CHAPTER 1 | MEASUREMENT

Introduction

- Everything in physics and in engineering is based on the results of *experiments* that are designed and performed for the *measurements* and *comparisons* of (physical) quantities. Among these quantities are length, mass, time, temperature, current, force, pressure, velocity, acceleration, etc.

- A quantity is measured in terms of its own **unit**, which is a special name assigned to the measure of that quantity. For example, meter (m) is a unit of length, second (s) is a unit of time, and meter per second-squared (m/s^2) is a unit of acceleration.

- The measurement of a quantity is made by comparing it with a **standard**, which is precisely one unit of that quantity.

- We can specify a unit and its standard for a quantity in whatever way we like. The point here, however, is that the standard for a unit must be practical, accessible, transportable, duplicatable, and above all, they must be invariable and must be accepted or adopted internationally by the majority of scientists, authorities, and people.

- Whenever we have adopted a standard for mass, for example, we can use it in measuring and comparing the mass of any object, like the mass of a soccer ball, a book, your computer, an electron, a building, etc. Most of the time, however, we cannot make a comparison directly: it is impossible to measure the mass of an electron or a building with the help of a pair of scales; they can be determined only by secondary meanings.

- Although there are many quantities to measure and compare, they are most of the time expressible in terms of a few **base quantities** and their **base standards**, whose uses are internationally adopted. Quantities length, mass, and time are such three base quantities; many other quantities are expressed in terms of them and their standards. Force, for example, is specified as "mass times length per time squared."

- In 1971, the use of the international system of units, abbreviated SI[1], has been adopted, where seven quantitites were chosen to be the base quantities. We tabulate these base quantities, along with their names and symbols, in Table 1. This unit system is sometimes referred to as the **metric system**, and these units are said to be based on a "human scale;" that is, they are within the mental capabilities of the human.

Table 1: The SI base quantities. (The last two units are the SI supplementary units.)

Quantity	Unit Symbol	Unit Name
length	m	meter
mass	kg	kilogram
time	s	second
electric current	A	ampere
temperature	K	kelvin
luminous intensity	cd	candela
amount of substance	mol	mole
2D plane angle	rad	radian
3D solid angle	sr	steradian

- In PHYS 101, the first part of your introductory physics courses, four base quantities in Table 1 will be sufficient for our studies. They are, with their unit names and symbols, length–meter (m), time–second (s), mass–kilogram (kg), and 2D plane angle–radian (rad). In the next semester, in PHYS 102, electric current–ampere (A) will be included in this list. You will use the remaining quantities in Table 1 in your other science and engineering courses.

[1]From French: *Le système international d'unités*

- As our course advances, we will frequently encounter **derived units**, which will be defined in terms of the base units given in Table 1. For example, we will call the SI unit of force as the **newton** (N), and will define it to be as[2]

$$1 \text{ newton} = 1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2.$$

- Be warned at the outset: we will almost always work with SI units throughout this course. This means that before attempting to solve a problem, you should check the given quantities; if they are not expressed in SI units, you must first convert them into their SI equivalents.

Length

- The speed of light is given *exactly* as

$$c = 299\ 792\ 458 \text{ m/s}.$$

Such an accurate quantity has allowed scientists, in 1983, to use the speed of light in defining the **meter**, the SI unit of length: *The meter is the length of the path traveled by light in vacuum during a time interval of* $1/299\ 792\ 458$ *of a second.*[3]

Time

- There is a lot that can be said about the concept of time, much of them is abstract or philosphical. It is enough for the time being to recognize that time gives answers to the following two questions about an event: "When did it happen?" and "How much did it last?"

- We can accept any *periodic* occurrence as our time standard. The rotation of the Earth about its own axis, which specifies the duration of a day, is such an periodic occurence. Another example is the pendulum of wall clock. The common problem with these periodic occurences is that their calibration is not precise enough. (Nevertheless, the notion of pendulum or "simple harmonic motion" had been the base for the clocks for many centuries.)

[2]The units in the last part of this equation is read as "kilogram-meter per second squared."

[3]From the NIST (National Institute of Standards and Technolgy) page. Do not memorize this definition; just try to grasp its meaning.

