ME201 STATICS

CHAPTER 6
Analysis of Structures

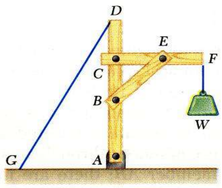
Contents

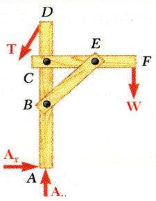
Application

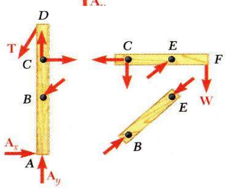
Design of support structures requires knowing the loads, or forces, that each member of the structure will experience.

Functional elements, such as the holding force of this plier, can be determined from concepts in this section.

Introduction

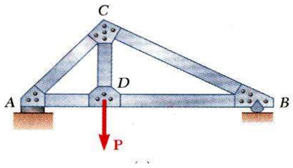


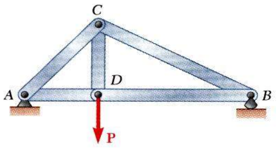


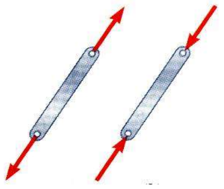


- For the equilibrium of structures made of several connected parts, the ***internal forces*** as well the ***external forces*** are considered.
- In the interaction between connected parts, **Newton's 3rd Law states that the *forces of action and reaction*** between bodies in contact have the same magnitude, same line of action, and opposite sense.
- Three categories of engineering structures are considered:
  - ***a) Trusses***: formed from ***two-force members***, i.e., straight members with end point connections and forces that act only at these end points.
  - ***b) Frames***: contain **at least one one multi-force member**, i.e., member acted upon by 3 or more forces.
  - ***c) Machines***: structures containing moving parts designed to transmit and modify forces.

