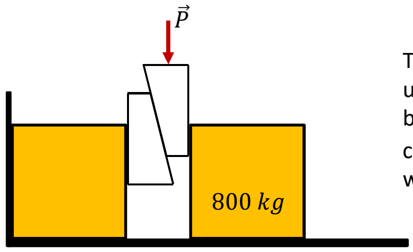
$$(\uparrow +) \quad \sum \vec{F}_y = 0 \qquad V - \frac{1}{2}w_0L + \frac{w_0(L-x)}{L}x + \frac{1}{2}\frac{w_0x^2}{L} = 0$$

$$\boxed{V = \frac{w_0(L-x)^2}{2L}}$$

$$(\circlearrowleft +) \sum \vec{M}_c = 0 \qquad M + \frac{1}{6}w_0L^2 - \frac{1}{2}w_0Lx + \frac{w_0(L-x)}{L}x\frac{x}{2} + \frac{1}{2}w_0\frac{x^2}{L}\frac{2}{3}x = 0$$

$$\boxed{M = -\frac{w_0}{6L}(L-3)^3}$$

Example 4

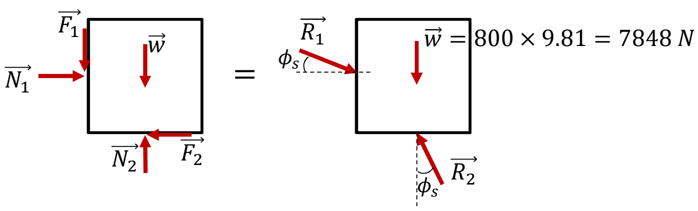


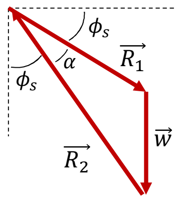
Two 8° wedges of negligible weight are used to move and position the 800 kg block. $\mu_s = 0.25$ at all surfaces of contact. Determine the smallest force $\vec{P}$ which should be applied as shown.

$$\mu_s = 0.25$$

$$\phi_s = \tan^{-1}(\mu_s) = 14.04°$$

FBD of the block



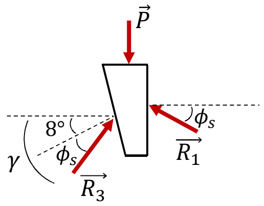


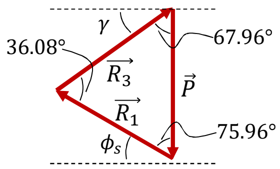
$$\alpha = 90 - 2\phi_s = 90 - 2 \times 14.04 = 61.92°$$

Law of sines

$$\frac{R_1}{\sin \phi_s} = \frac{w}{\sin \alpha} \qquad R_1 \approx 2158\ N$$

FBD of the wedge





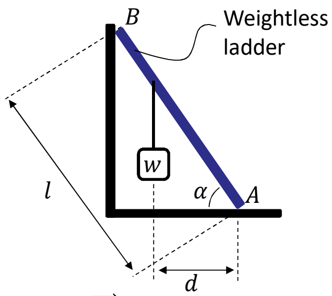
Law of sines

$$\frac{P}{\sin 36.08} = \frac{R_1}{\sin 67.96}$$

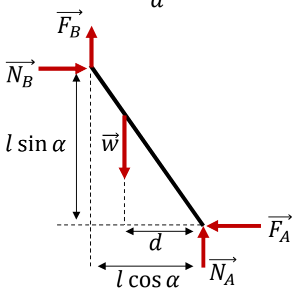
$$\boxed{\vec{P} = 1371\ N(\downarrow)}$$

One can solve the same problem by considering the FBD of 800 kg block & the wedge and writing $\sum F_x = 0$ & $\sum F_y = 0$.

Example 5



What is the maximum value of d in terms of μ_s, l and α? μ_s is the same for both surfaces.