- The need for the extreme accuracy for the modern technology and science always demanded scientists to resort to other meanings for the time standard. In 1967, scientists adopted the following definition for the **second**, the SI unit of time: *The second is the duration of* 9 192 631 770 *periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.*[4]

Mass

- The **kilogram**, the SI unit of mass, is defined to be equal to *the mass of the international prototype of the kilogram,*[5] which is a platinum-iridium cylinder, kept at Le Bureau International des Poids et Mesures (International Bureau of Weights and Measures) in Sèvres, France.

- This bureau duplicates the standard for the kilogram as precisely as possible and send the so-obtained copies to other countries for their own uses.

- We have another unit of mass, called **atomic mass unit** (amu), which is heavily used to express atomic and molecular masses. It is defined as *one twelfth of the mass of the carbon*-12 *atom.* The conversion between amu and kg is done via

$$1 \text{ amu} = 1.660\ 538\ 86 \times 10^{-27} \text{ kg}.$$

  Why do we need a second unit of mass? The answer is that we can compare the mass of an atom with one another much more accurately than we can compare it with the standard kilogram.

- As it is obvious from the above SI definitions, we have at the moment an extremely accurate lenght standard, the meter. Perhaps the same can be said for the time standard, the second. But for the mass standard, the kilogram, scientists could not reach such an accuracy, and they strive ceaselessly to develop techniques in order to obtain a more *operational* definition that can be checked in a laboratory at any time and at any place. The atomic mass unit mentioned in the preceding paragraph is such an operational definition.

[4]The NIST definition, not to be memorized.
[5]The NIST definition.

Scientific Notation

- We will use this notation when we keep track of very large and very small numbers. It deals with powers of 10:

$$\begin{aligned}
&\vdots \\
10^{-5} &= 0.00001 = \frac{1}{10\times 10\times 10\times 10\times 10} \\
10^{-4} &= 0.0001 = \frac{1}{10\times 10\times 10\times 10} \\
10^{-3} &= 0.001 = \frac{1}{10\times 10\times 10} \\
10^{-2} &= 0.01 = \frac{1}{10\times 10} \\
10^{-1} &= 0.1 = \frac{1}{10} \\
10^{0} &= 1 \\
10^{1} &= 10 \\
10^{2} &= 100 = 10\times 10 \\
10^{3} &= 1000 = 10\times 10\times 10 \\
10^{4} &= 10,000 = 10\times 10\times 10\times 10 \\
10^{5} &= 100,000 = 10\times 10\times 10\times 10\times 10 \\
&\vdots
\end{aligned}$$

- In the general form of 10^n, n is referred to as the **exponent** of 10.

- Numbers in the form

$$\boxed{a \times 10^n}$$

are said to be written in **scientific notation**, where a is between 1 and 10. Examples are

$$10\ 419\ 500\ 000 = 1.04195 \times 10^{10}$$

and[6]

$$0.000262 = 2.62 \times 10^{-4}.$$

[6] Numbers in the form of 123.45678 are in general referred to as **decimal numbers** (or **floating numbers**). Here each number is called a **digit**, and the point "." is called as the **decimal point**. The number of digits after the decimal point is referred to as the **decimal places**, so that the number 123.45678 has *five* decimal places.

- Especially in some pocket scientific calculators, scientific notation can appear like 6.567E9 and 0.273E-4, where the letter "E" is used to mean "exponent of 10". Thus, if you happen to see such numbers in your calculator, you should immediately translate them as

$$6.567\text{E}9 = 6.567 \times 10^9 \quad \text{and} \quad 0.273\text{E-}4 = 0.273 \times 10^{-4}.$$

- If two or more numbers written in scientific notation are multiplied or divided, the rules to be followed are

$$\boxed{\begin{aligned} 10^m \times 10^n &= 10^{m+n} \\ \frac{10^m}{10^n} &= 10^{m-n} \end{aligned}}$$

where the latter actually follows from the former.