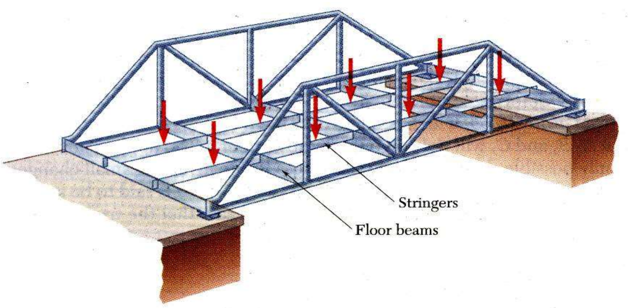
Definition of a Truss



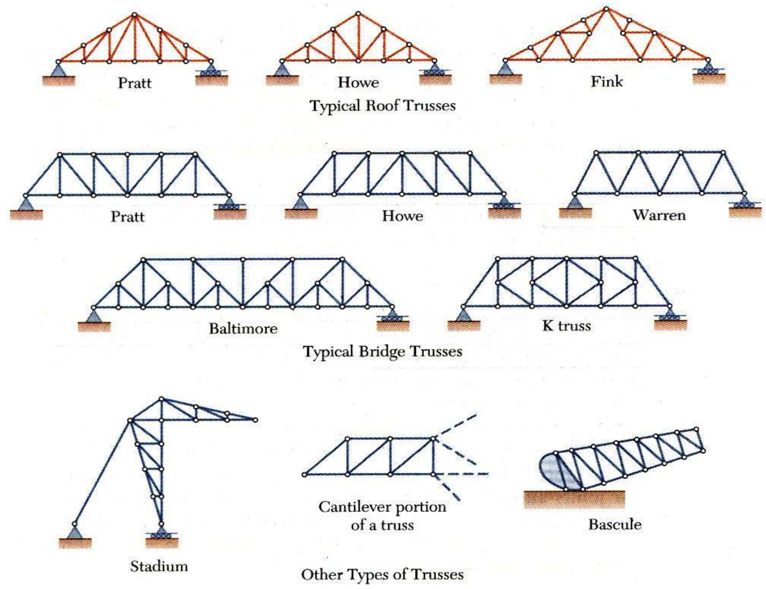




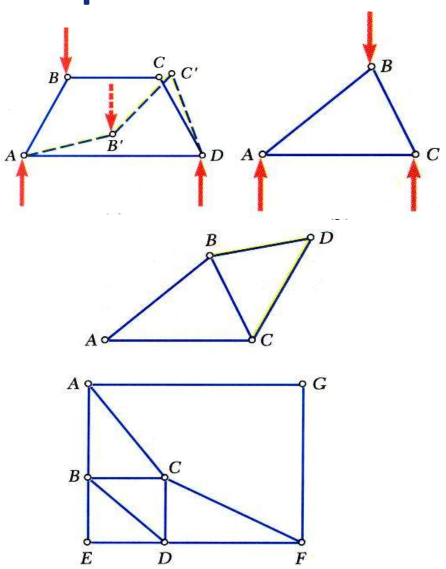
- A truss consists of **straight members** connected at joints. No member is continuous through a joint.
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as **a two-dimensional structure**.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. **Only *two-force members* are considered.**
- **When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.**



Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.



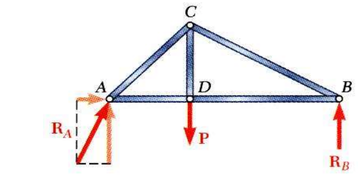
Simple Trusses

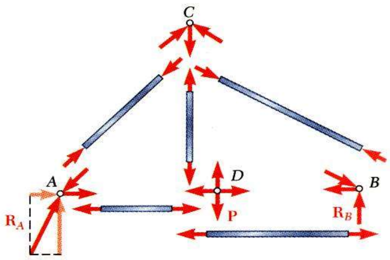


- A *rigid truss* will not collapse under the application of a load.

- **A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss**.

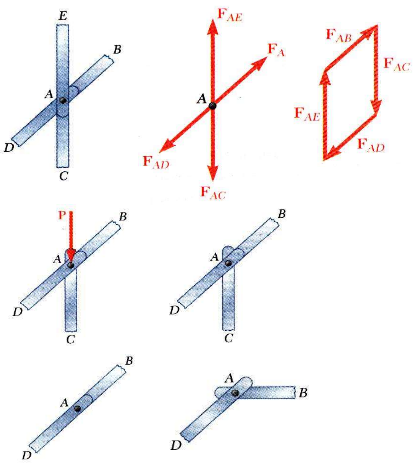
Analysis of Trusses by the Method of Joints



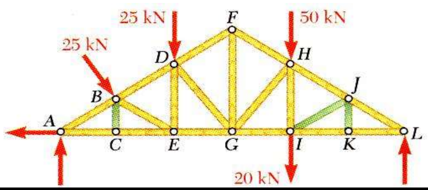


- Dismember the truss and create a freebody diagram for each member and pin.
- Conditions for equilibrium for the entire truss can be used to solve for **3 support reactions**.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- **Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite**.
- Conditions of equilibrium are used to solve for **2 unknown forces at each pin (or joint),** giving a total of 2*n* solutions, **where *n*=number of joints**. Forces are found by solving for unknown forces while moving from joint to joint sequentially.

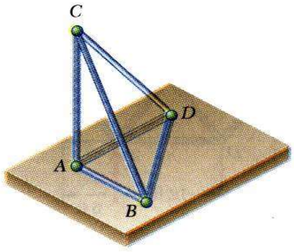
Joints Under Special Loading Conditions

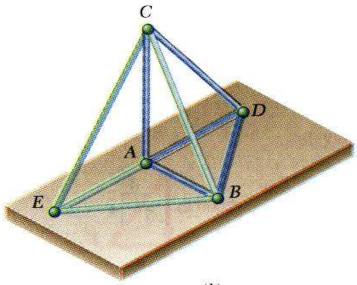


- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.



