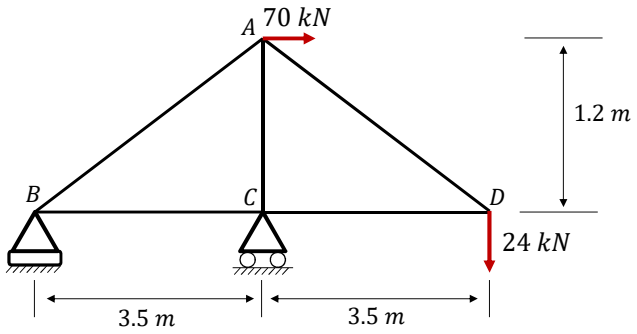
ME201 – Statics

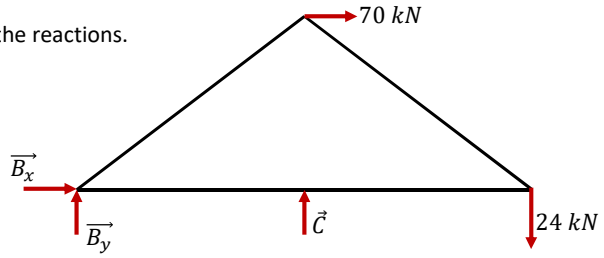
Chapter 6 – Recitation 05

Example 1



Determine the forces in each member.

First find the reactions.



$(\rightarrow +)\quad \sum \overrightarrow{F_x} = 0$
$B_x + 70 = 0$
$B_x = -70\ kN\ (\leftarrow)$

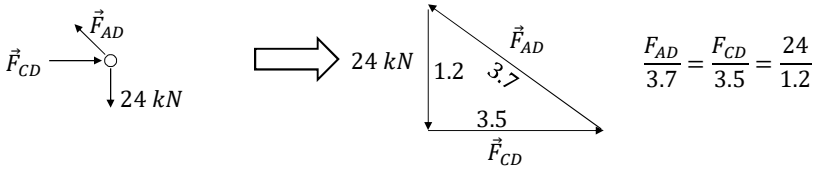
$(\circlearrowleft +) \sum \overrightarrow{M_B} = 0$
$3.5 \times C - 70 \times 1.2 - 7 \times 24 = 0$
$C = 72\ kN\ (\uparrow)$

$(\uparrow +)\quad \sum \overrightarrow{F_y} = 0$
$B_y + C - 24 = 0$
$B_y = -48\ kN\ (\downarrow)$

Joint *D*

Force applied on pin D by AD

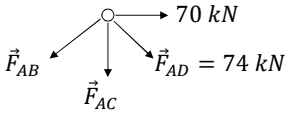
Force applied on AD by pin D



$$\frac{F_{AD}}{3.7} = \frac{F_{CD}}{3.5} = \frac{24}{1.2}$$

$$F_{AD} = 74\ kN\ \ (T)$$
$$F_{CD} = 70\ kN\ \ (C)$$

Joint A



F_{AB} and F_{AC} are assumed as tension.

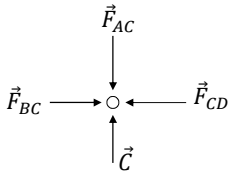
$(\rightarrow +)\quad \sum \overrightarrow{F_x} = 0 \qquad 70 + 74\frac{3.5}{3.7} - F_{AB}\frac{3.5}{3.7} = 0$

$$\boxed{F_{AB} = 148\ kN\ (T)}$$

$(\uparrow +)\quad \sum \overrightarrow{F_y} = 0 \qquad -74\frac{1.2}{3.7} - 148\frac{1.2}{3.7} - F_{AC} = 0$

$$\boxed{\begin{matrix} F_{AC} = -72\ kN \\ F_{AC} = 72\ kN\ (C) \end{matrix}}$$

Joint C

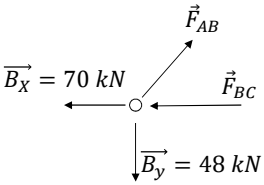


BC has to be in compression to satisfy the equilibrium at pin C. Take F_{BC} as compression.

