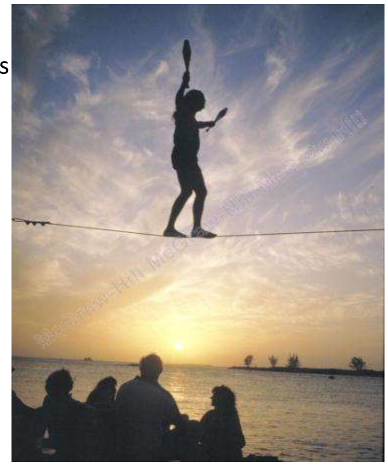
ME201 STATICS

CHAPTER 7
FORCES IN BEAMS AND CABLES

Contents

Application

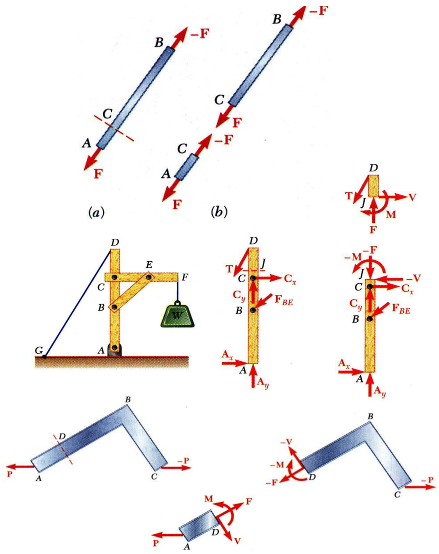
Forces that are *internal* to the structural members beams are the subject of this chapter



Introduction

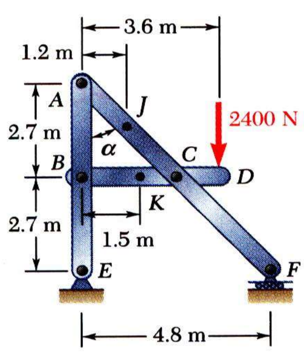
- Preceding chapters dealt with:
  a) determining **external** forces acting on a structure and
  b) determining forces which hold together the various members of a structure. Lets call joint forces!
- The current chapter is concerned with determining the ***internal forces*** (i.e., tension/compression, shear, and bending) which hold together the various parts of a given member.
- Focus is on:
  a) Beams - usually long, straight, prismatic members designed to support loads applied at various points along the member.

Internal Forces in Members



- Straight two-force member *AB* is in equilibrium under application of *F* and -*F*.
- ***Internal forces*** equivalent to *F* and -*F* are required for equilibrium of free-bodies *AC* and *CB*.
- Multiforce member *ABCD* is in equilibrium under application of cable and member contact forces.
- Internal forces equivalent to a force-couple system are necessary for equilibrium of free-bodies *JD* and *ABCJ*.
- An internal force-couple system is required for equilibrium of two-force members which are not straight.

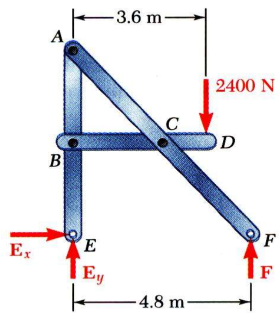
Sample Problem 7.1



Determine the internal forces *(a)* in member *ACF* at point *J* and *(b)* in member *BCD* at *K*.

SOLUTION:

- Compute reactions and forces at connections for each member.
- **Cut member *ACF* at *J*.** The internal forces at *J* are represented by equivalent force-couple system which is determined by considering equilibrium of either part.
- **Cut member *BCD* at *K*.** Determine force-couple system equivalent to internal forces at *K* by applying equilibrium conditions to either part.



SOLUTION:

- Compute reactions and connection forces.

