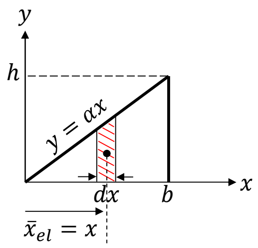
AE261 – Statics

Chapter 5 – Recitation 04

Example 1

Derive the centroid of a right triangle.



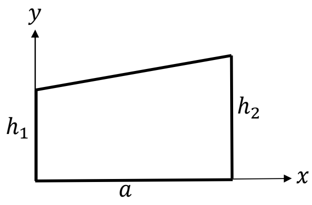
$$\bar{x}A = \int_0^b \bar{x}_{el}\, dA \qquad (dA = ydx)$$

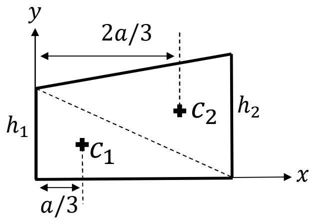
$$\bar{x}A = \int_0^b x\, y\, dx$$

$$\bar{x}\frac{1}{2}\alpha b^2 = \int_0^b x\, \alpha x\, dx \qquad A = \frac{1}{2}bh = \frac{1}{2}b(\alpha b) = \frac{1}{2}\alpha b^2$$

$$\bar{x}\frac{1}{2}\alpha b^2 = \alpha \frac{x^3}{3}\Big|_0^b = \alpha\frac{b^3}{3} \Longrightarrow \bar{x}\frac{1}{2}b^2 = \frac{b^3}{3} \qquad \boxed{\bar{x} = \frac{2}{3}b}$$

Example 2





Determine the x coordinate of the centroid of the trapezoid in terms of h_1, h_2 and a.

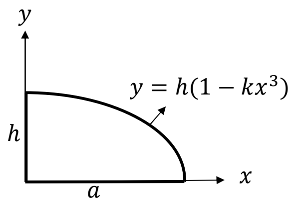
	A	$\bar{x}$	$A\bar{x}$
1	$\frac{1}{2}h_1a$	$\frac{1}{3}a$	$\frac{1}{6}h_1a^2$
2	$\frac{1}{2}h_2a$	$\frac{2}{3}a$	$\frac{2}{6}h_2a^2$

$$\bar{x} = \frac{\sum \bar{x}A}{\sum A} = \frac{\frac{1}{6}a^2(h_1 + 2h_2)}{\frac{1}{2}a\,(h_1 + h_2)}$$

$$\boxed{\bar{x} = \frac{1}{3}a\frac{h_1 + 2h_2}{h_1 + h_2}}$$

Example 3

Determine the x and y coordinate of the centroid of the given shape in terms of h and a.

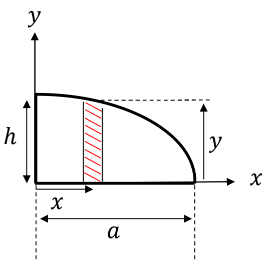


$y = h(1 - kx^3)$

For $x = a, y = 0$

$0 = h(1 - kx^3)$ ⇨ $k = 1/a^3$ which gives,

$$y = h\left(1 - \frac{x^3}{a^3}\right)$$



$\bar{x}_{el} = x$, $\bar{y}_{el} = \frac{y}{2}$, $dA = ydx$

$$A = \int_0^a dA = \int_0^a ydx = \int_0^a h\left(1 - \frac{x^3}{a^3}\right)\,dx = \left[x - \frac{x^4}{4a^3}\right]\Big|_0^a$$

$$A = \frac{3}{4}ah$$

$$\bar{x}A = \int_0^a \bar{x}_{el} dA = \int_0^a xh\left(1 - \frac{x^3}{a^3}\right) dA = \int_0^a h\left(x - \frac{x^4}{a^3}\right) dA$$

$$= h\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]\Big|_0^a = \frac{3}{10}a^2 h$$

$$\bar{x}A = \bar{x}\frac{3}{4}ah = \frac{3}{10}a^2 h \quad \Rightarrow \quad \boxed{\bar{x} = \frac{2}{5}a}$$

