ME201 STATICS

CHAPTER 2

Contents

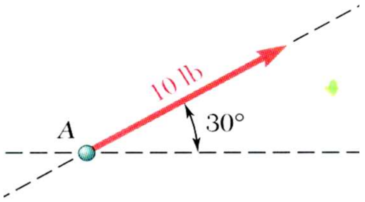
Application

The tension in the cable supporting this person can be found using the concepts in this chapter

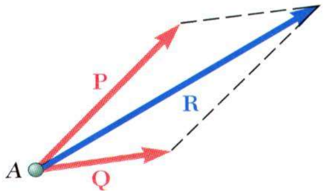
Introduction

- The objective for the current chapter is to investigate the effects of forces on particles:
  - **replacing multiple forces** acting on a particle with a single equivalent **or *resultant* force,**
  - **relations between forces** acting on a particle that is in a state of ***equilibrium***.
- The focus on ***particles*** **does not imply a restriction to miniscule bodies**. Rather, the study is restricted to analyses in which the **size and shape of the bodies is not significant** so that **all forces may be assumed** to be applied **at a single point**.

Resultant of Two Forces

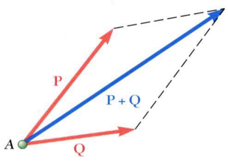


- **force**: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense.*



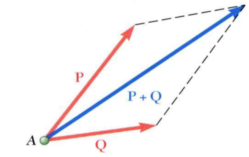
- Experimental evidence shows that the combined effect of two forces may be represented by a single ***resultant*** force.
- The resultant is **equivalent** to the **diagonal** of a **parallelogram** which contains the two forces in adjacent legs.
- **Force is a *vector* quantity**.

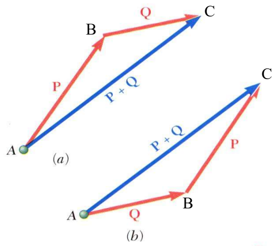
Vectors

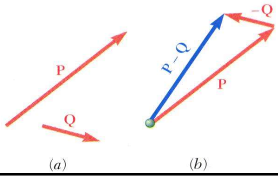


- ***Vector***: parameters possessing **magnitude and direction which add according to the parallelogram law**. Examples: **displacements, velocities, accelerations**.
- ***Scalar***: parameters possessing magnitude but **not direction**. Examples: **mass, volume, temperature**
- Vector classifications:
  - ***Fixed* or *bound*** vectors have well defined **points of application that cannot be changed without affecting an analysis**.
  - ***Free* vectors may be freely moved** in space without changing their effect on an analysis.
  - ***Sliding* vectors** may be applied anywhere **along their line of action** without affecting an analysis.

Addition of Vectors







- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

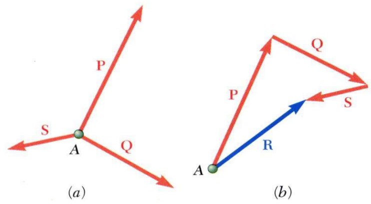
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

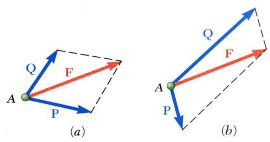
- Vector addition is commutative,

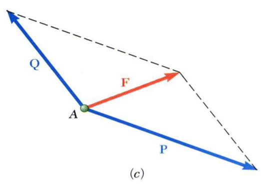
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Resultant of Several Concurrent Forces





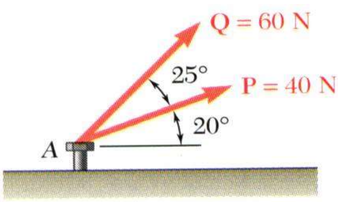


- ***Concurrent forces***: set of forces which all pass through the **same point**.

  A set of concurrent forces applied to a particle may be replaced by a **single resultant force** which is the vector **sum of the applied forces**.

- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Sample Problem 2.1

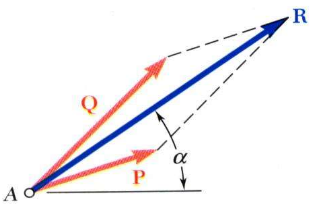


The two forces act on a bolt at *A*. Determine their resultant.

SOLUTION:

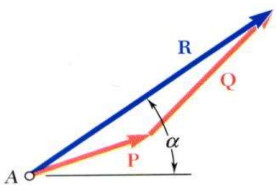
- Graphical solution - construct a parallelogram with sides in the same direction as **P** and **Q** and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the the diagonal.
- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

Sample Problem 2.1



- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

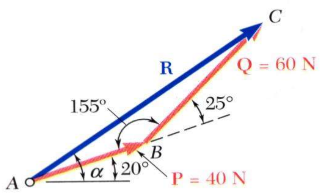
$\mathbf{R} = 98 \text{ N} \quad \alpha = 35°$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$\mathbf{R} = 98 \text{ N} \quad \alpha = 35°$

Sample Problem 2.1



- Trigonometric solution - Apply the triangle rule. From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\,\text{N})^2 + (60\,\text{N})^2 - 2(40\,\text{N})(60\,\text{N})\cos 155°$$

$$R = 97.73\,\text{N}$$

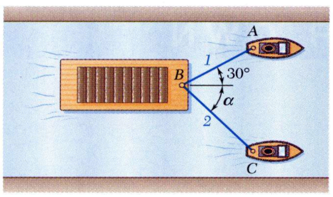
From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\,\text{N}}{97.73\,\text{N}}$$

$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\alpha = 35.04°$$

Sample Problem 2.2



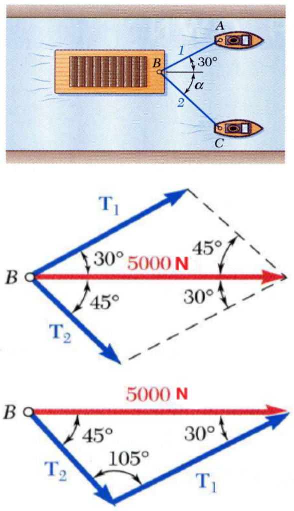
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 N directed along the axis of the barge, determine the tension in each of the ropes for α = 45°.

SOLUTION:

- Find a graphical solution by applying the Parallelogram Rule for vector addition. The parallelogram has sides in the directions of the two ropes and a diagonal in the direction of the barge axis and length proportional to 5000 N.

- Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.

Sample Problem 2.2



- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

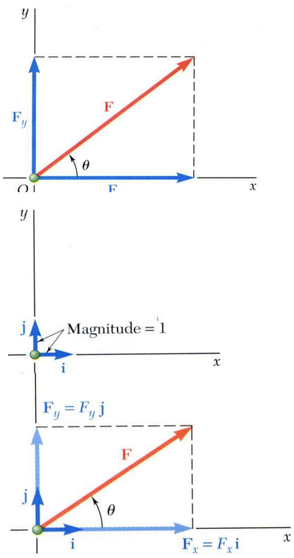
$T_1 = 3700\ \text{N} \quad T_2 = 2600\ \text{N}$

- Trigonometric solution - Triangle Rule with Law of Sines

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\ \text{N}}{\sin 105°}$$

$T_1 = 3660\ \text{N} \quad T_2 = 2590\ \text{N}$

Rectangular Components of a Force: Unit Vectors



- It's possible to resolve a force vector into **perpendicular components** so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as ***rectangular vector components*** and