Examples

1. Decimal numbers and their forms in scientific notation:

$$\begin{aligned} -0.000298564 &= -2.98564 \times 10^{-4} \\ 0.0739458 &= 7.39458 \times 10^{-2} \\ 82\ 789 &= 8.2789 \times 10^{4} \\ -1\ 928.254 &= -1.928254 \times 10^{3} \\ 225\ 568\ 252 &= 2.25568252 \times 10^{8} \end{aligned}$$

2. Multiplication and division in scientific notation:

$$\begin{aligned} \left(4 \times 10^2\right)\left(3 \times 10^4\right) &= 12 \times 10^6 = 1.2 \times 10^7 \\ \left(3.1 \times 10^{-2}\right)\left(6.2 \times 10^7\right) &= 19.22 \times 10^5 = 1.922 \times 10^6 \\ \frac{6.3 \times 10^{-3}}{3 \times 10^{-7}} &= 210 \times 10^2 = 2.1 \times 10^4 \\ \frac{2.2 \times 10^{-5}}{(8.8 \times 10^{-3})(25 \times 10^{-1})} &= 0.01 \times 10^{-1} = 1 \times 10^{-3} \end{aligned}$$

Prefixes

- Some of the most frequently used prefixes for SI units for the various powers of ten and their symbols are listed in Table 2; just memorize those which are unfamiliar to you. These prefixes are especially useful when dealing with very large and very small numbers and when we change a unit to another one.

Table 2: Prefixes that will be used frequently throughout the course.

Power	Prefix	Symbol	Power	Prefix	Symbol
10^{-15}	femto	f	10^{-2}	centi	c
10^{-12}	pico	p	10^{3}	kilo	k
10^{-9}	nano	n	10^{6}	mega	M
10^{-6}	micro	μ	10^{9}	giga	G
10^{-3}	milli	m	10^{12}	tera	T

- Each prefix in Table 2 stands for a certain power of 10. When we attach a prefix to a unit, it behaves as a multiplication factor, mulyiplying that unit with the corresponding factor, as in

$$2.34 \text{ km} \equiv 2.34 \text{ kilometers} = 2.34 \times 10^3 \text{ m}$$

and[7]

$$6.41 \text{ ms} \equiv 6.41 \text{ milliseconds} = 6.41 \times 10^{-3} \text{ s}.$$

[7] Here an important point is in order: in these lecture notes and in your main textbook, all mathematical symbols are written in *slanted* letters and all numbers and units in plane letters, just like $F = 4.56$ N. Thus, when we write the force on an object in terms of the gravitational force as 2.23 mg, this is not to be confused with the mass of an object written as 2.23 mg.

Changing Units

- Armed with scientific notation and prefixes, we are now ready to convert a given unit into any other unit via the method of **chain-link conversion**. As an example, we know that 1 day amounts to 24 hours, so that

$$1 \text{ day} = 24 \text{ hours}.$$

  This means we can legitimately write

$$\frac{1 \text{ day}}{24 \text{ hours}} = 1 \quad \text{or} \quad \frac{24 \text{ hours}}{1 \text{ day}} = 1$$

  Each of these ratios are called a **conversion factor**[8]. Since they are unity, we can insert a conversion factor wherever we like in an equation. But the rule of thumb nonetheless is that try to use them so as to cancel unwanted units.

Examples

1. How many seconds are in one year?

   SOLUTION:

$$1 \text{ year} = 1 \cancel{\text{year}} \times \frac{365 \cancel{\text{days}}}{1 \cancel{\text{year}}} \times \frac{24 \cancel{\text{h}}}{1 \cancel{\text{day}}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{h}}} \times \frac{60 \text{ s}}{1 \cancel{\text{min}}} = 3.1536 \times 10^7 \text{ s}.$$

2. Perform the following conversion

$$a = 90.36 \, \frac{\text{km/h}}{\text{s}} = ? \text{ m/s}^2.$$

   SOLUTION:

$$\begin{aligned} a &= 90.36 \, \frac{\text{km/h}}{\text{s}} = 90.36 \, \frac{\cancel{\text{km}}}{\cancel{\text{h}} \cdot \text{s}} \times \frac{1 \times 10^3 \text{ m}}{1 \cancel{\text{km}}} \times \frac{1 \cancel{\text{h}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ s}} \\ &= 25.1 \text{ m/s}^2. \end{aligned}$$

[8] It is to be noted that $\frac{1 \text{ day}}{24 \text{ hours}} = 1$ does *not* imply that $\frac{1}{24} = 1$ or $1 = 24$. You *cannot* separate the number given for a quantity from its unit; they are meaningful *only* when they are together.