$(\rightarrow +)\quad \sum \overrightarrow{F_x} = 0 \qquad F_{BC} - 70 = 0$

$$\boxed{F_{BC} = 70\ kN\ (C)}$$

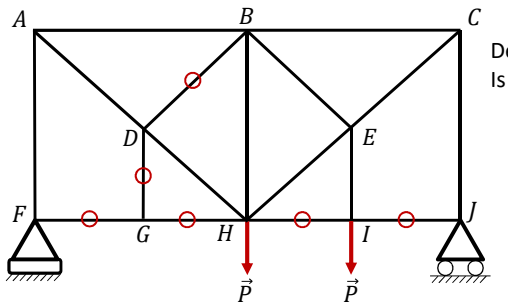
Joint B (to check)



$(\uparrow +)\quad \sum \overrightarrow{F_y} = 0 \qquad 148\frac{1.2}{3.7} - 48 = 0$ ✓

$(\rightarrow +)\quad \sum \overrightarrow{F_x} = 0 \qquad 148\frac{3.5}{3.7} - 70 - 70 = 0$ ✓

Example 2



Determine the zero force members.
Is the truss a simple truss?

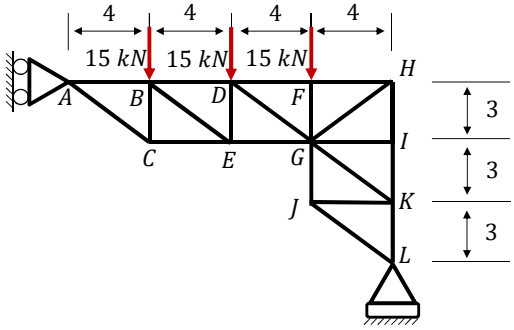
Since there is no horizontal actual force.

$\sum \overrightarrow{F_x} = 0$

Joint G $\rightarrow$ $F_{GD} = 0$
Joint D $\rightarrow$ $F_{DB} = 0$
Joint F $\rightarrow$ $F_{FG} = 0$ $\rightarrow$ $F_{HG} = 0$
Joint I $\rightarrow$ $F_{GJ} = F_{IH} = 0$

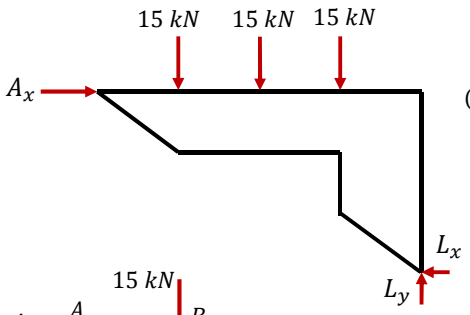
$m = 2n - 3$
$m = 17$
$n = 10$
$17 = 2 \times 10 - 3$
It is a simple truss !

Example 3



Determine the forces in members BD, BE and CE of the truss structure.

Check zero force members.

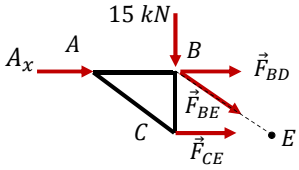


$(\circlearrowleft+)\sum \overrightarrow{M_L} = 0$

$15 \times 12 + 15 \times 8 + 15 \times 4 - A_x \times 9 = 0$

$\boxed{A_x = 40\ kN\ (\rightarrow)}$

No need to determine L_x & L_y.



$(\circlearrowleft+)\sum \overrightarrow{M_B} = 0 \rightarrow \quad F_{CE} \times 3 = 0$ $\quad \boxed{\vec{F}_{CE} = 0}$

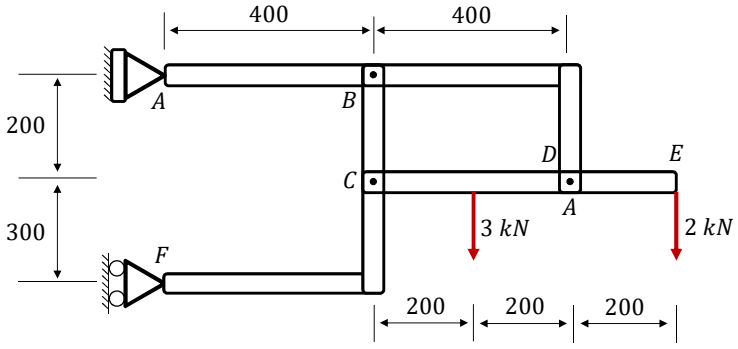
$(\circlearrowleft+)\sum \overrightarrow{M_E} = 0 \rightarrow \quad 15 \times 4 - F_{BD} \times 3 - A_x \times 3 = 0$