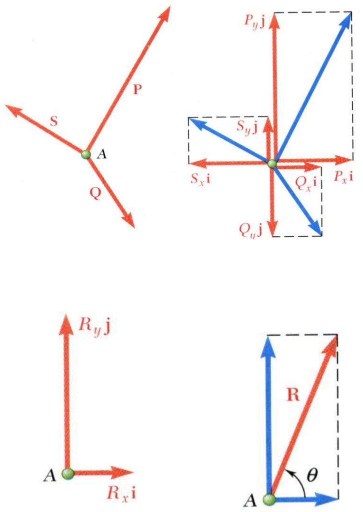
$$\vec{F} = \vec{F}_x + \vec{F}_y$$

- **Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.**

- Vector components **may be expressed as products of the unit vectors** with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$



- **To find the resultant of 3 (or more) concurrent forces,**
  $\vec{R} = \vec{P} + \vec{Q} + \vec{S}$
- **Resolve each force into rectangular components**, then add the components in each direction:

$$R_x\vec{i} + R_y\vec{j} = P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j}$$
$$= (P_x + Q_x + S_x)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j}$$

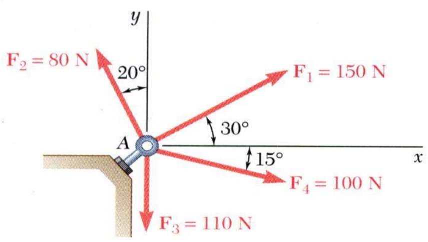
- The scalar components of the resultant vector are equal to the sum of the corresponding scalar components of the given forces.

$$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$

- To find the **resultant magnitude and direction**,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Sample Problem 2.3

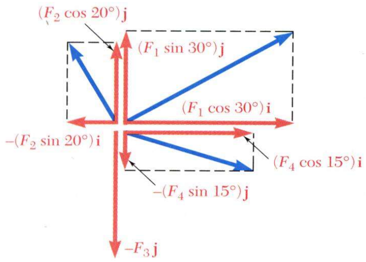


Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

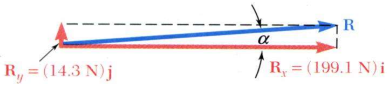
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components in the x and y directions.
- Calculate the magnitude and direction of the resultant.

SOLUTION:



- Resolve each force into rectangular components.

force	*mag*	*x − comp*	*y − comp*
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$



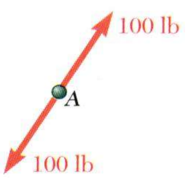
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction.

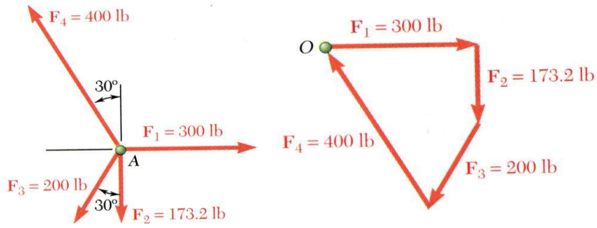
$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\,\text{N}}$$

$$\tan \alpha = \frac{14.3\,\text{N}}{199.1\,\text{N}} \qquad \boxed{\alpha = 4.1°}$$

Equilibrium of a Particle

- **When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.**
- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.



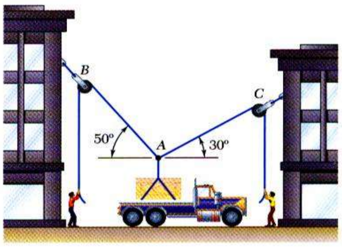


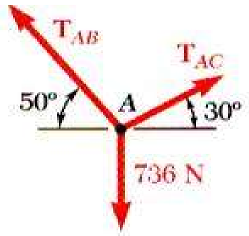
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense
- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Free-Body Diagrams

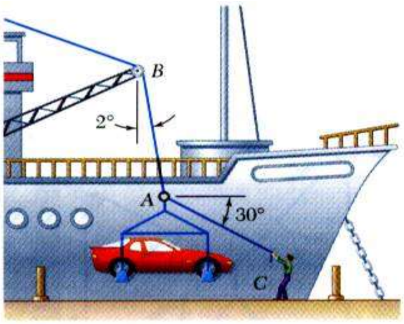




Space Diagram: A sketch showing the physical conditions of the problem, usually provided with the problem statement, or represented by the actual physical situation.