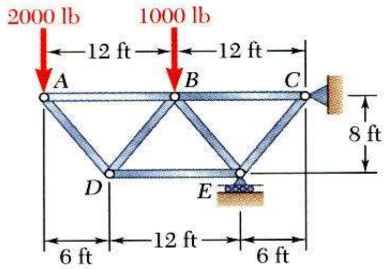
Space Trusses





- An *elementary space truss* consists of **6 members** connected at **4 joints to form a tetrahedron**.
- A *simple space truss* is formed and can be extended when **3 new members and 1 joint are added at the same time.**
- In a simple space truss, $m = 3n - 6$ where ***m* is the number of members and *n* is the number of joints**.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

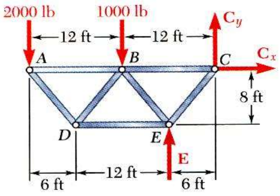
Sample Problem 6.1



Using the method of joints, determine the force in each member of the truss.

- DRAW THE FREE BODY DIAGRAM FOR THE ENTIRE TRUSS (always first) and solve for the 3 support reactions

Sample Problem 6.1

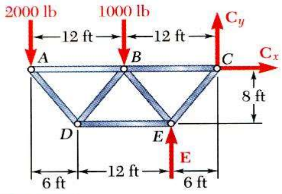


- Based on a free body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.

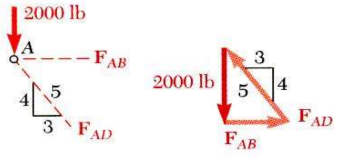
$$\sum M_C = 0$$
$$= (2000\ \text{lb})(24\ \text{ft}) + (1000\ \text{lb})(12\ \text{ft}) - E(6\ \text{ft})$$
$$\boxed{E = 10{,}000\ \text{lb} \uparrow}$$

$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -2000\ \text{lb} - 1000\ \text{lb} + 10{,}000\ \text{lb} + C_y$$
$$\boxed{C_y = 7000\ \text{lb} \downarrow}$$



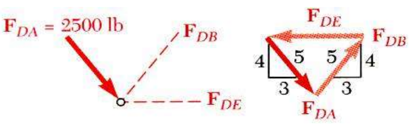
- We now solve the problem by moving sequentially from joint to joint and solving the associated FBD for the unknown forces.



- Joints *A* or *C* are equally good because each has only 2 unknown forces. Use joint A and draw its FBD and find the unknown forces.

$$\frac{2000\ \text{lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

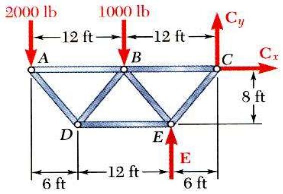
$$\boxed{\begin{aligned} F_{AB} &= 1500\ \text{lb}\ \ T \\ F_{AD} &= 2500\ \text{lb}\ \ C \end{aligned}}$$

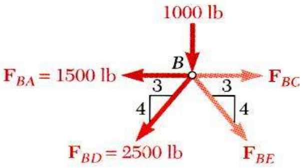


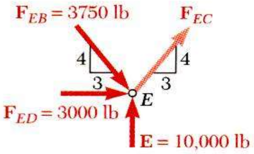
- Joint D, since it has 2 unknowns remaining (joint B has 3). Draw the FBD and solve.

$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$\boxed{\begin{aligned} F_{DB} &= 2500\ \text{lb}\ \ T \\ F_{DE} &= 3000\ \text{lb}\ \ C \end{aligned}}$$







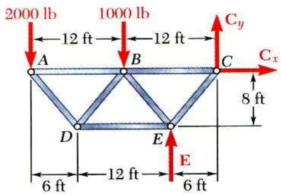
- There are now only two unknown member forces at joint B. Assume both are in tension.

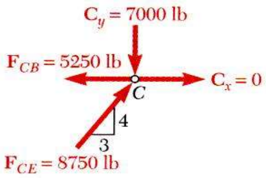
$$\sum F_y = 0 = -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5}F_{BE}$$
$$F_{BE} = -3750 \text{ lb} \qquad \boxed{F_{BE} = 3750 \text{ lb } C}$$

$$\sum F_x = 0 = F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750)$$
$$F_{BC} = +5250 \text{ lb} \qquad \boxed{F_{BC} = 5250 \text{ lb } T}$$

- There is one remaining unknown member force at joint *E* (or *C*). Use joint E and assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 3000 + \tfrac{3}{5}(3750)$$
$$F_{EC} = -8750 \text{ lb} \qquad \boxed{F_{EC} = 8750 \text{ lb } C}$$

