INDUSTRIAL AUTOMATION & ROBOTICS TECHNOLOGY

Robot Control

M Umer khan

Department of Mechatronics Engineering

ATILIM University

UMER.KHAN@ATILIM.EDU.TR

Contents

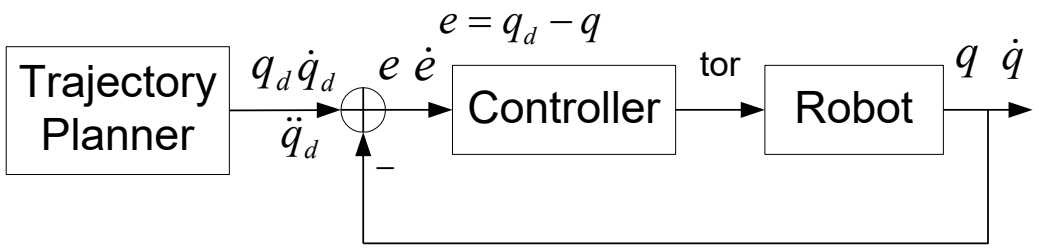
- Chapter 9, Dynamics and Control, Peter Corke

Robot Manipulator Control

- Robot System:

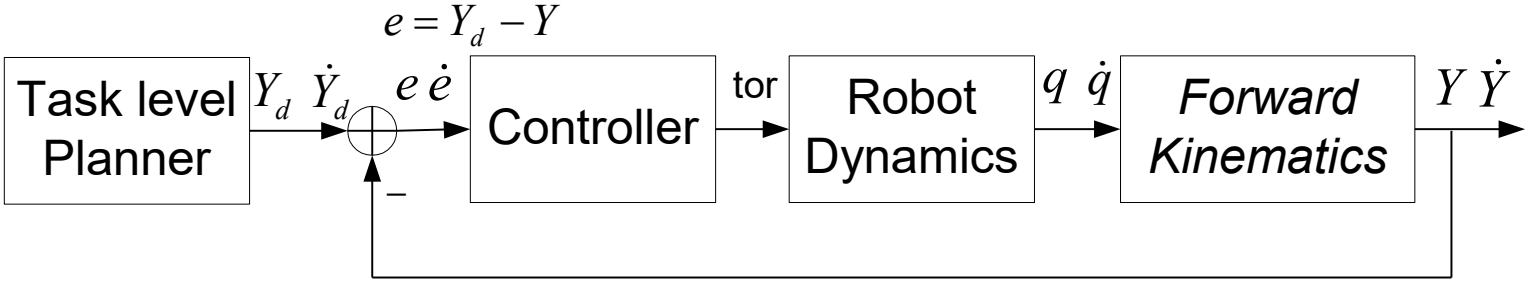
$$\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$$

- Joint Level Controller



Find a control input (tor), $q \to q_d$ as $t \to \infty$

- Task Level Controller



Find a control input (tor), $Y \to Y_d$ as $t \to \infty$ $e = Y_d - Y \to 0$

Robot Manipulator Control

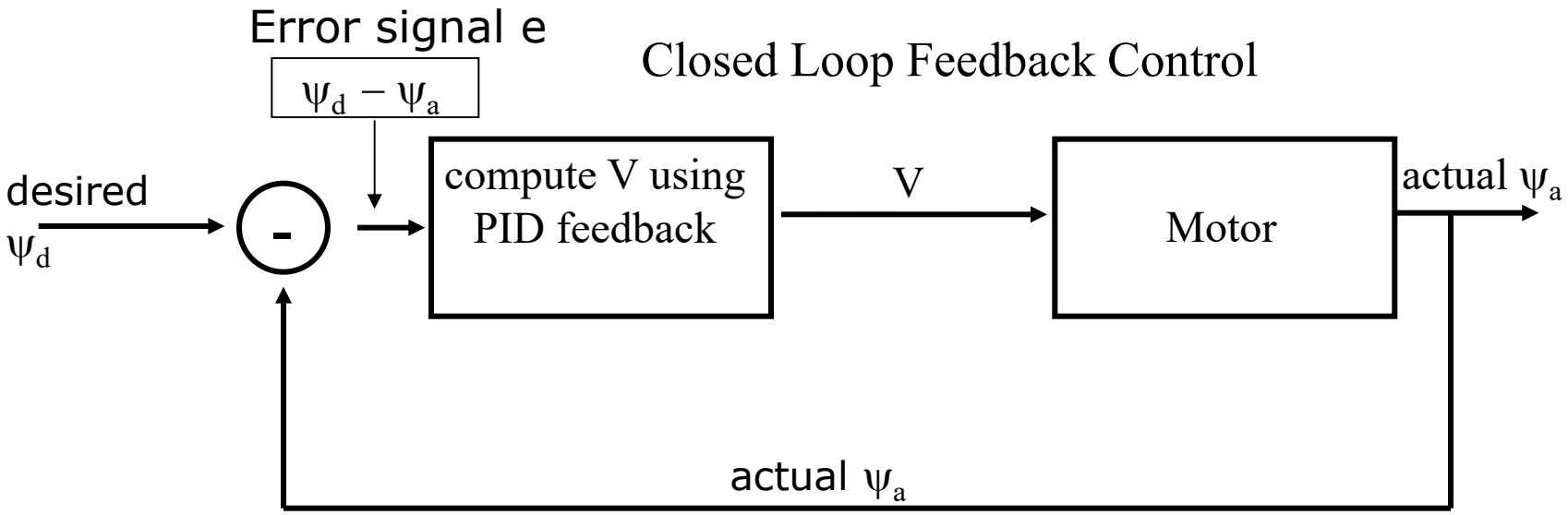
- Control Methods
  - Conventional Joint PID Control
    - Widely used in industry
  - Advanced Control Approaches
    - Computed torque approach
    - Nonlinear feedback
    - Adaptive control
    - Variable structure control
    - ….

