CHAPTER 2 | MOTION ALONG A STRAIGHT LINE

Introduction

- The branch of physics describing the motion of bodies *without* considering the causes of the motion is called **kinematics**[1].
- In this chapter we study only **one-dimensional (1D) motion**; that is, we consider the motion of bodies along a *straight* line. This line might be horizontal, vertical, or even slanted, but it will always be straight throughout this chapter.
- Later on we discuss two– and three–dimensional motions, which all depend on a solid understanding of the basic concepts of 1D motion; they are *displacement*, *velocity*, and *acceleration*.
- The objects in this chapter will be considered as **particles**. A particle might be *point-like* such as a small bead, or it might be a *rigid* body, like a car, whose every point moves in the same direction at the same rate; that is, it moves like a particle.

[1]This term comes from the Greek word *kinema*, meaning "motion" or "movement."

Position

- Consider an object which undergoes a 1D motion, along, say, the x-axis shown in Fig. 1. The **position** of the object is given by its x-coordinate with respect to the **origin**. Then, if the object is said to be located at $x = 2$ m, we understand it is 2 m in the *positive direction* from the origin. It it is at $x = -3$ m, this time it is 3 m in the *negative direction* from the origin.

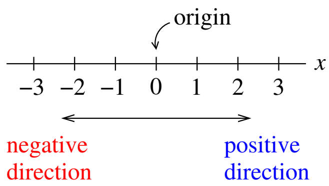

Figure 1: An x-axis for the position of an object.

Displacement

- The change in the position of the object is called its **displacement** Δx, which is given by

$$\Delta x \equiv x_2 - x_1 \qquad \text{(m)}$$

  where x_1 the object's position at time t_1, and x_2 at a *later* time t_2.

- The displacement of an object may be negative, positive, or zero. Put in another way, displacement is actually a **vector quantity**, which has both *magnitude* and *direction*. For 1D motion, we do not, however, need to use the vector properties of displacement; plus (+) and (−) signs will be sufficient.

Average Velocity and Average Speed

- We will frequently show the time-dependent position $x(t)$ of an object as a position versus time graph, as shown in Fig. 2.

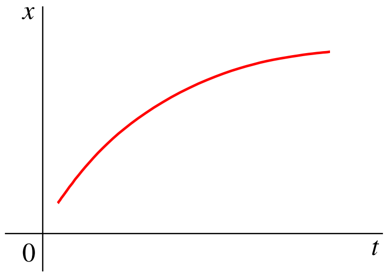

Figure 2: The time-dependent position $x(t)$ of a moving object.

- The **average velocity** v_{avg} gives us a rough estimate of "how fast" an object undergoes a displacement over a definite time interval, which is defined as

$$v_{\text{avg}} \equiv \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad \text{(m/s)}$$

- As Fig. 3 also shows, *average velocity amounts to the slope of the line passing through points* (t_1, x_1) *and* (t_2, x_2). As a result, we can extend the preceding equation as

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \tan\theta$$

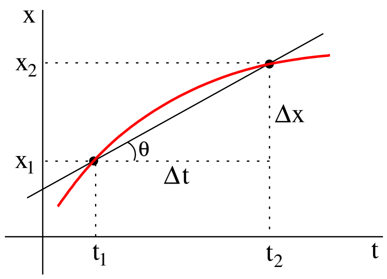

Figure 3: Average velocity & slope.

When this slope is positive, we say the object has moved to the right; when it is negative, the object has moved to the left. When the slope is zero, the object has not moved at all.

- Just like displacement, average velocity is also a vector quantity since it has both magnitude and direction. However, in 1D motion, we will use plus (+) or minus (−) signs to indicate its direction.

- **Average speed** s_{avg} is defined as the ratio of the *total distance* covered by an object to the time interval Δt:

$$s_{\text{avg}} \equiv \frac{\text{total distance}}{\Delta t} \qquad \text{(m/s)}$$

- Since "total distance" is always a positive quantity, so is the average speed. In other words, s_{avg} does *not* have direction so that it is *not* a vector quantity.

Instantaneous Velocity and Speed

- The (**instantaneous**) **velocity** $v(t)$ of an object gives us the information about how fast it is at any given instant t. It is the limiting value of the average velocity as Δt goes to zero at that instant:

$$v(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \text{(m/s)}$$

  Therefore, $v(t)$ is the time-derivative of position $x(t)$; i.e., $v(t)$ is the *rate*[2] at which position $x(t)$ changes with time at any given instant.