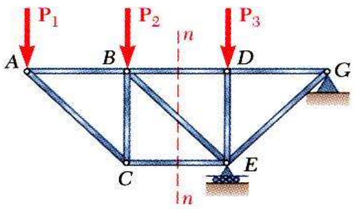




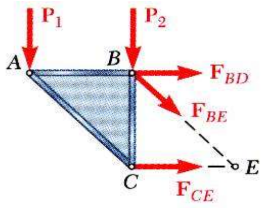
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -5250 + \tfrac{3}{5}(8750) = 0 \quad (\text{checks})$$
$$\sum F_y = -7000 + \tfrac{4}{5}(8750) = 0 \quad (\text{checks})$$

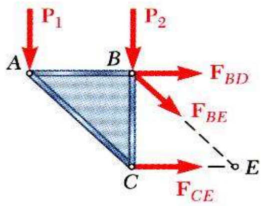
Analysis of Trusses by the Method of Sections

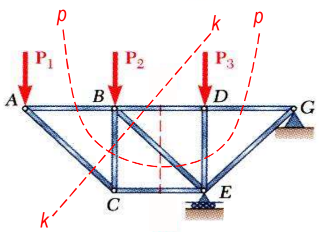


- **When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.**



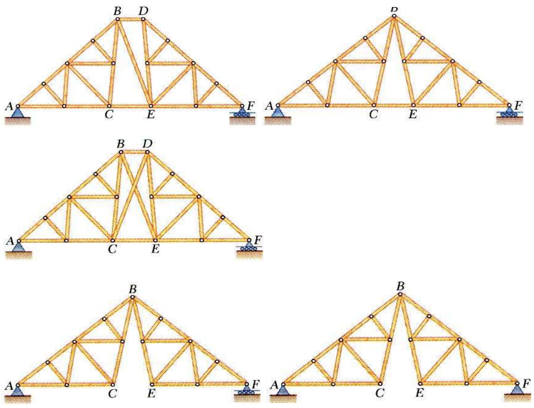
- To determine the force in member *BD,* form a *section* by "cutting" the truss at *n-n* and create a free body diagram for the left side.
- **Notice that the exposed internal forces are all *assumed* to be in tension.**
- With only 3 members cut by the section, the equations for static equilibrium may be applied to determine the unknown forces, including F_{BD}.





- Assume that the initial section cut was made using line *k-k*.
- **Notice that *any* cut may be chosen, so long as the cut creates a separated section.**
- So, for example, this cut with line *p-p* is acceptable.

Trusses Made of Several Simple Trusses



$m < 2n - 3$ $m < 2n - 4$

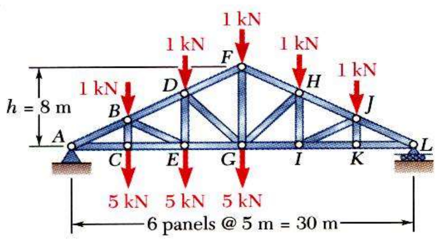
- *Compound trusses* are statically determinant, rigid, and completely constrained (m member number, n joint). $m = 2n - 3$
- Truss contains a *redundant member* and is *statically indeterminate.*

$$m > 2n - 3$$

- Additional reaction forces may be necessary for a rigid truss.
- Necessary but insufficient condition for a compound truss to be statically determinant, rigid, and completely constrained,

$$m + r = 2n$$

Sample Problem 6.3

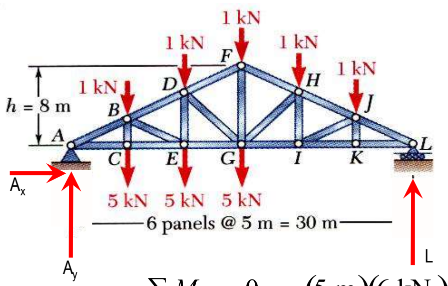


Determine the force in members *FH*, *GH*, and *GI*.