Consider entire frame as a free-body, and apply equilibrium conditions:

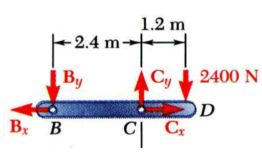
$$\sum M_E = 0:$$

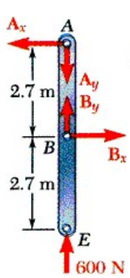
$$-(2400\ \text{N})(3.6\ \text{m}) + F(4.8\ \text{m}) = 0 \qquad F = 1800\ \text{N}$$

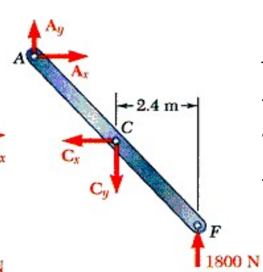
$$\sum F_y = 0:$$

$$-2400\ \text{N} + 1800\ \text{N} + E_y = 0 \qquad E_y = 600\ N$$

$$\sum F_x = 0: \qquad E_x = 0$$







Drawing the FBD for member BCD:

- Why are forces at B and C drawn in these directions? Is there a choice on the directions?

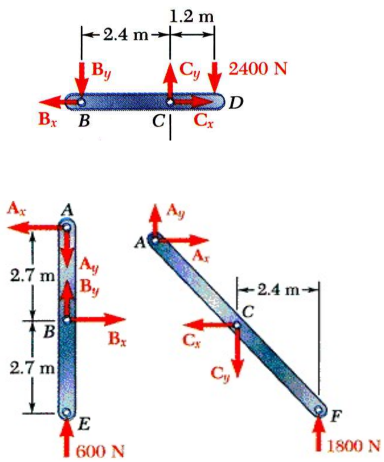
- Why are there two force components at each point instead of just a single force?

Drawing the FBD for member ABE:

- Why are forces at B in these directions? Is there a choice on the directions?

- Why are there two force components at A instead of just a single force?

Finally, the FBD for member ACF.



Consider member *BCD* as free-body:

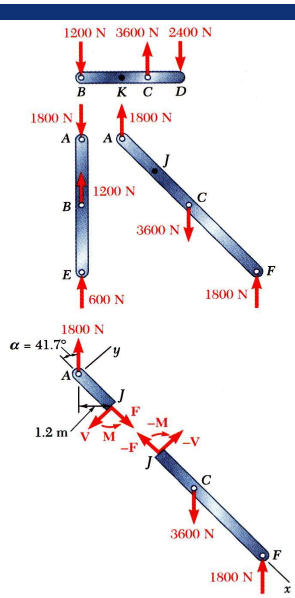
$$\sum M_B = 0:$$
$$-(2400\ \text{N})(3.6\ \text{m}) + C_y(2.4\ \text{m}) = 0 \qquad C_y = 3600\ \text{N}$$
$$\sum M_C = 0:$$
$$-(2400\ \text{N})(1.2\ \text{m}) + B_y(2.4\ \text{m}) = 0 \qquad B_y = 1200\ \text{N}$$
$$\sum F_x = 0: \qquad -B_x + C_x = 0$$

Consider member *ABE* as free-body:

$$\sum M_A = 0: \qquad B_x(2.4\ \text{m}) = 0 \qquad B_x = 0$$
$$\sum F_x = 0: \qquad B_x - A_x = 0 \qquad A_x = 0$$
$$\sum F_y = 0: \qquad -A_y + B_y + 600\ \text{N} = 0 \qquad A_y = 1800\ \text{N}$$

From member *BCD*,

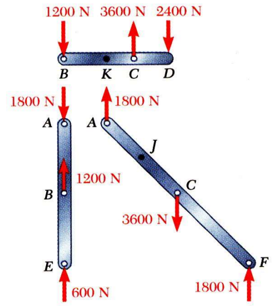
$$\sum F_x = 0: \qquad -B_x + C_x = 0 \qquad C_x = 0$$

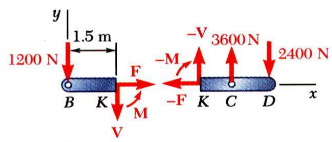


- Cut member *ACF* at *J*. The internal forces at *J* are represented by equivalent force-couple system.

Consider free-body *AJ*:

$$\sum M_J = 0:$$
$$-(1800\ \text{N})(1.2\ \text{m}) + M = 0 \qquad \boxed{M = 2160\ \text{N}\cdot\text{m}}$$
$$\sum F_x = 0:$$
$$F - (1800\ \text{N})\cos 41.7^\circ = 0 \qquad \boxed{F = 1344\ \text{N}}$$
$$\sum F_y = 0:$$
$$-V + (1800\ \text{N})\sin 41.7^\circ = 0 \qquad \boxed{V = 1197\ \text{N}}$$





- Cut member *BCD* at *K*. Determine a force-couple system equivalent to internal forces at *K* .

