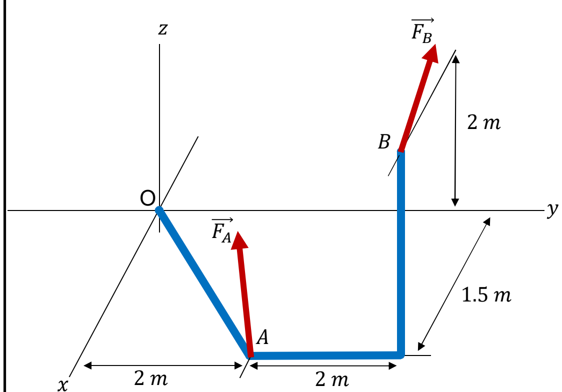
ME201 – Statics

Chapter 3 – Recitation

Example 1



$$\vec{F_A} = -20\vec{i} - 10\vec{j} + 25\vec{k}\ [N]$$

$$\vec{F_B} = -10\vec{i} + 25\vec{j} + 20\vec{k}\ [N]$$

Replace the force system by a resultant force and a couple at point O.

$$\overrightarrow{F_R} = \overrightarrow{F_A} + \overrightarrow{F_B}$$
$$= -20\vec{\imath} - 10\vec{\jmath} + 25\vec{k} + -10\vec{\imath} + 25\vec{\jmath} + 20\vec{k}$$
$$= -30\vec{\imath} + 15\vec{\jmath} + 40\vec{k}\ [N]$$

Position vectors $\overrightarrow{r_A}$ and $\overrightarrow{r_B}$

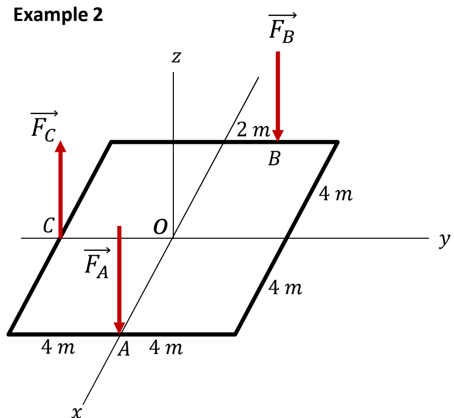
$$\overrightarrow{r_A} = 1.5\vec{\imath} + 2\vec{\jmath}\ [m]$$
$$\overrightarrow{r_B} = 1.5\vec{\imath} + 4\vec{\jmath} + 2\vec{k}\ [m]$$

Resultant moment about O,

$$\sum M_O = \overrightarrow{r_A} \times \overrightarrow{F_A} + \overrightarrow{r_B} \times \overrightarrow{F_B}$$
$$= (1.5\vec{\imath} + 2\vec{\jmath}\,) \times \left(-20\vec{\imath} - 10\vec{\jmath} + 25\vec{k}\right)$$
$$+\left(1.5\vec{\imath} + 4\vec{\jmath} + 2\vec{k}\right) \times (-10\vec{\imath} + 25\vec{\jmath} + 20\vec{k})$$
$$= -15\vec{k} - 37.5\vec{\jmath} + 40\vec{k} + 50\vec{\imath} + 37.5\vec{k} - 30\vec{\jmath} + 40\vec{k} + 80\vec{\imath} - 20\vec{\jmath} - 50\vec{\imath}$$
$$= 80\vec{\imath} - 87.5\vec{\jmath} + 117.5\vec{k}\ [N.m]$$

$$\overrightarrow{F_R} = -30\vec{\imath} + 15\vec{\jmath} + 40\vec{k}\ [N]$$
$$\overrightarrow{M_O} = 80\vec{\imath} - 87.5\vec{\jmath} + 117.5\vec{k}\ [N.m]$$

Example 2



$$\overrightarrow{F_A} = -300\vec{k}\ \ [N]$$
$$\overrightarrow{F_B} = -500\vec{k}\ \ [N]$$
$$\overrightarrow{F_C} = 100\vec{k}\ \ [N]$$
$$\overrightarrow{r_A} = 4\vec{\imath}\ \ [m]$$
$$\overrightarrow{r_B} = -4\vec{\imath} + 2\vec{\jmath}\ [m]$$
$$\overrightarrow{r_C} = -4\vec{\jmath}\ [m]$$

Replace the force system by a resultant force **only** and specify its points of application.