$F_{BD} = -20\ kN$

$\boxed{F_{BD} = 20\ kN\ (C)}$

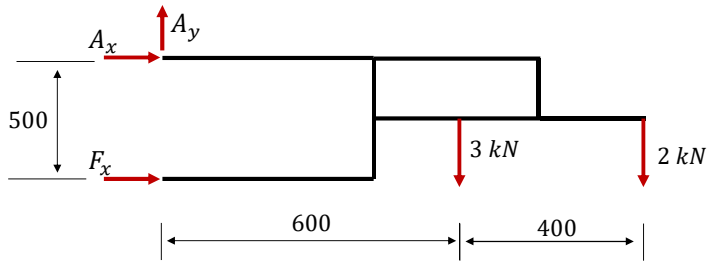
$(\uparrow +)\sum \overrightarrow{F_y} = 0 \quad \rightarrow \quad -15 - \frac{3}{5}F_{BE} = 0 \quad F_{BE} = -25\ kN$ $\quad \boxed{F_{BE} = 25\ kN\ (C)}$

Example 4



For the frame and the loading shown determine the components of all forces acting on member ABD.

FBD of the entire structure

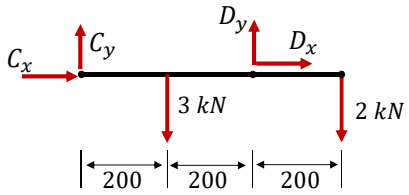


$(\circlearrowleft+)\sum\overrightarrow{M_A}=0 \rightarrow F_x\times 500-3\times 600-2\times 1000=0$ $\boxed{F_x=7.6\ kN\ (\rightarrow)}$

$(\rightarrow+)\sum\overrightarrow{F_x}=0 \rightarrow A_x+F_x=0$ $\boxed{A_x=7.6\ kN\ (\leftarrow)}$

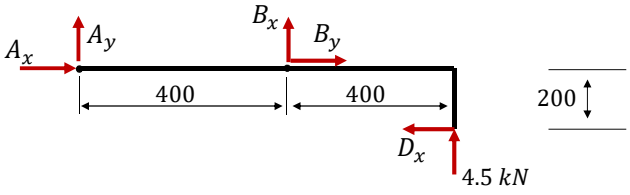
$(\uparrow+)\sum\overrightarrow{F_y}=0 \rightarrow$ $\boxed{A_y=5\ kN\ (\uparrow)}$

FBD of CDE



$(\circlearrowleft+)\sum\overrightarrow{M_C}=0 \rightarrow D_y\times 400-3\times 200-2\times 600=0$ $\boxed{D_y=4.5\ kN\ (\uparrow)}$

FBD of ABD



$(\circlearrowleft+)\sum\overrightarrow{M_B}=0 \rightarrow \quad -D_x\times 200-4.5\times 400-5\times 400=0$

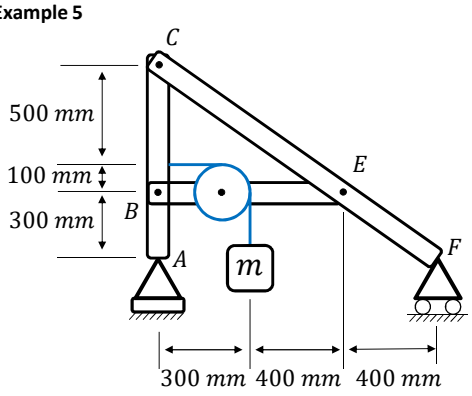
$D_x=-19\ kN$

$\boxed{D_x=19\ kN\ (\rightarrow)}$

$(\rightarrow+)\sum\overrightarrow{F_x}=0 \quad \rightarrow \quad B_x+19-7.6=0$ $\quad \boxed{B_x=11.4\ kN\ (\leftarrow)}$

$(\uparrow+)\sum\overrightarrow{F_y}=0 \quad \rightarrow \quad B_y+5-4.5=0$ $\quad \boxed{B_y=0.5\ kN\ (\uparrow)}$

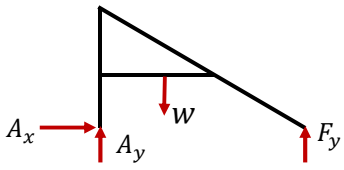
Example 5



The pin support B will safely support a force of 24 kN magnitude. Based on this criteria what is the largest mass m that the frame will safely support.

