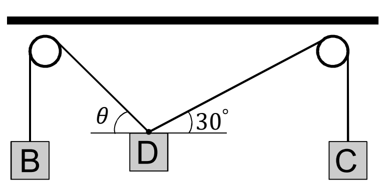
ME201 – Statics

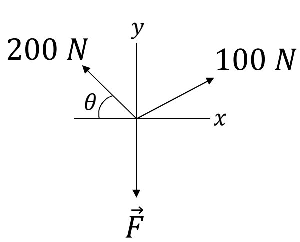
Chapter 1&2 – Recitation

Example 1



$w_B = 200\ N$
$w_C = 100\ N$

What is the weight of D and angle θ?



$\sum \overrightarrow{F_x} = 0$

$100 \cos 30 - 200 \cos \theta = 0$

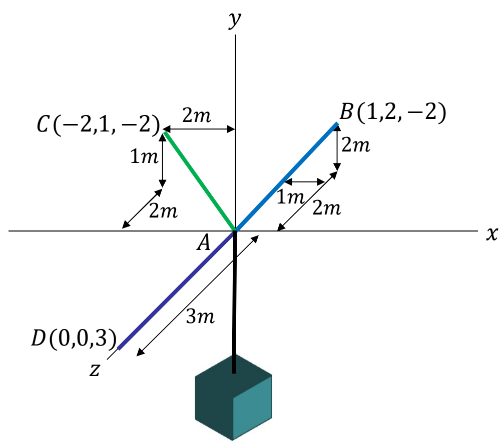
$\underline{\theta = 64.34^{\circ}}$

$\sum \overrightarrow{F_y} = 0$

$100 \sin 30 + 200 \sin 64.34 - w_D = 0$

$\underline{w_D = 230\ \text{N}}$

Example 2



Determine the forces in cables AB, AC and AD required to support the 300 N crate.

Example 2

$$\overrightarrow{F_{AB}} = \frac{F_{AB}}{d_{AB}}\left[(x_B - x_A)\vec{\imath} + (y_B - y_A)\vec{\jmath} + (z_B - z_A)\vec{k}\right]$$

$$\overrightarrow{F_{AB}} = \frac{F_{AB}}{\sqrt{(1-0)^2+(2-0)^2(-2-0)^2}}\left[(1-0)\vec{\imath} + (2-0)\vec{\jmath} + (-2-0)\vec{k}\right]$$

$$\boxed{\overrightarrow{F_{AB}} = F_{AB}\left(\frac{1}{3}\vec{\imath} + \frac{2}{3}\vec{\jmath} - \frac{2}{3}\vec{k}\right)}$$

$$\overrightarrow{F_{AC}} = \frac{F_{AC}}{d_{AC}}\left[(x_C - x_A)\vec{\imath} + (y_C - y_A)\vec{\jmath} + (z_C - z_A)\vec{k}\right]$$

$$\overrightarrow{F_{AC}} = \frac{F_{AC}}{\sqrt{(-2)^2+(1)^2(-2)^2}}\left[-2\vec{\imath} + \vec{\jmath} - 2\vec{k}\right]$$

$$\boxed{\overrightarrow{F_{AC}} = F_{AC}\left(-\frac{2}{3}\vec{\imath} + \frac{1}{3}\vec{\jmath} - \frac{2}{3}\vec{k}\right)}$$

$$\boxed{\overrightarrow{F_{AD}} = F_{AD}\vec{k}} \qquad \boxed{\vec{w} = -300\vec{\jmath}}$$

Example 2

Equations of equilibrium

$$\sum \vec{F} = 0$$

$$F_{AB}\frac{1}{3}\vec{\imath} + F_{AB}\frac{2}{3}\vec{\jmath} - F_{AB}\frac{2}{3}\vec{k} - F_{AC}\frac{2}{3}\vec{\imath} + F_{AC}\frac{1}{3}\vec{\jmath} - F_{AC}\frac{2}{3}\vec{k}$$

$$+F_{AD}\vec{k} - 300\vec{\jmath} = 0$$

Equating $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ components yields,

$$F_{AB}\frac{1}{3} - F_{AC}\frac{2}{3} = 0 \quad (1) \Longrightarrow F_{AB} = 2F_{AC}$$

$$F_{AB}\frac{2}{3} + F_{AC}\frac{1}{3} - 300 = 0 \quad (2) \Longrightarrow F_{AC}\frac{5}{3} = 300 \quad \boxed{F_{AC} = 180\ N}$$

$$-F_{AB}\frac{2}{3} - F_{AC}\frac{2}{3} + F_{AD} = 0 \quad (3) \qquad \boxed{F_{AB} = 360\ N}$$