Find the resultant force

$$\overrightarrow{F_R} = \overrightarrow{F_A} + \overrightarrow{F_B} + \overrightarrow{F_C}$$
$$= (-300 - 500 + 100)\vec{k}\ \ [N]$$
$$= -700\vec{k}\ \ [N]$$

Determine the moments of forces with respect to point $\boldsymbol{O}$.

$$\sum \overrightarrow{M_O} = \overrightarrow{r_A} \times \overrightarrow{F_A} + \overrightarrow{r_B} \times \overrightarrow{F_B} + \overrightarrow{r_C} \times \overrightarrow{F_C}$$
$$= (4\vec{\imath}) \times (-300\vec{k}) + (-4\vec{\imath} + 2\vec{\jmath}) \times (-500\vec{k}) + (-4\vec{\jmath}) \times (100\vec{k})$$
$$= 1200\vec{\jmath} - 2000\vec{\jmath} - 1000\vec{\imath} - 400\vec{\imath}$$
$$= -1400\vec{\imath} - 800\vec{\jmath}\ [N.m]$$

Then, the equivalent force-couple system at point $\boldsymbol{O}$ is

$$\overrightarrow{F_R} = -700\vec{k}\ \ [N]$$
$$\overrightarrow{M_R} = -1400\vec{\imath} - 800\vec{\jmath}\ [N.m]$$

We are asked to replace with a single force only. Therefore, the position of the resultant force should be such that

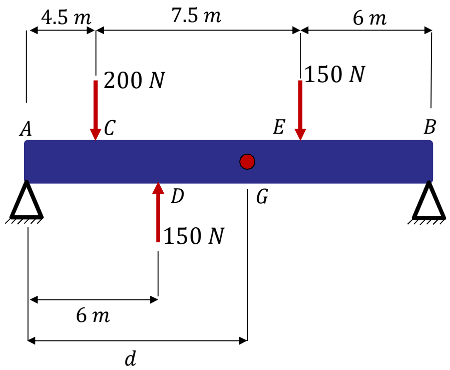
$$\overrightarrow{r_P} \times \overrightarrow{F_R} = \overrightarrow{M_R}$$
$$(x_P\vec{\imath} + y_P\vec{\jmath}) \times (-700\vec{k}) = -1400\vec{\imath} - 800\vec{\jmath}$$
$$700x_P\vec{\jmath} - 700y_P\vec{\imath} = -1400\vec{\imath} - 800\vec{\jmath}$$

$$x_P = \frac{-800}{700} = 1.14\ [m]$$
$$y_P = \frac{1400}{700} = 2\ [m]$$

It is also possible to find x_P and y_P without computing $\sum \overrightarrow{M_O}$. We can directly sum moments of $\overrightarrow{F_A}$, $\overrightarrow{F_B}$, $\overrightarrow{F_C}$ about the x and y axes and equate them to moment of resultant $\overrightarrow{F_R}$ at (x_P, y_P) about x and y axis, i.e.

$$\underbrace{-700y_P}_{(M_R)_x} = \underbrace{-100 * 4 - 500 * 2}_{\sum M_x} \rightarrow y_P = 2\ [m]$$
$$\underbrace{700x_P}_{(M_R)_y} = \underbrace{300 * 4 - 500 * 2}_{\sum M_y} \rightarrow x_P = 1.14[m]$$

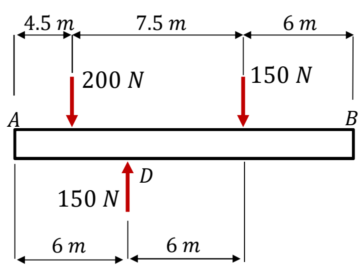
Example 3

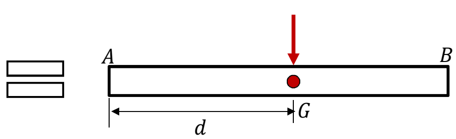


A force and a couple are applied to a beam as shown.