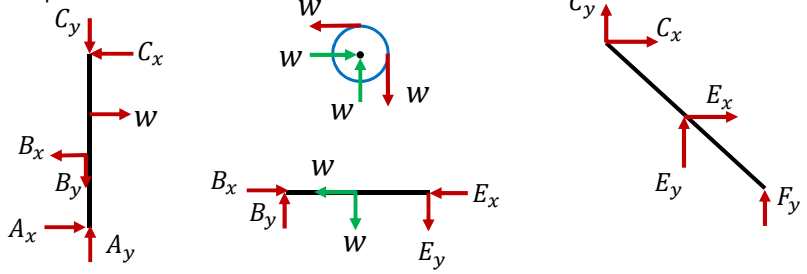
$mg=w$

$w=9.81m$



$(\rightarrow+)\sum\overrightarrow{F_x}=0 \quad \rightarrow \quad A_x=0$

Draw separate FBD's



Element ABC

$(\circlearrowleft+)\sum\overrightarrow{M_A}=0 \rightarrow \quad B_x \times 0.3 - w \times 0.4 + C_x \times 0.9 = 0$

$0.3B_x + 0.9C_x = 0.4w \quad (\mathbf{1})$

$(\rightarrow +)\sum\overrightarrow{F_x}=0 \quad \rightarrow \quad A_x - B_x + w - C_x = 0$

$B_x + C_x = w \quad (\mathbf{2})$

$\Rightarrow$

$\boxed{C_x = \frac{w}{6}}$

$\boxed{B_x = \frac{5w}{6}}$

Element BE

$(\circlearrowleft+)\sum\overrightarrow{M_E}=0 \rightarrow \quad -B_y \times 0.7 + w \times 0.4 = 0$ $\quad \boxed{B_y = \frac{4w}{7}}$

$$\sqrt{{B_x}^2 + {B_y}^2} = 24\ kN$$

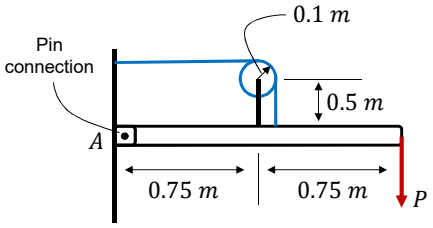
$$\sqrt{\left(\frac{5w}{6}\right)^2 + \left(\frac{4w}{7}\right)^2} = 24\ kN$$

$w = 23.752\ kN \quad \rightarrow \quad mg = 23.752$

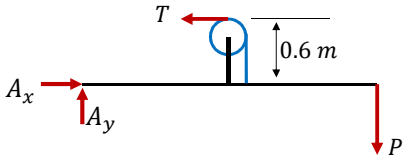
$m\ 9.81 = 23.752$

$\boxed{m = 2421.2\ kg}$

Example 6



Determine the greatest force P that can be applied to the frame if the largest force resultant acting at A can have a magnitude of 2 kN.



$(\circlearrowleft+)\sum \overrightarrow{M_A} = 0 \rightarrow \quad T \times 0.6 - P \times 1.5 = 0$

$T = \frac{1.5}{0.6}P$

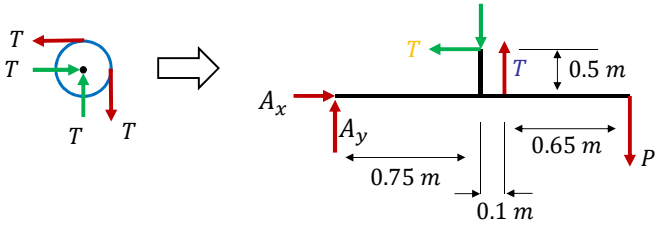
$(\rightarrow+)\sum \overrightarrow{F_x} = 0 \quad \rightarrow \quad A_x - T = 0$

$A_x = \frac{1.5}{0.6}P$

$(\uparrow+)\sum \overrightarrow{F_y} = 0 \quad \rightarrow \quad A_y = P$

$\sqrt{{A_x}^2 + {A_y}^2} = 2\ kN \rightarrow \sqrt{(2.5P)^2+P^2} = 2$ $\boxed{P = 0.743\ N}$

Alternative Solution



$(\circlearrowleft+)\sum \overrightarrow{M_A} = 0 \rightarrow \quad T \times 0.85 - T \times 0.75 + T \times 0.5 - P \times 1.5 = 0$