3. The density of aluminum, near room temperature, is $2.70\ \text{g}\cdot\text{cm}^{-3}$. Express this density in units of $\text{kg}\cdot\text{m}^{-3}$.

   SOLUTION:

$$
\begin{aligned}
2.70\ \text{g}\cdot\text{cm}^{-3} &= 2.70\ \frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times \frac{1\ \text{kg}}{1\times 10^3\ \cancel{\text{g}}} \times \left(\frac{1\ \cancel{\text{cm}}}{1\times 10^{-2}\ \text{m}}\right)^3 \\
&= 2700\ \text{kg}/\text{m}^3 = 2700\ \text{kg}\cdot\text{m}^{-3}.
\end{aligned}
$$

- As the examples above clearly show, units in conversions must obey the ordinary algebraic rules. It might also be useful to note in the last example that cm^3 means $(\text{cm})^3$, just like $25^3 = (25)^3$. You will frequently encounter the same case, so you just have to get used to it.

Significant Figures

- In any measurement process there are many factors that affect the measured values. Among these factors are the accuracy and quality of the experimental apparatus, the ability and experience of the experimenter, the temperature and humidity of the laboratory, etc. Put in another way, all the measured values are subjected to unawoidable *experimental uncertainties.*

- Suppose that in PHYS 101 laboratory you are to measure the dimensions of a rectangular prism and that the meter stick you use is an ordinary one, with an accuracy of ± 1 mm $= \pm 0.1$ cm. This means that the meter stick limits the measured values: when you measure the length of the prism as, say, 21.3 cm, you will be aware of the fact that the actual value is between 21.2 cm and 21.4 cm. You *cannot* say anything more.

  Let's suppose that you measured the length, width, and height of the rectangular prism as

$$\ell = 21.3\ \text{cm} \pm 0.1\ \text{cm},\ w = 18.5\ \text{cm} \pm 0.1\ \text{cm},\ \text{and}\ h = 9.7\ \text{cm} \pm 0.1\ \text{cm},$$

  where the former two values have *three* and the latter has *two* significant figures. We can then say

the first estimated digit, which imparts the experimental uncertainty, is contained in significant figures.

Now let's calculate the volume of this prism using a pocket calculator; it would give

$$V = \ell wh = (21.3\text{ cm})(18.5\text{ cm})(9.7\text{ cm}) = 3822.285\text{ cm}^3.$$

But there is a problem here: this result, with its seeming seven significant figures, is more accurate than the given data, and this is of course absolutely nonsense.

- To overcome this nonsenseness, we state the following important rule about significant figures:

> Consider a multiplication or division process involving two or more data. The number of significant figures in the final result must be the same as the *least* number of significant figures in the given data (which corresponds to the *least accurate* of the given quantities).

Then, we should round[9] the result for the volume of the above prism to *two* significant figures. To repeat,

$$V = (21.5\text{ cm})(18.5\text{ cm})(9.7\text{ cm}) = 3822.285\text{ cm}^3 \approx 3800\text{ cm}^3.$$

Note that we can still legitimately claim that the volume can range roughly between

$$(21.2\text{ cm})(18.4\text{ cm})(9.6\text{ cm}) \approx 3700\text{ cm}^3$$

and

$$(21.4\text{ cm})(18.6\text{ cm})(9.8\text{ cm}) \approx 3900\text{ cm}^3.$$

[9] Do you remember from your high school days how to round a number? Suppose that you are given a number 8.5014936 which is to be rounded. Notice in the followings how the bold digits are treated:

$$\begin{aligned}
8.5014936 &= 8.501493|\mathbf{6} \approx 8.501494\\
8.5014936 &= 8.50149|\mathbf{3}6 \approx 8.50149\\
8.5014936 &= 8.5014|\mathbf{9}36 \approx 8.5015\\
8.5014936 &= 8.501|\mathbf{4}936 \approx 8.501\\
8.5014936 &= 8.50|\mathbf{1}4936 \approx 8.50\\
8.5014936 &= 8.5|\mathbf{0}14936 \approx 8.5\\
8.5014936 &= 8.|\mathbf{501}4936 \approx 9
\end{aligned}$$

Pay attention especially to the last line.