- It is very important to note that $v(t)$ *equals the slope of* x-t *curve at any given instant*, as shown in Fig. 4. If this slope is positive at any instant, we say that it *is* moving to the right at that instant; if it is negative, the object *is* moving to the left. When the slope is zero, the object *is* not moving at that instant.

- The magnitude (or absolute value) of velocity is called **speed**. This means that speed is *always* positive. What your car's speedometer measures is its speed, *not* its velocity. Can you explain why?

- As it should be obvious now that velocity is another vector quantity, with its magnitude and direction.

[2] You will frequently encounter this word; it will usually mean the change in a given quantity with time.

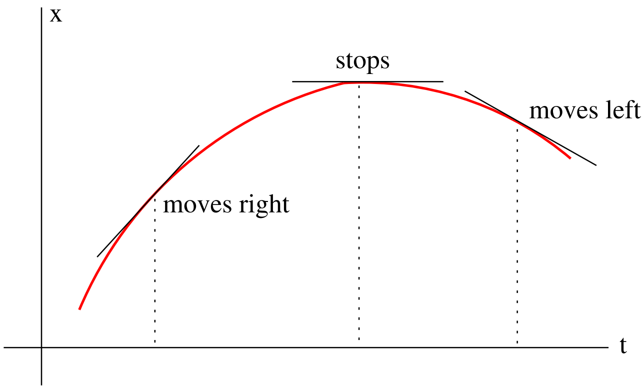

Figure 4: $v(t)$ equals the slope of x-t curve at any given instant.

Acceleration

- If the velocity of an object changes with time, we say that the object *accelerates*, and the rate at which its velocity changes is called its **acceleration**.

  Let the object have velocity v_1 at time t_1 and velocity v_2 at a *later* time t_2. Its **average acceleration** a_{avg} is defined by

$$a_{\text{avg}} \equiv \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \qquad (\text{m/s}^2)$$

- As we did in defining instantaneous velocity, we define the (**instantaneous**) **acceleration** of an object at any given time as

$$a(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (\text{m/s}^2)$$

  The acceleration $a(t)$ of an object gives the information about how fast its velocity is changing at any given instant t; in other words, the slope of $v(t)$ at any point gives the acceleration of the object at that point.

- A closer look at its definition above, acceleration is the *second derivative of displacement*:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- Although we shall usually use $\mathrm{m/s^2}$ for the SI unit of acceleration, a more natural looking unit is $\frac{\mathrm{m/s}}{\mathrm{s}} \equiv \mathrm{m/s/s}$, which are both to be read as "*meters per second* (pause a bit) *per second*." This notation for the unit of acceleration conveys its meaning clearly[3].

- Since acceleration is such a quantity that has both magnitude and direction, acceleration is also yet another vector quantity, just like displacement and velocity.

- In Fig. 5 we show successively the curves of the acceleration, velocity, and displacement of an object, as an example. You should note how the velocity curve is obtained from the displacement curve, and the acceleration curve from the velocity curve.

- Sometimes in some texts, including our textbook, the word "deceleration" is used when an object is slowing down, i.e., when its acceleration is negative. The student is recommended not to use this word, you had better employ simply positive or negative acceleration.

- It is important to notice that a *negative acceleration does not necessarily convey the meaning "slowing down."* If the initial velocity and acceleration of an object are both negative, the object *speeds up* towards the negative direction. We can draw a more general conclusion: Whenever acceleration and velocity have the same sign, the object under question speeds up for all time, and whenever they have opposite signs, the object first *slows down*, then stops momentarily, and speeds up towards the opposite direction.

- Sometimes we will express accelarations with large values in terms of the so-called ***g*-units**:

  $$\boxed{1\ g \equiv 9.80\ \mathrm{m/s^2}}$$

  The meaning of this constant will be clear shortly.

[3]Do not be surprised if you happen to see a unit like $\frac{\mathrm{mi/h}}{\mathrm{s}} \equiv \mathrm{mi/h/s}$ for the unit of acceleration. What matters is that the unit of acceleration be in the form of length/time/time.