$$-240 - 120 + F_{AD} = 0$$

$$\boxed{F_{AD} = 360\ N}$$

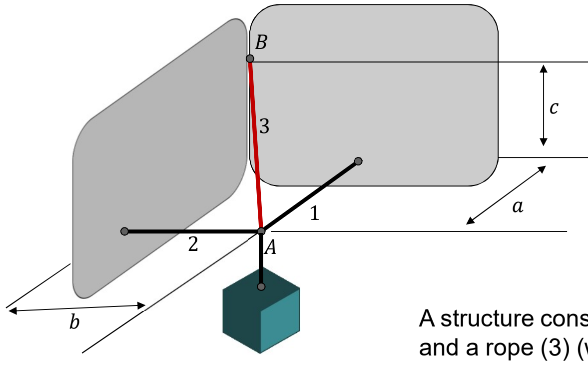
Example 2

$$\boxed{\overrightarrow{F_{AB}} = 120\vec{\imath} + 240\vec{\jmath} - 240\vec{k}}$$

$$\boxed{\overrightarrow{F_{AC}} = -120\vec{\imath} + 60\vec{\jmath} - 120\vec{k}}$$

$$\boxed{\overrightarrow{F_{AD}} = 360\vec{k}}$$

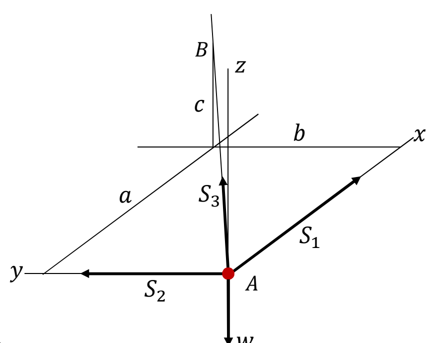
Example 3



A structure consists of two bars (1 and 2) and a rope (3) (weights negligible). It is loaded in A by a box weight W. Determine the forces in the bars and the rope.

Example 3

Isolate the pin A



$$\overrightarrow{S_1} = S_1\vec{\imath} \qquad \overrightarrow{S_2} = S_2\vec{\jmath}$$

$$\overrightarrow{S_3} = ?$$

$$\overrightarrow{r_{AB}} = a\vec{\imath} + b\vec{\jmath} + c\vec{k} \quad \Rightarrow \quad \overrightarrow{S_3} = S_3\frac{a\vec{\imath} + b\vec{\jmath} + c\vec{k}}{\sqrt{a^2+b^2+c^2}}$$

$$\overrightarrow{e_{AB}} = \frac{a\vec{\imath} + b\vec{\jmath} + c\vec{k}}{\sqrt{a^2+b^2+c^2}}$$

Example 3

Equilibrium equations

$$\overrightarrow{S_1} + \overrightarrow{S_2} + \overrightarrow{S_3} + \vec{w} = 0$$

$$S_1\vec{\imath} + S_2\vec{\jmath} + S_3\frac{a\vec{\imath}+b\vec{\jmath}+c\vec{k}}{\sqrt{a^2+b^2+c^2}} - w\vec{k} = 0$$

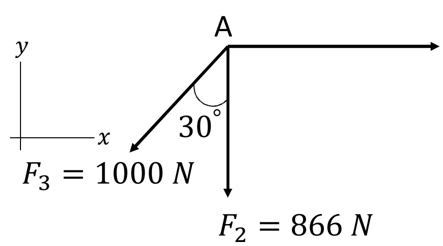
$$\sum \overrightarrow{F_x} = 0 \Rightarrow S_1 + S_3\frac{a}{\sqrt{a^2+b^2+c^2}} = 0$$

$$\sum \overrightarrow{F_y} = 0 \Rightarrow S_2 + S_3\frac{b}{\sqrt{a^2+b^2+c^2}} = 0$$

$$\sum \overrightarrow{F_z} = 0 \Rightarrow S_3\frac{c}{\sqrt{a^2+b^2+c^2}} - w = 0$$

$$S_3 = \frac{w\sqrt{a^2+b^2+c^2}}{c}$$
$$S_2 = -\frac{b}{c}w$$
$$S_1 = -\frac{a}{c}w$$

Example 4



$F_1 = 1500\ N$

There are 4 forces (F_1, F_2, F_3, F_4) acting on point A. 3 of the forces are shown in the figure. If the total forces acting on A is zero, what should be $\overrightarrow{F_4}$?

$$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \overrightarrow{F_4} = \sum \vec{F} = 0$$
$$R_x = F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$$
$$R_y = F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

$$\overrightarrow{F_1} = 1500\ N\ \vec{\imath} \qquad \overrightarrow{F_2} = -866\ N\vec{\jmath}$$
$$\overrightarrow{F_3} = 1000\cos 240\,\vec{\imath} + 1000\sin 240\,\vec{\jmath} = -500N\ \vec{\imath} - 866N\ \vec{\jmath}$$
$$\overrightarrow{F_4} = F_{4x}\vec{\imath} - F_{4y}\vec{\jmath}$$