SOLUTION:

1. Draw the FBD for the entire truss. Apply the equilibrium conditions and solve for the reactions at *A* and *L*.
2. Make a cut through members *FH*, *GH*, and *GI* and take the right-hand section as a free body (the left side would also be good).
3. Apply the conditions for static equilibrium to determine the desired member forces.

Sample Problem 6.3



SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilib-rium to solve for the reactions at *A* and *L*.

$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$

$$-(20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$

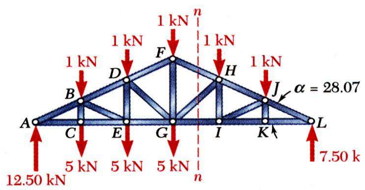
$$L = 7.5\text{ kN} \uparrow$$

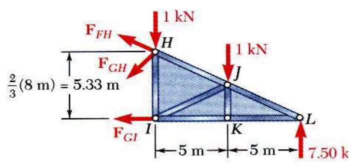
$$\sum F_y = 0 = -20\text{ kN} + L + A_y$$

$$A_y = 12.5\text{ kN} \uparrow$$

$$\sum F_x = 0 = A_x$$

Sample Problem 6.3





- Make a cut through members *FH*, *GH*, and *GI* and take the right-hand section as a free body. Draw this FBD.
- What is the one equilibrium equation that could be solved to find F_{GI}? Confirm your answer with a neighbor.

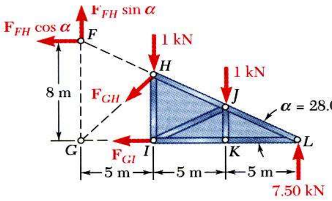
- Sum of the moments about point H:

$$\sum M_H = 0$$

$$(7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) = 0$$

$$F_{GI} = +13.13\text{ kN}$$

$$F_{GI} = 13.13\text{ kN} \quad T$$



- F_{FH} is shown as its components. What one equilibrium equation will determine F_{FH}?

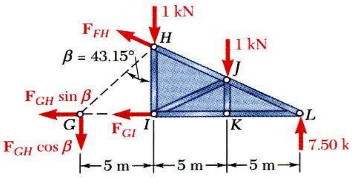
$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m})$$

$$+ (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.82\text{ kN} \qquad \boxed{F_{FH} = 13.82\text{ kN } C}$$



- There are many options for finding F_{GH} at this point (e.g., $\Sigma F_x=0$, $\Sigma F_y=0$). Here is one more:

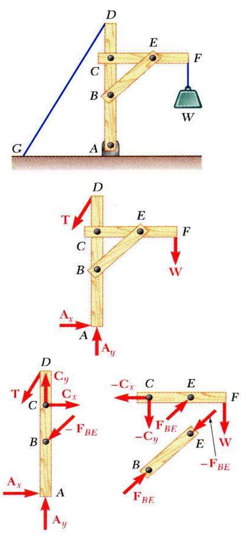
$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

$$\sum M_L = 0$$

$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(10\text{ m}) = 0$$

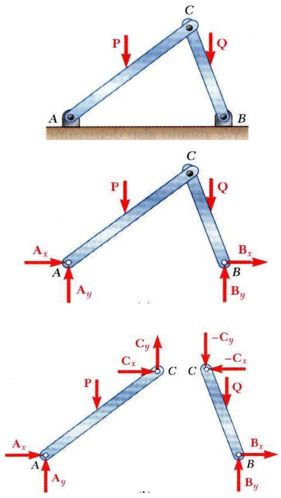
$$F_{GH} = -1.371\text{ kN} \qquad \boxed{F_{GH} = 1.371\text{ kN } C}$$

Analysis of Frames



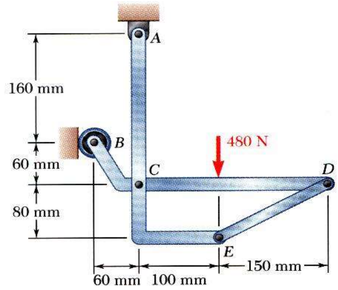
- ***Frames* and *machines* are structures with at least one *multiforce* (>2 forces) member.** Frames are designed to support loads and are usually stationary. Machines contain moving parts and transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. **They must be represented with two unknown components.**
- Forces between connected components are equal, have the same line of action, and opposite sense.