Free Body Diagram: A sketch showing only the forces on the selected particle. <u>This must be created by you</u>.

Sample Problem 2.4

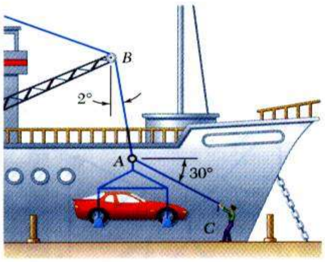


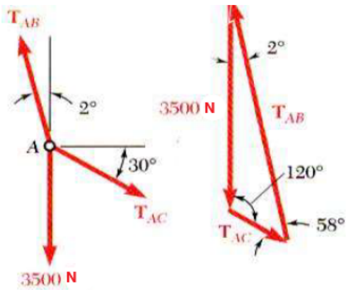
In a ship-unloading operation, a 3500N automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

- Construct a free body diagram for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

Sample Problem 2.4





SOLUTION:

- Construct a free body diagram for the particle at A, and the associated polygon.
- Apply the conditions for equilibrium and solve for the unknown force magnitudes.

Law of Sines:

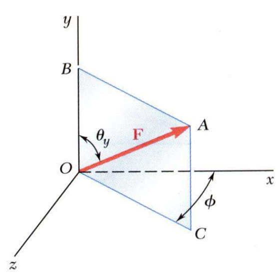
$$\frac{T_{AB}}{\sin 120^\circ} = \frac{T_{AC}}{\sin 2^\circ} = \frac{3500\text{ N}}{\sin 58^\circ}$$

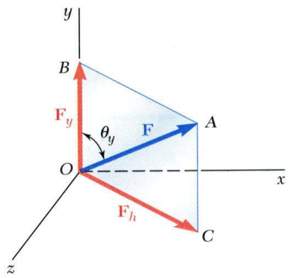
$T_{AB} = 3570\text{ N}$

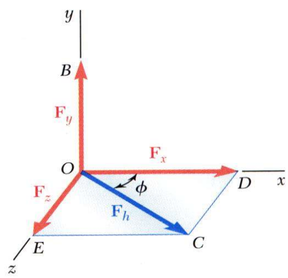
$T_{AC} = 144\text{ N}$

Expressing a Vector in 3-D Space

If angles with some of the axes are given:







- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

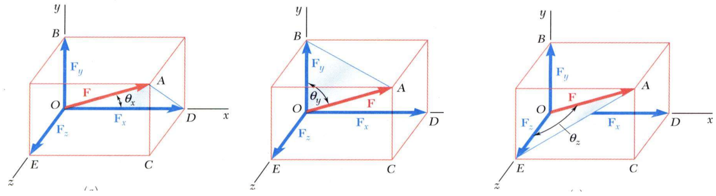
$$F_y = F \cos\theta_y$$

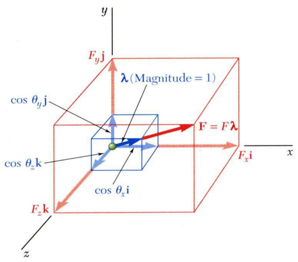
$$F_h = F \sin\theta_y$$

- Resolve F_h into rectangular components

$$F_x = F_h \cos\varphi$$
$$= F \sin\theta_y \cos\varphi$$
$$F_z = F_h \sin\varphi$$
$$= F \sin\theta_y \sin\varphi$$

If the direction cosines are given:





- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

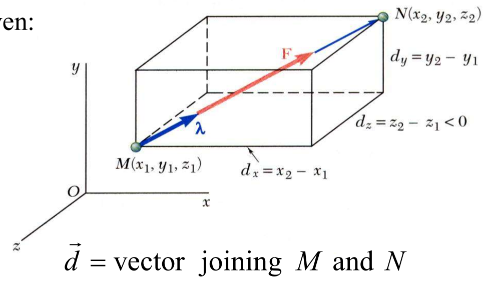
$$= F\vec{\lambda}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