$\bar{y}_{el} = \frac{y}{2}$, $dA = ydx$

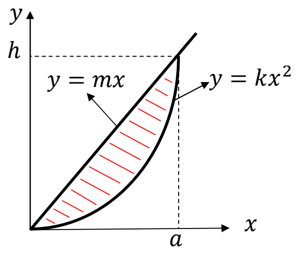
$$\bar{y}A = \int_0^a \bar{y}_{el} dA = \int_0^a \frac{y}{2} ydx = \frac{1}{2}\int_0^a y^2 dx = \frac{1}{2}\int_0^a h^2\left(1 - \frac{x^3}{a^3}\right)^2 dx$$

$$= \frac{h^2}{2}\int_0^a \left(\frac{x^6}{a^6} - \frac{2x^3}{a^3} + 1\right) dx = \frac{h^2}{2}\left[\frac{x^7}{7a^6} - \frac{x^4}{2a^3} + x\right]\Big|_0^a = \frac{h^2}{2}\left(\frac{a}{7} - \frac{a}{2} + a\right) = \frac{h^2}{2}\left(\frac{9a}{14}\right)$$

$$= \frac{9ah^2}{14} \quad \Rightarrow \quad \bar{y}A = \bar{y}\frac{3}{4}ah = \frac{9ah^2}{14}$$

$$\boxed{\bar{y} = \frac{3}{7}h}$$

Example 4



Determine the centroid of the area by direct integration in terms of a and h.

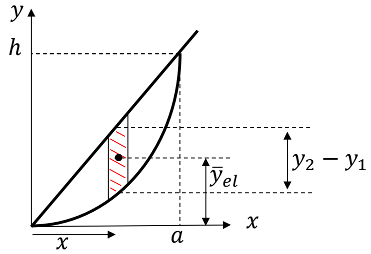
$y_1 = kx^2$ $y_2 = mx$

$@\, x = a$ $@\, x = a$

$y = h$ $y = h$

$k = \dfrac{h}{a^2}$ $m = \dfrac{h}{a}$

$y_1 = \dfrac{h}{a^2}x^2$ $y_2 = \dfrac{h}{a}x$



$$\bar{x}_{el} = x, \quad \bar{y}_{el} = \frac{y_2 + y_1}{2}$$

$$dA = (y_2 - y_1)dx = \left(\frac{h}{a}x - \frac{h}{a^2}x^2\right)dx$$

$$= \frac{h}{a^2}(ax - x^2)dx$$

$$A = \int dA = \int_0^a \frac{h}{a^2}(ax - x^2)dx$$

$$= \frac{h}{a^2}\left[\frac{ax^2}{2} - \frac{x^3}{3}\right]\Bigg|_0^a = \frac{h}{a^2}\left[\frac{a^3}{2} - \frac{a^3}{3}\right] = \frac{h}{a^2}\frac{a^3}{6} \Rightarrow A = \frac{ah}{6}$$

$$Q_y = \int \bar{x}_{el}\, dA = \int_0^a x\frac{h}{a^2}(ax - x^2)dx$$

$$= \frac{h}{a^2}\int_0^a (ax^2 - x^3)dx = \frac{h}{a^2}\left[\frac{ax^3}{3} - \frac{x^4}{4}\right]\Bigg|_0^a$$

$$= \frac{h}{a^2}\left(\frac{a^4}{3} - \frac{a^4}{4}\right) = \frac{h}{a^2}\frac{a^4}{12}$$

$$Q_y = \frac{a^2h}{12}$$

$$Q_x = \int \bar{y}_{el}\, dA = \int_0^a \frac{y_1 + y_2}{2} \frac{h}{a^2}(ax - x^2)dx$$

$$= \int_0^a \frac{1}{2}\left(\frac{h}{a^2}x^2 + \frac{h}{a}x\right)\frac{h}{a^2}x^2(ax^2 - x^3)dx = \frac{h^2}{2a^4}\int_0^a (x^2 + ax)(ax - x^2)dx$$

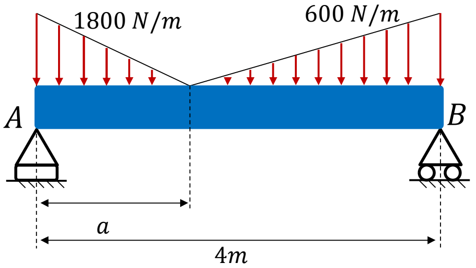
$$= \frac{h^2}{2a^4}\int_0^a (a^2x^2 - x^4)dx = \frac{h^2}{2a^4}\left[\frac{a^2x^3}{3} - \frac{x^5}{5}\right]\Big|_0^a$$

$$= \frac{h^2}{2a^4}\left(\frac{a^5}{3} - \frac{a^5}{5}\right) = \frac{h^2}{2a^4}\frac{2a^5}{15} \qquad Q_x = \frac{ah^2}{15}$$