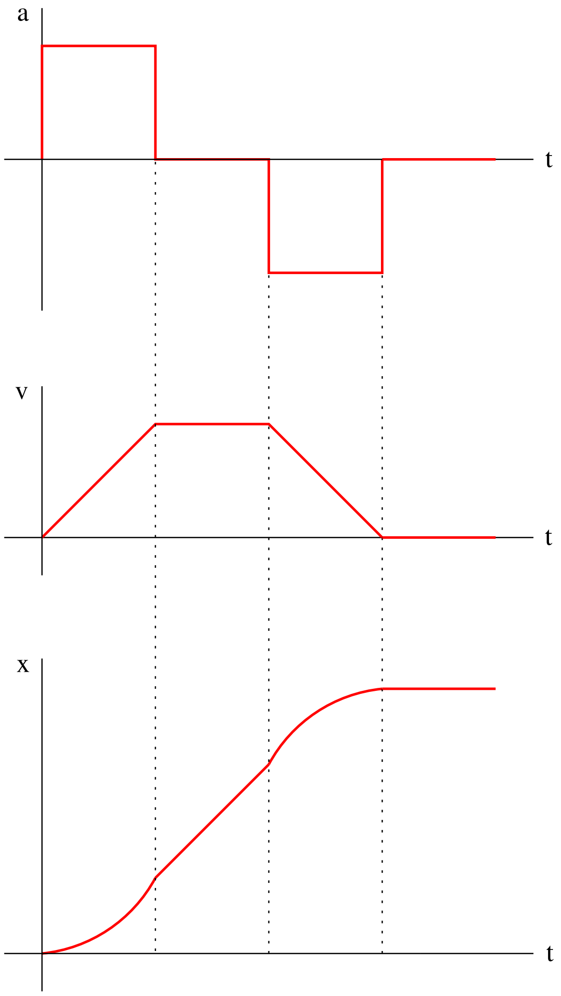

Figure 5: Successive graphs of acceleration, velocity, and displacement for an object.

1D Motion with Constant Acceleration

- In many daily-life problems, the object under question moves with constant acceleration (or, its acceleration can be assumed to be approximately constant). Figure 6 below shows successively the curves of $a(t)$, $v(t)$, and $x(t)$ for an object moving with (positive) constant acceleration.

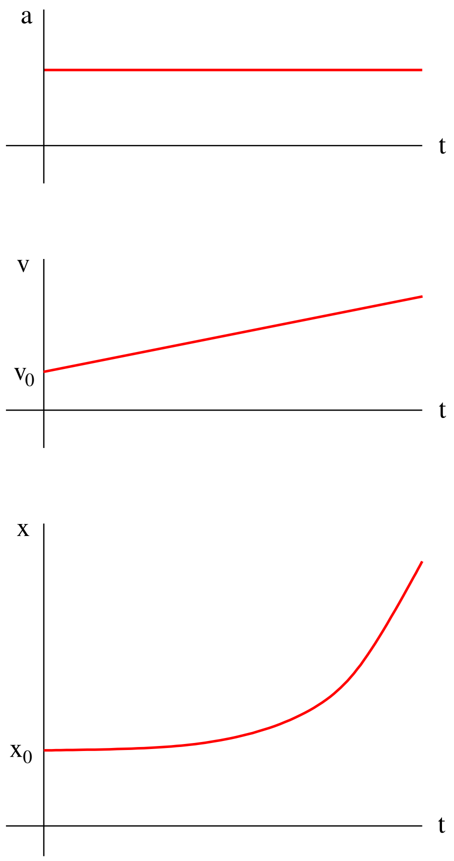

Figure 6: Representative graphs of $a(t)$, $v(t)$, and $x(t)$ for an object moving with constant acceleration.

In the following, we will derive some formulas that can be employed in solving 1D problems involving constant acceleration. The serious student should be aware from the outset of the fact thet *the formulas we will obtain shortly are valid only for constant acceleration*; if acceleration is time-varying, you *cannot* use them. Please pay heed to this advice in order not to harm your engineering carreer from the very beginning!

- Let's first find the expression for velocity $v(t)$ via integrating acceleration $a(t)$:

$$a = \frac{dv}{dt} = \text{const.}$$

then

$$\int_{v_0}^{v} dv = \int_0^t a\, dt \quad \Rightarrow \quad v\Big|_{v_0}^{v} = at\Big|_0^t \quad \Rightarrow \quad v - v_0 = a(t-0)$$

and

$$\boxed{v = v_0 + at}$$

where v_0 is the **initial velocity** at time $t = 0$ and $v = v(t)$ is the velocity at *any* later time t.

- The position $x(t)$ can be obtained by integrating the velocity obtained above, as

$$v = \frac{dx}{dt}$$

so that

$$\int_{x_0}^{x} dx = \int_0^t v\, dt = \int_0^t (at + v_0)\, dt$$

whence

$$x\Big|_{x_0}^{x} = \left(\tfrac{1}{2}at^2 + v_0 t\right)\Big|_0^t \quad \Rightarrow \quad x - x_0 = \tfrac{1}{2}a\left(t^2 - 0^2\right) + v_0(t-0)$$

and

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2}at^2}$$

where x_0 is the **initial position** at time $t = 0$ and $x = x(t)$ is the position at *any* later time t.