(a) Replace this system with a single force $\vec{F}$ applied at point G, and determine the distance d.

(b) Solve part (a) assuming that the direction of 150 N forces are reversed.





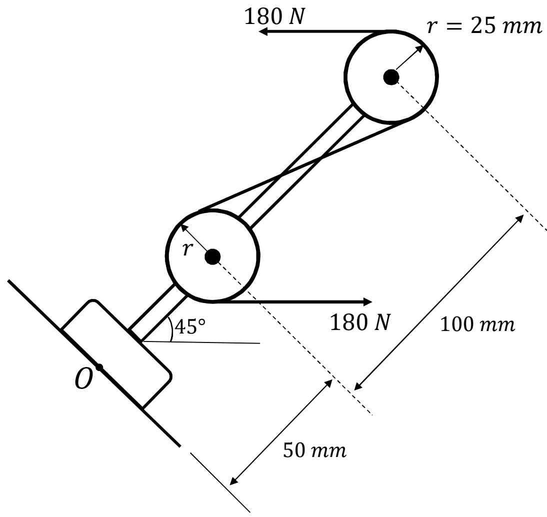
(a) $\sum M_G = 0$ (↻+) ⇨ $200\,[N](d - 4.5\,[m]) - 150\,[N]\,6\,[m] = 0$

$d = 9\,[m]$

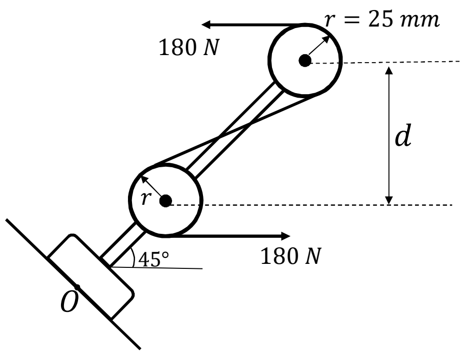
(b) $\sum M_G = 0$ (↻+) ⇨ $200\,[N](d - 4.5\,[m]) + 150\,[N]\,6\,[m] = 0$

$d = 0\,[m]$

Example 4



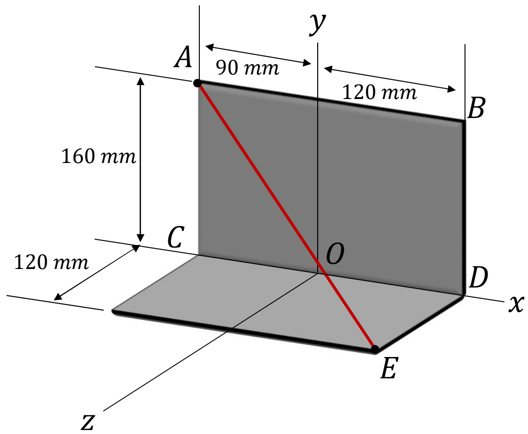
Determine the equivalent force-couple system at point O.



$$d = 25 + 25 + 100\ \sin 45$$
$$d = 25 + 25 + 70.71$$
$$d = 120.71\ [mm]$$
$$M_0 = dF$$
$$M_0 = 120.71\ *\ 180$$

$$M_0 = 21727.8\ [N.mm]$$
$$M_0 = 21.73\ [N.m]$$

Example 5



The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is $435\ [N]$, determine the moment about O of the force exerted by the wire **(a)** on corner A, **(b)** on corner E.