- Sometimes we encounter numbers in which the existence of zeros causes ambiguity in the number of the significant figures. When we are told that the measured distance between two points is $d = 4500$ m, we cannot determine how many significant figures this number has: it might be two, three, or four, all possible. The same situation is seen in a quantity like $t = 0.0034$ s, because it might be 0.00340 s or 0.003400 s; which one is the proper one?[10]

  To avoid such difficulties, you are recommended to always use scientific notation in giving the answer of a problem or the measured value of an experiment. With this in mind, we can write, for the examples in the preceding paragraph, a two-significant figure distance value as $d = 4.5\times10^2$ m and a four-significant figure time value as $t = 3.400\times10^{-2}$ s. Now we can draw the following conclusion:

  > Except front-zeros that locate the decimal point, we count any reliably known digit as a **significant figure**.

- When adding and subtracting quantities, we take the number of decimal places into account[11]:

  > In adding and subtracting quantities, the number of decimal places of the final result and the *smallest* number of decimal places of any term in the process should match.

  Examples are

$$\begin{aligned} 35 + 2.3 &= 37 \quad (\textit{not}\ \ 37.3) \\ 2 + 0.25 &= 2 \quad (\textit{not}\ \ 2.25) \\ 0.0008 + 2.7002 &= 2.7010 \quad (\textit{not}\ \ 2.701) \end{aligned}$$

[10]Note that the two zeros between the decimal point and the digit 3 in these three numbers *cannot* be regarded as significant figures; they are there merely to locate the decimal point.

[11]*Significant figures* and *decimal places* are not to be confused. (Re-read the footnote 6). Suppose that we are given the mass of an object as 5.6 μg. This quantity can also be given as 0.0056 mg or 0.0000056 g. These numbers respectively have one, four, seven decimal places, although they all have two significant numbers.

- Now let's facilitate things a bit. Here is the final points about significant figures and decimal places:

> Throughout the lecture notes at your hand and the two introductory physics courses, We will always assume that all the given quantities accurate enough to result in answers with *three* significant figures.
>
> We will always keep at least *five* decimal places in all mid-results and will use them in the following procedures. Only at the very end will we give the main answer with *three* significant figures.
>
> We will always take for granted the well-known constants and conversion factors with *three* significant figures.

Accordingly, you shall understand a distance of 10 m as 10.0 m or a force of 2 N as 2.00 N.

Likewise, you will frequently see the use of the following constants and conversion factors (with three significant numbers):

$$1 \text{ in.} = 2.54 \text{ cm} \qquad \text{(in.: inch)}$$
$$1 \text{ ft} = 30.5 \text{ cm} \qquad \text{(ft: foot)}$$
$$1 \text{ mi} = 1610 \text{ m} \qquad \text{(mi: mile)}$$

$$c = 3.00 \times 10^{8} \text{ m/s} \qquad \text{(speed of light)}$$
$$g = 9.80 \text{ kg} \cdot \text{m/s}^{2} \qquad \text{(gravitational acceleration)}$$
$$N_{\mathrm{A}} = 6.02 \times 10^{23} \text{ mol}^{-1} \qquad \text{(Avogadro number)}$$
$$e = 1.60 \times 10^{-19} \text{ C} \qquad \text{(elementary charge)}$$
$$m_{\mathrm{e}} = 9.11 \times 10^{-31} \text{ kg} \qquad \text{(electron mass)}$$
$$m_{\mathrm{p}} = 1.67 \times 10^{-27} \text{ kg} \qquad \text{(proton mass)}$$
$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ N} \cdot \text{m}^2/\text{C}^2 \qquad \text{(permittivity constant)}$$

Order-of-Magnitude Calculations

- When a number describing a quantity is written in scientific notation, the power of ten is referred to as its **order of magnitude**. For example, the orders of magnitude of quantities

$$m_1 = 6.2 \times 10^2 \text{ kg} \quad \text{and} \quad m_2 = 3.7 \times 10^4 \text{ kg}$$

  are 2 and 4, respectively.