Control Theory Review (I)

PID controller: Proportional / Integral / Derivative control

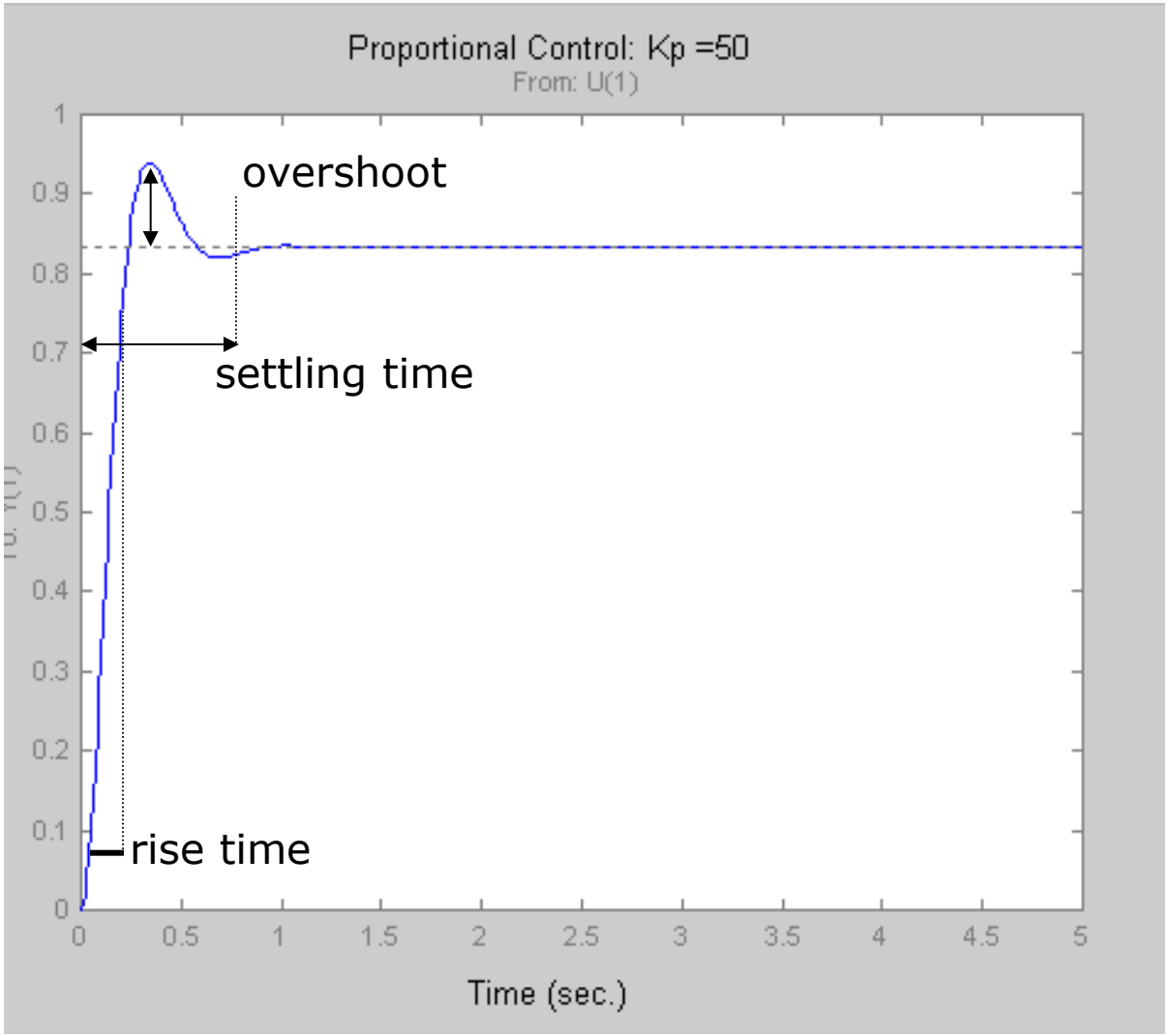
$$e = \psi_d - \psi_a$$

$$V = K_p \cdot e + K_i \int e\,dt + K_d \frac{de}{dt}$$



Reference book: Modern Control Engineering, Katsuhiko Ogata, ISBN0-13-060907-2

Evaluating the response



steady-state error