Consider free-body *BK*:

$\sum M_K = 0:$

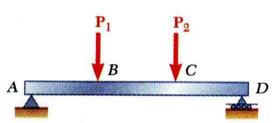
$(1200\text{ N})(1.5\text{ m}) + M = 0$ $\boxed{M = -1800\text{ N}\cdot\text{m}}$

$\sum F_x = 0:$ $\boxed{F = 0}$

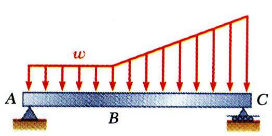
$\sum F_y = 0:$

$-1200\text{ N} - V = 0$ $\boxed{V = -1200\text{ N}}$

Various Types of Beam Loading and Support

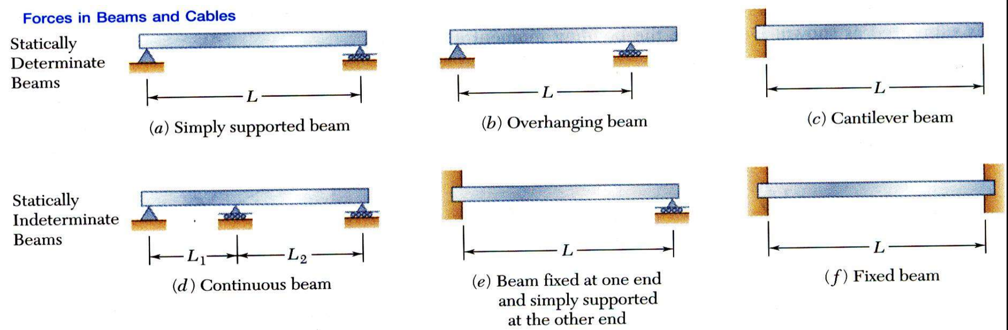


(*a*) Concentrated loads



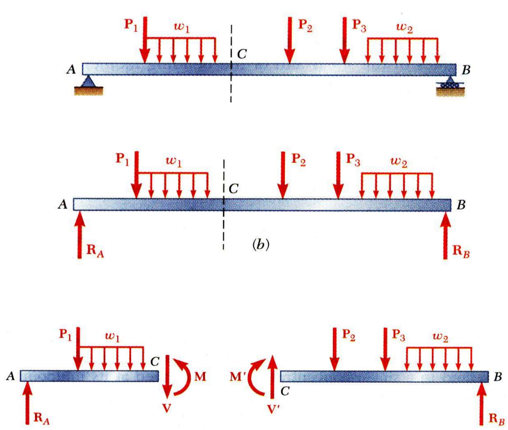
(*b*) Distributed load

- *Beam* - structural member designed to support loads applied at various points along its length.
- Beam can be subjected to ***concentrated*** loads or ***distributed*** loads or combination of both.
- *Beam design* is two-step process:
  1) determine shearing forces and bending moments produced by applied loads
  2) select cross-section best suited to resist shearing forces and bending moments



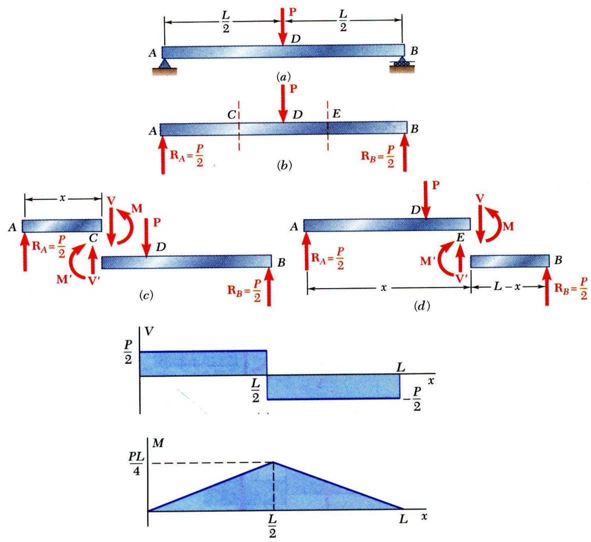
- Beams are classified according to way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.