- When the value of a quantity increases by, say, two orders of magnitude, we say that that value increases by a factor of 100, since $10^2 = 100$.

- We sometimes want to obtain an estimated answer, *not* an exact one, to a problem, maybe because we do not have enough information. Or, the calculation at hand might be a preliminary one, according to whose result we would proceed for a more precise answer or would not. In such cases, we usually perform simple and approximate calculations based on certain assumptions, and try to get the desired answer to the ***nearest*** **order of magnitude**[12]. As this self-explanatory expression suggests, the nearest orders of magnitude of the masses m_1 and m_2 mentioned above are 3 and 4, respectively.

- It is now clear that a result obtained from an order-of-magnitude calculation is reliable within a factor of 10.

Example

There is a 1070-page physics book on my desk, which is said to be published on acid-free paper. I want to learn the thickness of one single sheet of this book. I don't have any handy micrometer to measure such a small thickness. What available is only a ruler. When I measure the thickness of this 535-sheet book, I see it is about 3.2 cm. Then, the thickness of one single sheet, to the nearest order of magnitude, is

$$\text{thickness} = \frac{3.3 \times 10^{-2} \text{ m}}{535 \text{ sheet}} \approx 6 \times 10^{-5} \text{ m/sheet} = 10^{-4} \text{ m/sheet}.$$

Although this is a crude result, this estimate isn't bad at all, as can be easily checked from the web. (A much more accurate way to measure the thickness of a paper is to make use of an "air-wedge" formed between two thick glass plates which are in contact along one end and are separated along the other end by the sheet of paper. For further details, just google the phrase "air-wedge interference" in the web.)

[12]Sometimes referred to as "guesstimates" or "ball-park figures."

- I strongly recommend that the serious engineering student get into the habit of "Making an estimate before every calculation, trying a physical argument ... before every derivation, quessing the answer to every puzzle. Courage: no one else needs to know what the guess is."[13]

Density

- Now it's time to learn our first relation in physics. We define the **density** ρ of a substance as its mass per unit volume:

$$\boxed{\rho = \frac{m}{V} \qquad \left(\mathrm{kg/m^3}\right)}$$

- Although the SI unit of density is $\mathrm{kg/m^3}$, especially in chemistry books the unit $\mathrm{g/cm^3}$ is frequently employed.

- The density of water is worth knowing by heart:

$$\rho_{\text{water}} = 1.00 \ \mathrm{g/cm^3}.$$

  Since this is a "human scale" density, in order to "sense" the densities of other subtances, we compare them with that of water. Platinum is, for example, 21 times denser than water.

Key Words, Phrases, and Equations

- Unit
- Standard
- Base quantities, base units, base standards
- The metric (SI) system
- Length, mass, time, 2D plane angle
- Meter (m), kilogram (kg), second (s), radian (rad)
- Derived units

[13]E. Taylor and J.A. Wheeler, *Spacetime Physics*, San Fransisco, W.H. Freeman, 1966, p.60; quoted in *Physics for Scientists & Engineers with Modern Physics*, 3rd Edition, by R.A. Serway, Saunders College Publishing, 1990, p. 12.

- Speed of light (exactly):

$$c = 299\ 792\ 458 \text{ m/s}$$

- Atomic mass unit (amu):

$$1 \text{ amu} = 1.660\ 538\ 86 \times 10^{-27} \text{ kg}$$

- Scientific notation, exponent (of 10)
- Decimal (floating) numbers, digit, decimal point, decimal places
- The SI prefixes
- Chain-link conversion, conversion factor
- Significant figures
- Rounding a number
- Order of magnitude, nearest order of magnitude, order-of-magnitude calculations
- Density:

$$\rho = \frac{m}{V} \qquad \left(\text{kg/m}^3\right)$$