ss error -- difference from the system's desired value

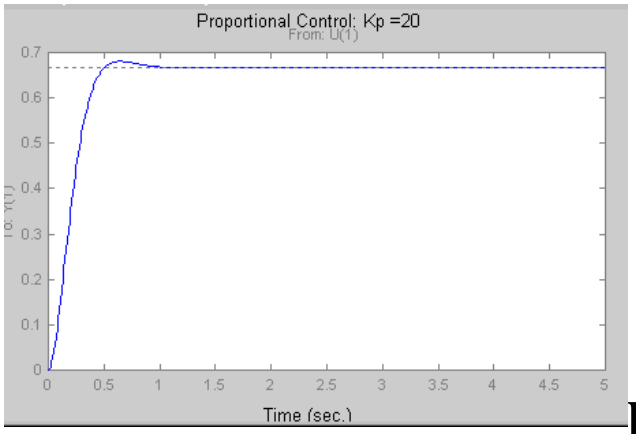
overshoot -- % of final value exceeded at first oscillation

rise time -- time to span from 10% to 90% of the final value

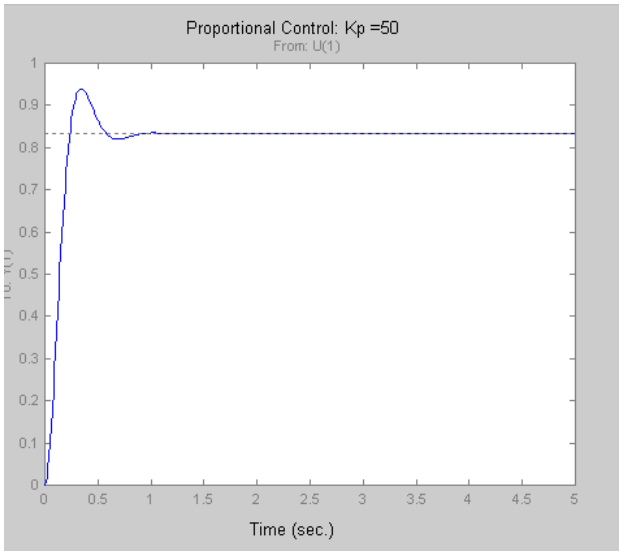
settling time -- time to reach within 2% of the final value

How can we eliminate the steady-state error?