$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

$(\uparrow +)\quad \sum \vec{F_y} = 0 \qquad F_B + N_A - w = 0$

$$\boxed{w = N_A + \mu_s N_B \qquad (1)}$$

$(\rightarrow +)\quad \sum \vec{F_x} = 0 \qquad N_B - F_A = 0$

$$\boxed{N_B = \mu_s N_A \qquad (2)}$$

$(\circlearrowright +)\sum \overrightarrow{M_A} = 0$
$wd - N_B l \sin\alpha - F_B l \cos\alpha = 0$
$wd = N_B l \sin\alpha - \mu_s N_B l \cos\alpha$

$$d = \frac{N_B l(\sin\alpha + \mu_s \cos\alpha)}{w} \qquad (3)$$

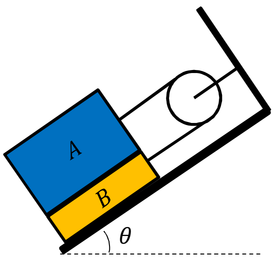
$(2) \rightarrow (1) \quad \mu_s N_B + \frac{1}{\mu_s} N_B = w$

$$w = N_B\left(\frac{1+\mu_s^2}{\mu_s}\right) \qquad (4)$$

$(4) \rightarrow (3) \quad \mu_s N_B + \frac{1}{\mu_s} N_B = w$

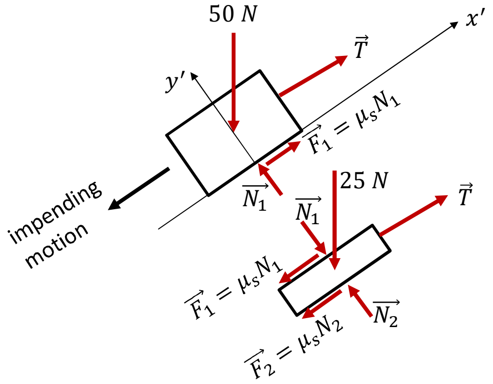
$$d = \frac{\mu_s l(\sin\alpha + \mu_s \cos\alpha)}{1+\mu_s^2}$$

Example 6



Block A weights $50\,N$ and block B weights $25\,N$. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which the motion is impending.

A direction of impending motion should be selected.



FBD of block A

$(\nwarrow +)\quad \sum \overrightarrow{F_{y'}} = 0 \qquad N_1 - 50\cos\theta = 0 \qquad \boxed{N_1 = 50\cos\theta \qquad (1)}$

$(\nearrow +)\quad \sum \overrightarrow{F_{x'}} = 0 \qquad T + F_1 - 50\sin\theta = 0$

$$\boxed{T = -F_1 + 50\sin\theta \qquad (2)}$$

$$F_1 = \mu_s N_1 = 0.15(50\cos\theta) = 7.5\cos\theta$$

Substitute into (2)

$$\boxed{T = 50\sin\theta - 7.5\cos\theta \qquad (3)}$$

FBD of block B

$(\nwarrow +)\quad \sum \overrightarrow{F_{y'}} = 0 \qquad N_2 - N_1 - 25\cos\theta = 0$

Substitute N_1 from (1)

$$\boxed{N_2 = 75\cos\theta \qquad (4)}$$

$(\nearrow +)\quad \sum \overrightarrow{F_{x'}} = 0$

$T - F_1 - F_2 - 25\sin\theta = 0$

$F_2 = 0.15(75\cos\theta) = 11.25\cos\theta$

$$\boxed{T = 25\sin\theta + 18.75\cos\theta \qquad (5)}$$

By equating (3) and (5)

$$50\sin\theta - 7.5\cos\theta = 25\sin\theta + 18.75\cos\theta$$

$$\tan\theta = \frac{26.25}{25}$$

$$\boxed{\theta \approx 46.4°}$$

If the opposite direction is chosen.

FBD of block A

$(\nwarrow +)\ \sum \overrightarrow{F_{y'}} = 0 \qquad N_1 - 50\cos\theta = 0 \qquad \boxed{N_1 = 50\cos\theta \qquad (1)}$

$(\nearrow +)\ \sum \overrightarrow{F_{x'}} = 0 \qquad T - F_1 - 50\sin\theta = 0$

$$\boxed{T = 7.5\cos\theta + 50\sin\theta \qquad (2)}$$

FBD of block B

$(\nwarrow +)\ \sum \overrightarrow{F_{y'}} = 0 \qquad N_2 - N_1 - 25\cos\theta = 0$

$$\boxed{N_2 = 75\cos\theta \qquad (3)}$$

$(\nearrow +)\ \sum \overrightarrow{F_{x'}} = 0$

$T + F_1 + F_2 - 25\sin\theta = 0$

$T + 7.5\cos\theta + 11.25\cos\theta - 25\sin\theta = 0$

$$\boxed{T = 25\sin\theta - 18.75\cos\theta \qquad (4)}$$

By equating (2) and (5) $\quad 7.5\cos\theta + 50\sin\theta = 25\sin\theta - 18.75\cos\theta$

$$\tan\theta = -\frac{26.25}{25}$$

$\boxed{\theta \approx -46.4°}$ Does not make sense!