If two points on the line of action are given:

Direction of the force is defined by the location of two points,
$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\vec{d} = \text{vector joining } M \text{ and } N$$

$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$

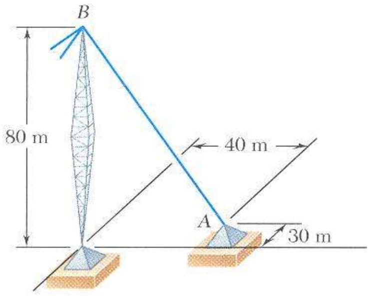
$$d_x = x_2 - x_1, d_y = y_2 - y_1, d_z = z_2 - z_1$$

$$\vec{F} = F\vec{\lambda}$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$F_x = \frac{Fd_x}{d} F_y = \frac{Fd_y}{d} F_z = \frac{Fd_z}{d}$$

Sample Problem 2.7



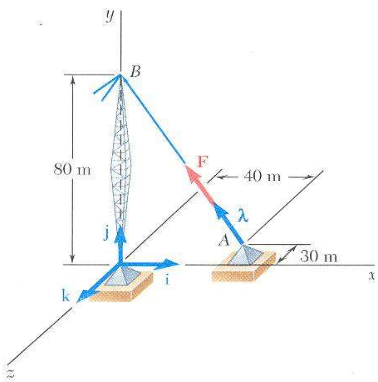
The tension in the wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at A,

b) the angles θ_x, θ_y, θ_z defining the direction of the force (the direction cosines)

SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.



SOLUTION:

- Determine the unit vector pointing from A towards B.

$$\overrightarrow{AB} = (-40\text{m})\vec{i} + (80\text{m})\vec{j} + (30\text{m})\vec{k}$$

$$AB = \sqrt{(-40\text{m})^2 + (80\text{m})^2 + (30\text{m})^2}$$

$$= 94.3\text{ m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$

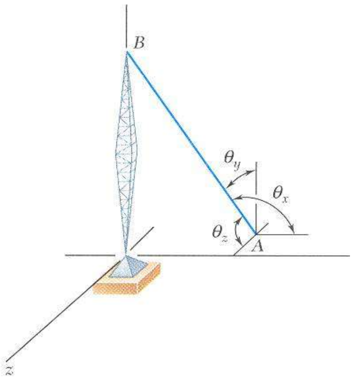
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

- Determine the components of the force.

$$\vec{F} = F\vec{\lambda}$$

$$= (2500\text{ N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$

$$= (-1060\text{ N})\vec{i} + (2120\text{ N})\vec{j} + (795\text{ N})\vec{k}$$

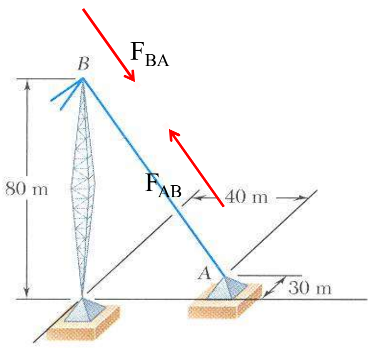


- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$
$$= -0.424\,\vec{i} + 0.848\,\vec{j} + 0.318\,\vec{k}$$

$$\theta_x = 115.1^\circ$$
$$\theta_y = 32.0^\circ$$
$$\theta_z = 71.5^\circ$$

What if…?



What are the components of the force in the wire at point B? Can you find it without doing any calculations?

SOLUTION:

- Since the force in the guy wire must be the same throughout its length, the force at B (and acting toward A) must be the same magnitude but opposite in direction to the force at A.

$$\vec{F}_{BA} = -\vec{F}_{AB}$$
$$= (1060\,\text{N})\vec{i} + (-2120\ \text{N})\vec{j} + (-795\ \text{N})\vec{k}$$