$0.6T = 1.5P$

$T = 2.5P$

$(\rightarrow+)\sum \overrightarrow{F_x} = 0 \quad \rightarrow \quad A_x - T = 0$

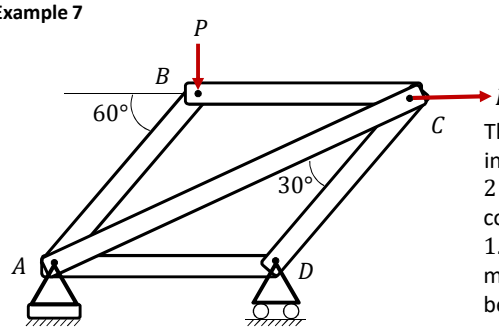
$A_x = T = 2.5P$

$(\uparrow+)\sum \overrightarrow{F_y} = 0 \quad \rightarrow \quad A_y - T + T - P = 0$

$A_y = P$

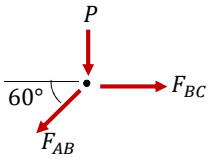
$\sqrt{{A_x}^2 + {A_y}^2} = 2\ kN \rightarrow \sqrt{(2.5P)^2+P^2} = 2$ $\boxed{P = 0.743\ N}$

Example 7



The maximum allowable tensile force in the members of truss is $(\mathrm{F_t})_{max} = 2\ kN$ and maximum allowable compressive force is $(\mathrm{F_c})_{max} = 1.2\ kN$. Determine the maximum magnitude P of the two loads that can be applied to the truss.

Joint B



$$(\uparrow +)\sum \overrightarrow{F_y} = 0 \quad \rightarrow \quad -P - F_{AB}\sin 60 = 0$$

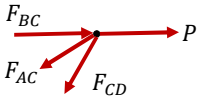
$$\boxed{F_{AB} = 1.1547\ P \quad (C)}$$

$$(\rightarrow +)\sum \overrightarrow{F_x} = 0 \quad \rightarrow \quad F_{BC} - F_{AB}\cos 60 = 0$$

$$F_{BC} = -0.5774\ P$$

$$\boxed{F_{BC} = 0.5774\ P \quad (C)}$$

Joint C



$$(\rightarrow +)\sum \overrightarrow{F_x} = 0 \quad \rightarrow \quad P + F_{BC} - F_{AC}\cos 30 - F_{CD}\cos 60 = 0$$

$$F_{BC} = 0.5774\ P$$

$$(\uparrow +)\sum \overrightarrow{F_y} = 0 \quad \rightarrow \quad -F_{AC}\sin 30 - F_{CD}\sin 60 = 0$$

$$F_{AC} = -1.732\ F_{CD}$$

$$P + 0.5774\ P - F_{AC}\ 0.866 - F_{CD}\ 0.5 = 0$$

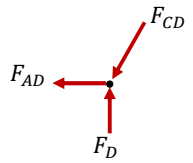
$$1.5774\ P = -1.732 \times 0.866\ F_{CD} - F_{CD}\ 0.5 = 0$$

$$F_{CD} = -1.577\ P$$

$$\boxed{F_{CD} = 1.577\ P \quad (C)}$$

$$\boxed{F_{AC} = 2.732\ P\ (T)}$$

Joint D



$(\rightarrow +)\sum \overrightarrow{F_x} = 0 \quad \rightarrow \quad F_{AD} + F_{CD}\cos 60 = 0$

$F_{AD} = F_{CD}/2$

$\boxed{F_{AD} = 0.7887\ P \quad (C)}$

Results

$F_{AB} = 1.1547\ P \quad (C)$
$F_{BC} = 0.5774\ P \quad (C)$
$F_{CD} = 1.577 \quad P \quad (C)$
$F_{AC} = 2.732 \quad P \quad (T)$
$F_{AD} = 0.7887\ P \quad (C)$

Assume $F_{AC} = 2\ kN$

$P = 732\ N$

$F_{CD} = 1154.4\ N < (\mathrm{F_c})_{max}\ (= 1.2\ kN)$ ✓

If $F_{CD} = 1.577\ P = 1.2\ kN$

$P = 760.94\ N$

$F_{AC} = 2078\ N > (\mathrm{F_t})_{max} (= 2\ kN)$

Not possible !