Frames Which Cease To Be Rigid When Detached From Their Supports



- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates **four unknown force components which cannot be determined from the three equilibrium conditions (statically indeterminate**).
- **The frame must be considered as two distinct, but related, rigid bodies**.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams show **6 unknown force components**.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations. Thus, taking the frame apart made the problem solvable.

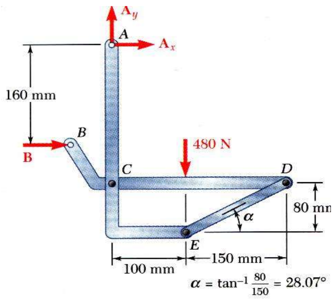
Sample Problem 6.4



Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

<u>SOLUTION</u>:

1. Create a free body diagram for the complete frame and solve for the support reactions.



SOLUTION:

1. Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\ \text{N}$$

$$\boxed{A_y = 480\ \text{N}\ \uparrow}$$

$$\sum M_A = 0 = -(480\ \text{N})(100\ \text{mm}) + B(160\ \text{mm})$$

$$\sum F_x = 0 = B + A_x$$

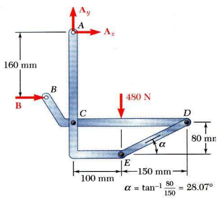
$$\boxed{B = 300\ \text{N}\ \rightarrow}$$

$$A_x = -300\ \text{N}$$

$$\boxed{A_x = 300\ \text{N}\ \leftarrow}$$

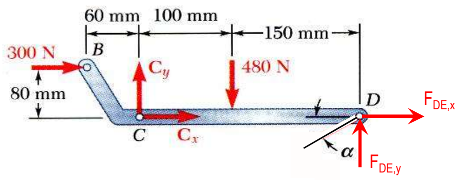
Note:

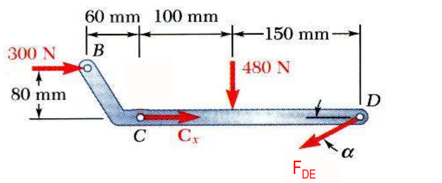
$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

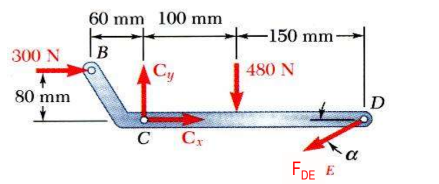


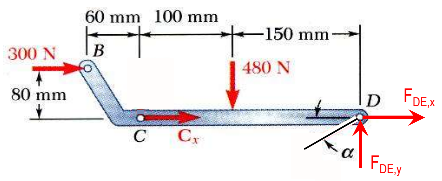
SOLUTION (cont.):

2. Create a free body diagram for member BCD (since the problem asked for forces on this body).

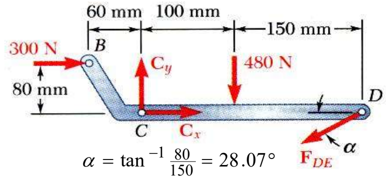








SOLUTION (cont.):



$$\sum M_C = 0 = (F_{DE}\sin\alpha)(250\text{ mm}) + (300\text{ N})(60\text{ mm}) + (480\text{ N})(100\text{ mm})$$
$$F_{DE} = -561\text{ N}$$

$$F_{DE} = 561\text{ N } C$$

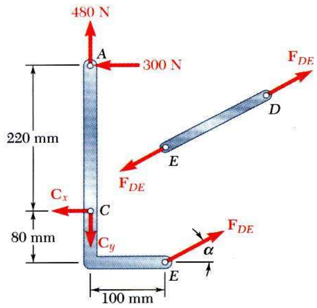
- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$
$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N}$$

$$C_x = -795\text{ N}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$
$$0 = C_y - (-561\text{ N})\sin\alpha - 480\text{ N}$$

$$C_y = 216\text{ N}$$



- With member *ACE* as a free body with no additional unknown forces, check the solution by summing moments about *A*.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

(checks)