Shear and Bending Moment in a Beam



- Wish to determine bending moment and shearing force at any point (for example, point C) in a beam subjected to concentrated and distributed loads.
- Determine reactions at supports by treating whole beam as free-body.
- Cut beam at *C* and draw free-body diagrams for *AC* and *CB*. **<u>By definition, positive sense for internal force-couple systems are as shown for each beam section</u>.**
- From equilibrium considerations, determine *M* and *V* or *M'* and *V'*.

Shear and Bending Moment Diagrams



- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,

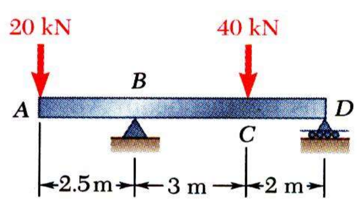
$$V = +P/2 \quad M = +Px/2$$

- Cut beam at *E* and consider member *EB*,

$$V = -P/2 \quad M = +P(L-x)/2$$

- For a beam subjected to <u>**concentrated loads**</u>**, shear is constant between loading points and moment varies linearly**.

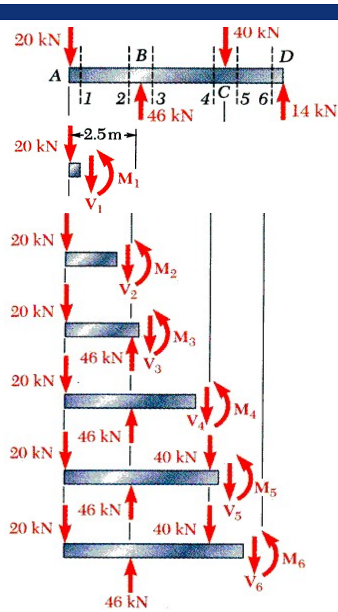
Sample Problem 7.2



Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results.



SOLUTION:

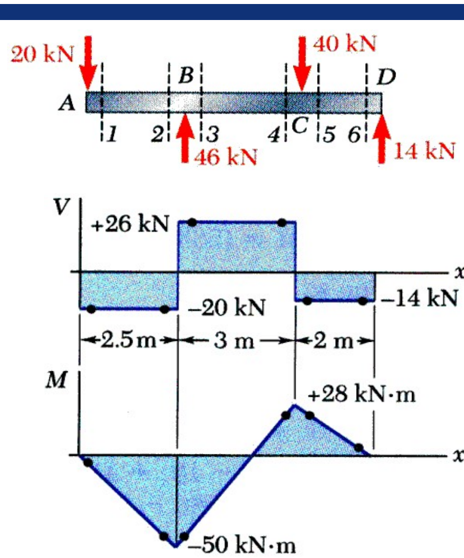
- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems at sections on either side of load application points.

$$\sum F_y = 0: \quad -20\text{ kN} - V_1 = 0 \qquad \boxed{V_1 = -20\text{ kN}}$$

$$\sum M_2 = 0: \quad (20\text{ kN})(0\text{ m}) + M_1 = 0 \qquad \boxed{M_1 = 0}$$

Similarly,

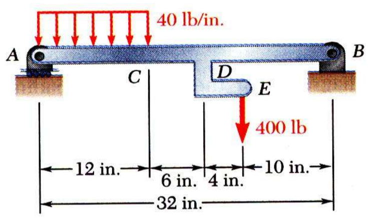
$V_2 = -20\text{ kN}$	$M_2 = -50\text{ kN}\cdot\text{m}$
$V_3 = 26\text{ kN}$	$M_3 = -50\text{ kN}\cdot\text{m}$
$V_4 = 26\text{ kN}$	$M_4 = +28\text{ kN}\cdot\text{m}$
$V_5 = -14\text{ kN}$	$M_5 = +28\text{ kN}\cdot\text{m}$
$V_6 = -14\text{ kN}$	$M_6 = 0\text{ kN}\cdot\text{m}$



- Plot results.

  Note that shear is of constant value between concentrated loads and bending moment varies linearly.