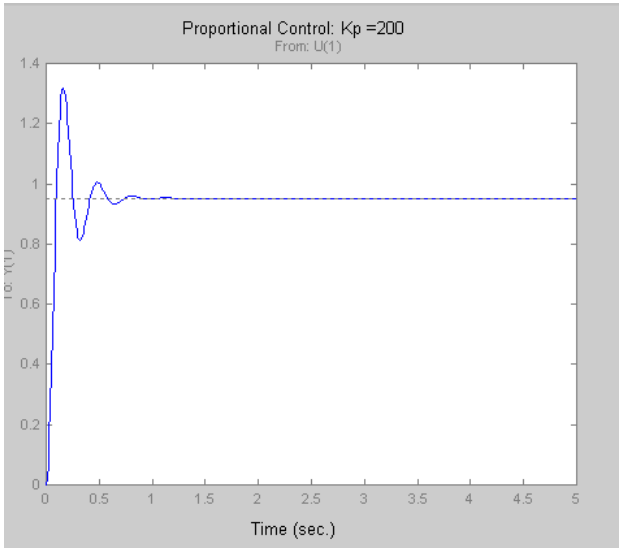
Control Performance, P-type



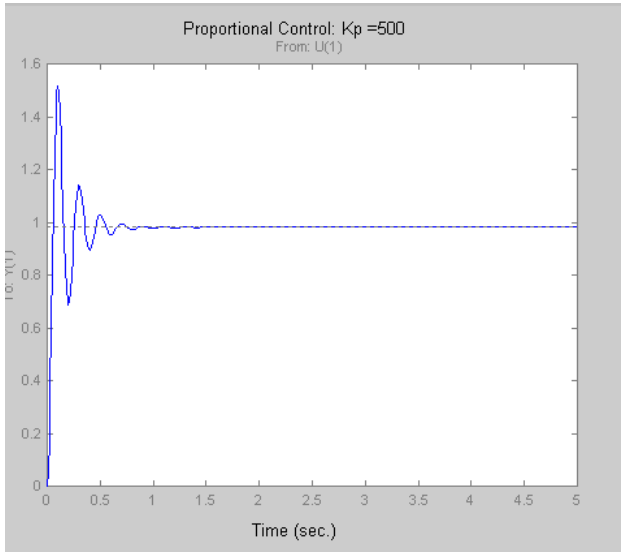
$K_p = 20$



$K_p = 50$



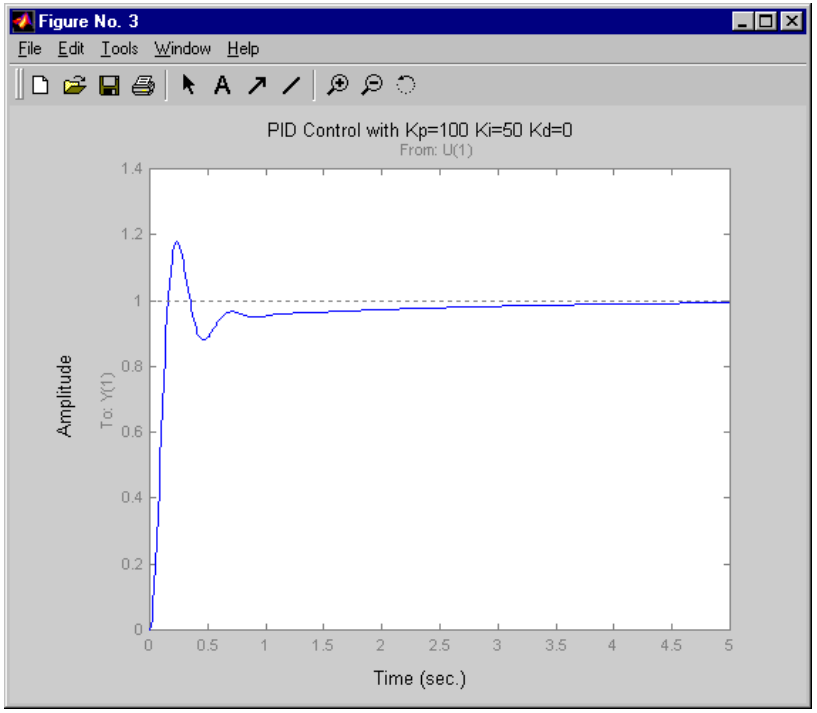
$K_p = 200$

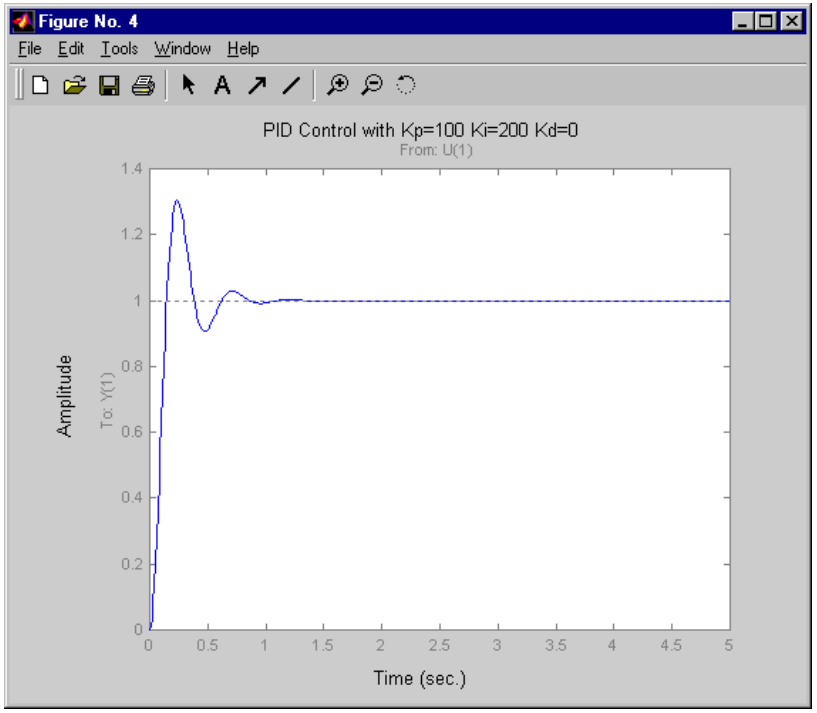