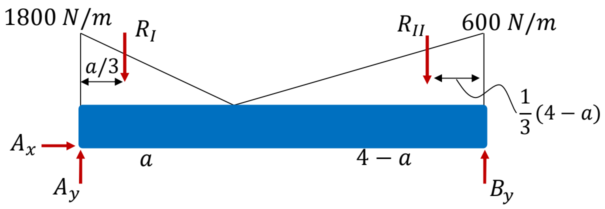
$$Q_x = \bar{y}A \qquad \frac{ah^2}{15} = \bar{y}\frac{ah}{6} \qquad \boxed{\bar{y} = \frac{2h}{5}}$$

$$Q_y = \bar{x}A \qquad \frac{a^2h}{12} = \bar{y}\frac{ah}{6} \qquad \boxed{\bar{x} = \frac{a}{2}}$$

Example 5



Determine **a)** the distance a such that the reaction at support B is minimum, **b)** the corresponding reactions at the supports.



$$R_I = \frac{1800a}{2} = 900a\ N$$

$$R_{II} = \frac{600(4-a)}{2}$$

$$R_{II} = 300(4-a)\ N$$

$\curvearrowleft + \quad \sum M_A = 0 \Longrightarrow -R_I\frac{a}{3} - R_{II}\left(a + 2\backslash 3\ (4-a)\right) + B_y 4 = 0$

$B_y 4 = R_I\frac{a}{3} + R_{II}\frac{a+8}{3}$

$B_y = \frac{1}{4}\left[900a\frac{a}{3} + 300(4-a)\frac{a+8}{3}\right]$

$B_y = \frac{1}{4}[300a^2 + 100(4-a)(a+8)]$

$B_y = \frac{1}{4}[300a^2 + 100(4a + 32 - a^2 - 8a)]$

$B_y = \frac{1}{4}[200a^2 - 400a + 3200]$

$B_y = 50a^2 - 100a + 900 \Longrightarrow \frac{dB_y}{da} = 100a - 100 = 0$ $\boxed{a = 1\ m}$

b) $B_y = 750\ N \uparrow$

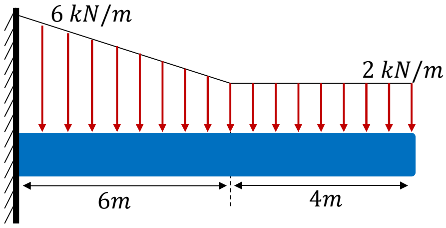
$\sum F_x = 0 \Longrightarrow A_x = 0$

$\sum F_y = 0 \Longrightarrow A_y + B_y - R_I - R_{II} = 0$

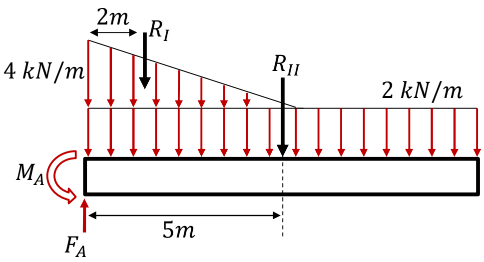
$A_y = 900 + 900 - 750$

$A_y = 1050\ N \uparrow$

Example 6



Determine the reactions at the beam supports for the given loadings.



$$R_I = \frac{1}{2}\left(4\frac{kN}{m}\right)(6\ m) = 12\ kN$$

$$R_{II} = (2\ kN)(10\ m) = 20\ kN$$

$$\sum F_x = 0 \quad (+\uparrow)$$

$$F_A - 12\ kN - 20\ kN = 0$$

$$\boxed{F_A = 32\ kN}$$

$$\sum M_A = 0 \quad (+\circlearrowleft)$$

$$M_A - R_I 2 - R_{II} 5 = 0$$

$$M_A = 2(12\ kN) + 5(20\ kN)$$

$$\boxed{M_A = 124\ kN.m}$$