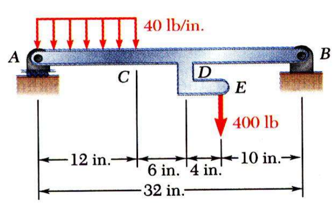
Sample Problem 7.3

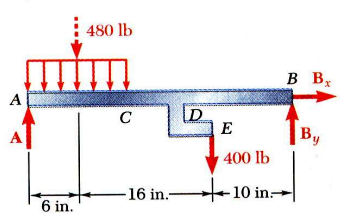


Draw the shear and bending moment diagrams for the beam *AB*. The distributed load of 40 lb/in. extends over 12 in. of the beam, from *A* to *C*, and the 400 lb load is applied at *E*.

SOLUTION:

- Taking entire beam as free-body, calculate reactions at *A* and *B*.
- Determine equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.
- Plot results.





SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *A* and *B*.

$\sum M_A = 0:$

$B_y(32\text{ in.}) - (480\text{ lb})(6\text{ in.}) - (400\text{ lb})(22\text{ in.}) = 0$

$B_y = 365\text{ lb}$

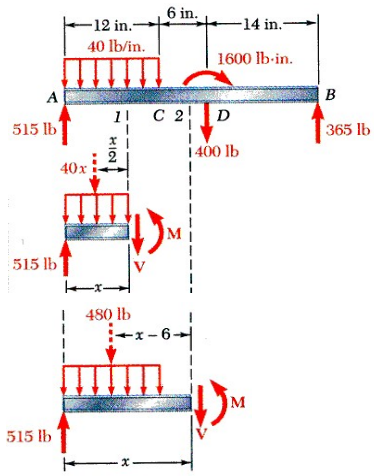
$\sum M_B = 0:$

$(480\text{ lb})(26\text{ in.}) + (400\text{ lb})(10\text{ in.}) - A(32\text{ in.}) = 0$

$A = 515\text{ lb}$

$\sum F_x = 0: \quad B_x = 0$

- Note: The 400 lb load at *E* may be replaced by a 400 lb force and 1600 lb-in. couple at *D*.



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From A to *C*:

$$\sum F_y = 0: \qquad 515 - 40x - V = 0$$

$$\boxed{V = 515 - 40x}$$

$$\sum M_1 = 0: \qquad -515x - 40x\left(\tfrac{1}{2}x\right) + M = 0$$

$$\boxed{M = 515x - 20x^2}$$

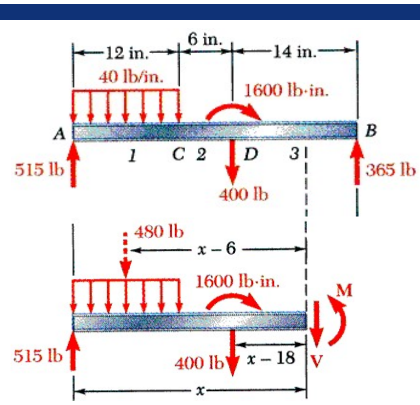
From *C* to *D*:

$$\sum F_y = 0: \qquad 515 - 480 - V = 0$$

$$\boxed{V = 35 \text{ lb}}$$

$$\sum M_2 = 0: \qquad -515x + 480(x - 6) + M = 0$$

$$\boxed{M = (2880 + 35x) \text{ lb} \cdot \text{in.}}$$



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From *D* to *B*:

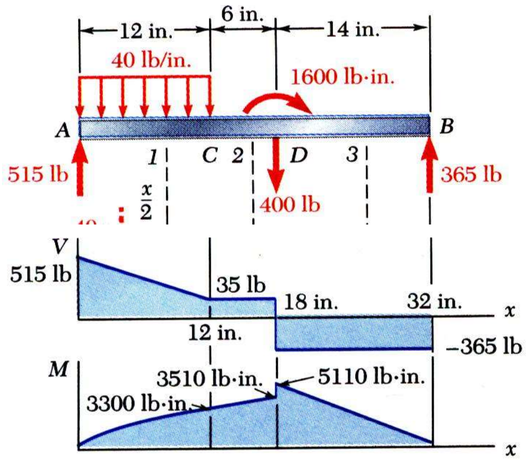
$$\sum F_y = 0: \qquad 515 - 480 - 400 - V = 0$$

$$\boxed{V = -365 \text{ lb}}$$

$$\sum M_2 = 0:$$

$$-515x + 480(x - 6) - 1600 + 400(x - 18) + M = 0$$

$$\boxed{M = (11{,}680 - 365x) \text{ lb} \cdot \text{in.}}$$



- Plot results.