Example 4

$$\sum \overrightarrow{F_x} = 1500\ N + 0 - 500\ N + F_{4x} = 0$$

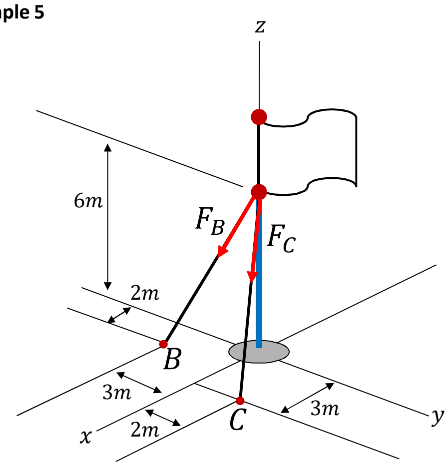
$$F_{4x} = -1000\ N$$

$$\sum \overrightarrow{F_y} = 0 - 866\ N - 866\ N + F_{4y} = 0$$

$$F_{4y} = 1732\ N$$

$$\boxed{\overrightarrow{F_4} = -1000\ N\ \vec{\imath} + 1732\ N\ \vec{\jmath}}$$

Example 5



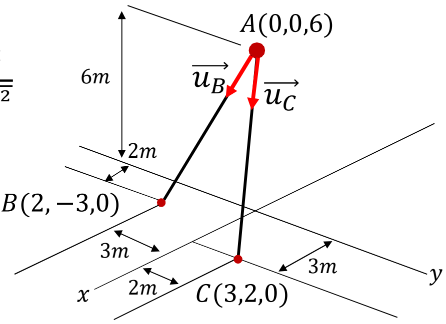
If $F_B = 560\ N$ and $F_C = 700\ N$, determine the magnitude and coordinate angles of the resultant force acting on the flag pole.

Example 5

The unit vectors $\overrightarrow{u_B}$ and $\overrightarrow{u_C}$ of $\overrightarrow{F_B}$ and $\overrightarrow{F_C}$ must be determined first.

$$\overrightarrow{u_B} = \frac{\overrightarrow{r_B}}{r_B} = \frac{(2-0)\vec{i}+(-3-0)\vec{j}+(0-6)\vec{k}}{\sqrt{(2-0)^2+(-3-0)^2+(0-6)^2}}$$
$$= \frac{2}{7}\vec{i} - \frac{3}{7}\vec{j} - \frac{6}{7}\vec{k}$$

$$\overrightarrow{u_C} = \frac{\overrightarrow{r_C}}{r_C} = \frac{(3-0)\vec{i}+(2-0)\vec{j}+(0-6)\vec{k}}{\sqrt{(3-0)^2+(2-0)^2+(0-6)^2}}$$
$$= \frac{3}{7}\vec{i} + \frac{2}{7}\vec{j} - \frac{6}{7}\vec{k}$$



Example 5

Thus, the force vectors $\overrightarrow{F_B}$ and $\overrightarrow{F_C}$ are given by

$$\overrightarrow{F_B} = F_B\overrightarrow{u_B} = 560(\frac{2}{7}\vec{i} - \frac{3}{7}\vec{j} - \frac{6}{7}\vec{k}) = \{160\vec{i}-240\vec{j}-480\vec{k}\}\ \mathrm{N}$$
$$\overrightarrow{F_C} = F_C\overrightarrow{u_C} = 700(\frac{3}{7}\vec{i} + \frac{2}{7}\vec{j} - \frac{6}{7}\vec{k}) = \{300\vec{i}+200\vec{j}-600\vec{k}\}\ \mathrm{N}$$

Resultant Force:

$$\overrightarrow{F_R} = \overrightarrow{F_B} + \overrightarrow{F_C} = \left(160\vec{i}-240\vec{j}-480\vec{k}\right) + \left(300\vec{i}+200\vec{j}-600\vec{k}\right)$$
$$= \left\{460\vec{i} - 40\vec{j} - 1080\vec{k}\right\}\ \mathrm{N}$$

The magnitude of $\overrightarrow{F_R}$ is

$$F_R = \sqrt{{(F_R)_x}^2 + {(F_R)_y}^2 + {(F_R)_z}^2}$$
$$= \sqrt{460^2 + (-40)^2+(-1080)^2} = 1174.56\ N = 1.17\ kN$$

Example 5

The coordinate direction angles of $\overrightarrow{F_R}$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9°$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0°$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157°$$