- Another very useful formula can be found by eliminating time t between the two boxed equations above. From the first one, we have

$$v = v_0 + at \quad \Rightarrow \quad at = v - v_0$$

Substituting this into the second equation, we get

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = \frac{v_0}{a}\,at + \frac{1}{2a}\,(at)^2$$

and

$$\begin{aligned}
2a\,(x - x_0) &= 2v_0 at + (at)^2 \\
&= (at)\,(2v_0 + at) \\
&= (v - v_0)\,(2v_0 + v - v_0) \\
&= (v - v_0)\,(v_0 + v) \\
&= -\,(v_0 - v)\,(v_0 + v) \\
&= -\left(v_0^2 - v^2\right) \\
&= v^2 - v_0^2
\end{aligned}$$

and finally

$$\boxed{v^2 = v_0^2 + 2a\,(x - x_0)}$$

- With the help of the three boxed equations above, you can solve any 1D motion problem with constant acceleration. Because of their importance, we will give them together:

$$\boxed{\begin{aligned}
a &= \text{const.} \\
v &= v_0 + at \\
x &= x_0 + v_0 t + \tfrac{1}{2}at^2 \\
v^2 &= v_0^2 + 2a\,(x - x_0)
\end{aligned}}$$

It is highly recommended that the serious student learn them by heart.

- Finally we note that the above equations are also valid for zero-acceleration, constant-velocity cases. We obtain the following straightforward equations:

$$\begin{aligned} a &= 0 \\ v &= v_0 \\ x &= x_0 + v_0 t \end{aligned}$$

Free-Fall Acceleration

- If an object is thrown up or down along a vertical axis near the surface of the earth, we see that it gravitates always towards the earth because of the downward gravitational force. Neglecting the effects of air, the object is seen to accelerate downward at a constant rate, which is called the **free-fall acceleration** (or, equivalently, **gravitational acceleration**). We will use the letter g for the magnitude of this acceleration.

- It is an empirical fact that the free-fall accelaration g of an object does *not* depend on its specific characteristics; that is to say, *all* objects undergo the very same downward acceleration.

- Although the value of g changes a little with altitude and latitude, its value near the surface of the earth is almost constant and given as

$$g = 9.80 \text{ m/s}^2$$

- Since the free-fall acceleration is constant, all of the formulas for constant-acceleration that we derived in the preceding section apply also to free-fall near the surface of the earth. For this we shall label the vertical axis the y-axis, with its *upward* positive direction. With this choice, we shall take the downward acceleration to be $a = -g$ in our formulas. As a result, we obtain the following formulas for free-fall:

$$\boxed{\begin{aligned} a &= -g \\ v &= v_0 - gt \\ y &= y_0 + v_0 t - \tfrac{1}{2} g t^2 \\ v^2 &= v_0^2 - 2g\,(y - y_0) \end{aligned}}$$

- The following consideration might be useful. If an object initially at rest is dropped off an elevation of h and hits the ground in time t, it is easy to show that

$$\boxed{h = \tfrac{1}{2} g t^2}$$

- Finally, in all the problems and examples that will be presented in the video sections and that will be studied in the class, we shall always neglect the effects of air. Will this be always legitimate? My answer is "yes," provided that the objects under consideration do *not* go down or up too fast.

Key Words, Phrases, and Equations

- Displacement:

$$\Delta x = x_2 - x_1 \qquad \text{(m)}$$

- Average velocity:

$$v_{\text{avg}} = \tan\theta = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad \text{(m/s)}$$

- Average speed:

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t} \qquad \text{(m/s)}$$

- (Instantaneous) velocity:

$$v(t) = \frac{dx}{dt} \qquad \text{(m/s)}$$

- Average acceleration:

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} \qquad \left(\text{m/s}^2\right)$$

- (Instantaneous) acceleration:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- 1D motion with constant acceleration:

$$\begin{aligned} a &= \text{const.} \\ v &= v_0 + at \\ x &= x_0 + v_0 t + \tfrac{1}{2}at^2 \\ v^2 &= v_0^2 + 2a\,(x - x_0) \end{aligned}$$

- 1D motion with zero acceleration:

$$\begin{aligned} a &= 0 \\ v &= v_0 \quad \text{(const.)} \\ x &= x_0 + v_0 t \end{aligned}$$

- Free-fall acceleration:

$$\begin{aligned} g &= 9.80 \text{ m/s}^2 \\ a &= -g \\ v &= v_0 - gt \\ y &= y_0 + v_0 t - \tfrac{1}{2}gt^2 \\ v^2 &= v_0^2 - 2g\,(y - y_0) \end{aligned}$$