From A to C:

$V = 515 - 40x$

$M = 515x - 20x^2$

From C to D:

$V = 35 \text{ lb}$

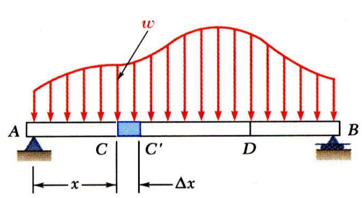
$M = (2880 + 35x)\,\text{lb} \cdot \text{in.}$

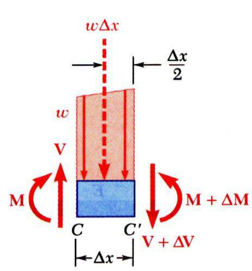
From D to B:

$V = -365 \text{ lb}$

$M = (11{,}680 - 365x)\,\text{lb} \cdot \text{in.}$

Relations Among Load, Shear, and Bending Moment





- Relations between load and shear:

$$V - (V + \Delta V) - w\Delta x = 0$$

$$\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx = -(\text{area under load curve})$$

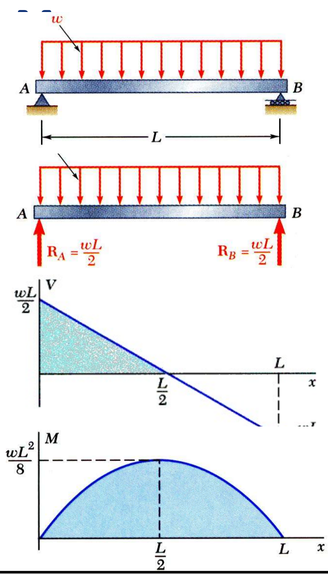
- Relations between shear and bending moment:

$$(M + \Delta M) - M - V\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\frac{dM}{dx} = \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \lim_{\Delta x \to 0} \left(V - \tfrac{1}{2} w \Delta x\right) = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx = (\text{area under shear curve})$$

Relations Among Load, Shear, and Bending



- Reactions at supports, $R_A = R_B = \frac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

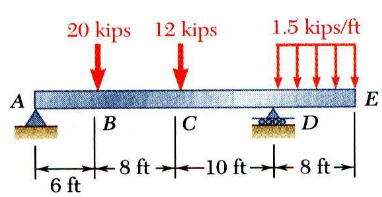
- Moment curve,

$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

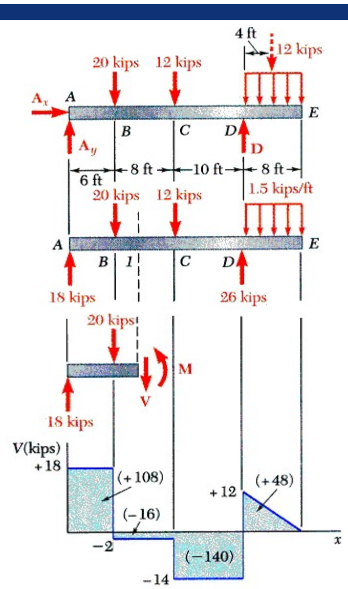
Sample Problem 7.4



Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free body, determine reactions at supports.
- Between concentrated load application points, and shear is constant. $dV/dx = -w = 0$
- With uniform loading between D and E, the shear variation is linear.
- Between concentrated load application points, The change in moment between load application points is equal to area under shear curve between points. $dM/dx = V = \text{constant}$
- With a linear shear variation between D and E, the bending moment diagram is a parabola.

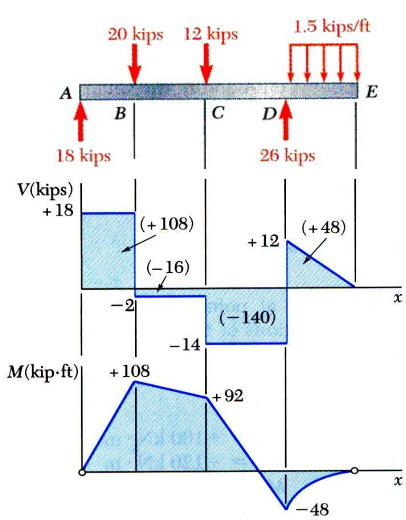


SOLUTION:

- Taking enatire beam as a free-body, determine reactions at supports.

$$\sum M_A = 0:$$
$$D(24\text{ ft}) - (20\text{ kips})(6\text{ ft}) - (12\text{ kips})(14\text{ ft}) - (12\text{ kips})(28\text{ ft}) = 0$$
$$\boxed{D = 26\text{ kips}}$$
$$\sum F_y = 0:$$
$$A_y - 20\text{ kips} - 12\text{ kips} + 26\text{ kips} - 12\text{ kips} = 0$$
$$\boxed{A_y = 18\text{ kips}}$$

- Between concentrated loads, $dV/dx = -w = 0$ and shear is constant and determined by appropriate section cut and solution.
- With uniform loading between *D* and *E*, the shear variation is linear.

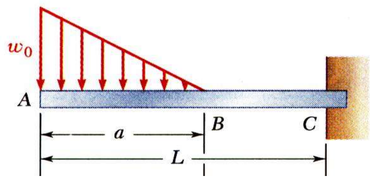


- Between concentrated load application points, The change in moment between load application points is equal to area under the shear curve between points. $dM/dx = V = \text{constant}$.

$$M_B - M_A = +108 \qquad M_B = +108\text{ kip}\cdot\text{ft}$$
$$M_C - M_B = -16 \qquad M_C = +92\text{ kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -48\text{ kip}\cdot\text{ft}$$
$$M_E - M_D = +48 \qquad M_E = 0$$

- With a linear shear variation between *D* and *E*, the bending moment diagram is a parabola.

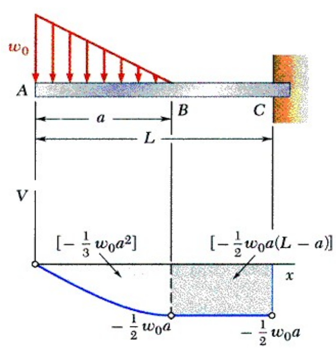
Sample Problem 7.6



Sketch the shear and bending-moment diagrams for the cantilever beam and loading shown.

SOLUTION:

- The change in shear between *A* and *B* is equal to the negative of area under load curve between points. The linear load curve results in a parabolic shear curve.
- With zero load, change in shear between *B* and *C* is zero.
- The change in moment between *A* and *B* is equal to area under shear curve between points. The parabolic shear curve results in a cubic moment curve.
- The change in moment between *B* and *C* is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.



SOLUTION:

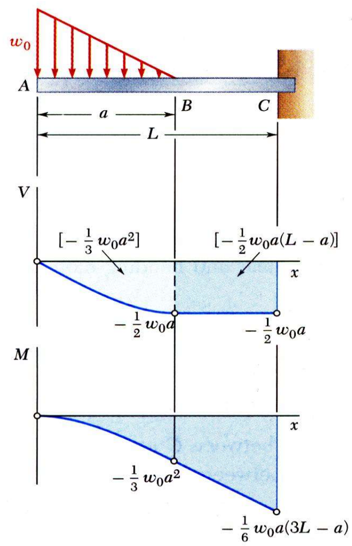
- The change in shear between *A* and *B* is equal to negative of area under load curve between points. The linear load curve results in a parabolic shear curve.

$$\text{at } A, \quad V_A = 0, \quad \frac{dV}{dx} = -w = -w_0$$

$$V_B - V_A = -\tfrac{1}{2} w_0 a \qquad\qquad V_B = -\tfrac{1}{2} w_0 a$$

$$\text{at } B, \quad \frac{dV}{dx} = -w = 0$$

- With zero load, change in shear between *B* and *C* is zero.



- The change in moment between *A* and *B* is equal to area under shear curve between the points. The parabolic shear curve results in a cubic moment curve.

$$\text{at } A, \quad M_A = 0, \quad \frac{dM}{dx} = V = 0$$

$$M_B - M_A = -\tfrac{1}{3}w_0a^2 \qquad M_B = -\tfrac{1}{3}w_0a^2$$

$$M_C - M_B = -\tfrac{1}{2}w_0a(L-a) \quad M_C = -\tfrac{1}{6}w_0a(3L-a)$$

- The change in moment between *B* and *C* is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.