(a) $\overrightarrow{F_A} = F_A \overrightarrow{r_{AE}} = F_A \frac{\overrightarrow{AE}}{AE}$

$$\overrightarrow{AE} = 0.21\vec{\imath} - 0.16\vec{\jmath} + 0.12\vec{k}\ [m]$$

$$AE = \sqrt{0.21^2 + 0.16^2 + 0.12^2} = 0.29\ [m]$$

$$\overrightarrow{F_A} = 435\ [N] \frac{0.21\vec{\imath} - 0.16\vec{\jmath} + 0.12\vec{k}}{0.29}$$

$$= 315\vec{\imath} - 240\vec{\jmath} + 180\vec{k}\ [N]$$

$$\overrightarrow{M_O} = \overrightarrow{r_{A/O}} \times \overrightarrow{F_A}$$

$$\overrightarrow{r_{A/O}} = -0.09\vec{\imath} + 0.16\vec{\jmath}\ [m]$$

$$\overrightarrow{M_O} = (-0.09\vec{\imath} + 0.16\vec{\jmath}) \times (315\vec{\imath} - 240\vec{\jmath} + 180\vec{k})$$

$$\overrightarrow{M_O} = 21.6\vec{k} + 16.2\vec{\jmath} - 50.4\vec{k} + 28.8\vec{\imath}$$

$$\boxed{\overrightarrow{M_O} = 28.8\vec{\imath} + 16.2\vec{\jmath} - 28.8\vec{k}\ [N.m]}$$

(b) $\overrightarrow{F_E} = -\overrightarrow{F_A} = -315\vec{\imath} + 240\vec{\jmath} - 180\vec{k}\ [N]$

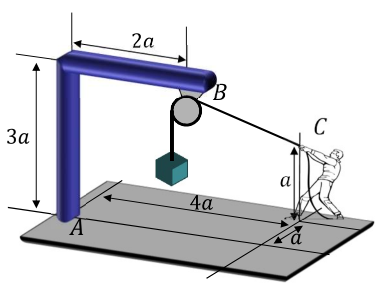
$$\overrightarrow{M_O} = \overrightarrow{r_{E/O}} \times \overrightarrow{F_E}$$

$$\overrightarrow{r_{E/O}} = 0.12\vec{\imath} + 0.12\vec{\jmath}\ [m]$$

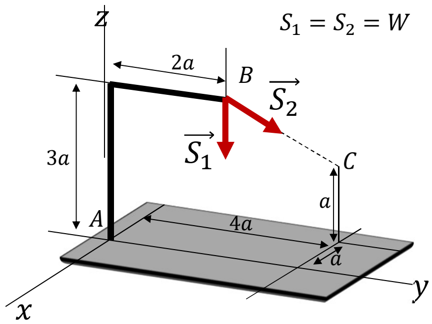
$$\overrightarrow{M_O} = (0.12\vec{\imath} + 0.12\vec{\jmath}) \times (-315\vec{\imath} + 240\vec{\jmath} - 180\vec{k})$$

$$\boxed{\overrightarrow{M_O} = -28.8\vec{\imath} - 16.2\vec{\jmath} + 28.8\vec{k}\ [N.m]}$$

Example 6



A rope passes over an ideal pully as shown in the figure. It carries a crate with weight W and held at point C. The radius of the pulley may be neglected. Determine the resultant moment of the forces in the rope about point A.



$S_1 = S_2 = W$

$\overrightarrow{M_A} = \overrightarrow{r_{AB}} \times \overrightarrow{S_2} + \overrightarrow{r_{AB}} \times \overrightarrow{S_1}$

$\overrightarrow{r_{AB}} = 2a\vec{j} + 3a\vec{k}$

$\overrightarrow{S_1} = -W\vec{k}$

$\overrightarrow{S_2} = \frac{\overrightarrow{BC}}{BC} W$

$= \frac{-a\vec{i} + 2a\vec{j} - 2a\vec{k}}{3a} W$

$\overrightarrow{S_2} = W(\frac{-1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{-2}{3}\vec{k})$

$$\overrightarrow{M_A} = (2a\vec{j} + 3a\vec{k}) \times \left(\frac{-1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{-2}{3}\vec{k}\right) W + (2a\vec{j} + 3a\vec{k}) \times (-W\vec{k}\ )$$

$$= \left(\frac{2}{3}\vec{k} + \frac{-4}{3}\vec{i} - 1\vec{j} - 2\vec{i} - 2\vec{i}\right) Wa$$

$$\boxed{\overrightarrow{M_A} = \left(-\frac{16}{3}\vec{i} - 1\vec{j} + \frac{2}{3}\vec{k}\right) Wa}$$