$K_p = 500$

Control Performance, PI - type

$K_p = 100$

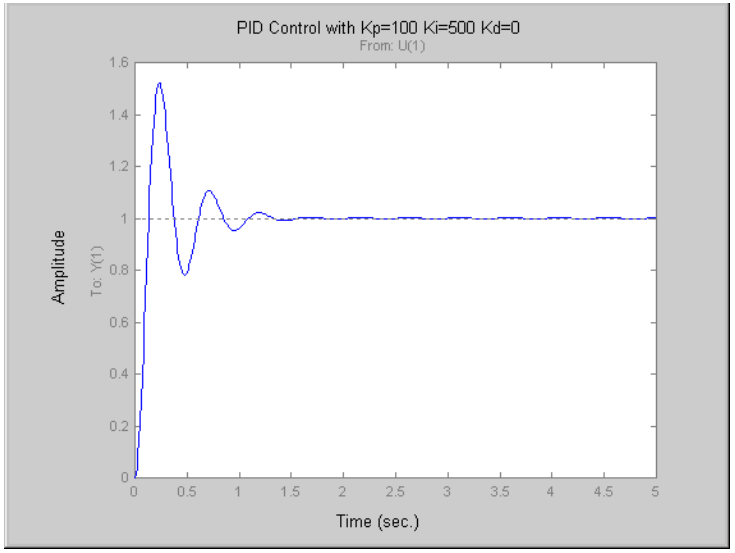


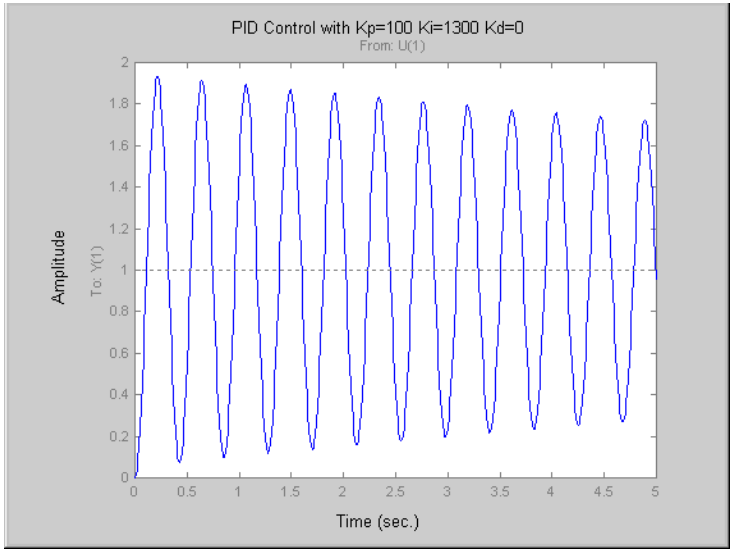


$K_i = 50$

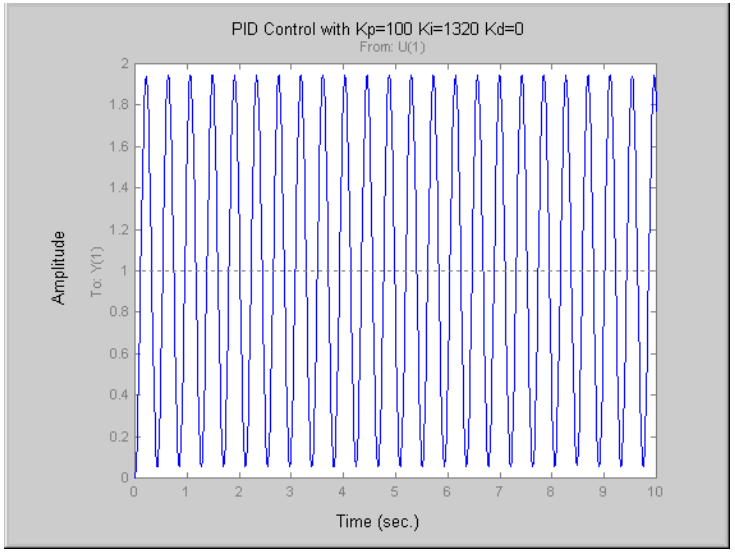
$K_i = 200$

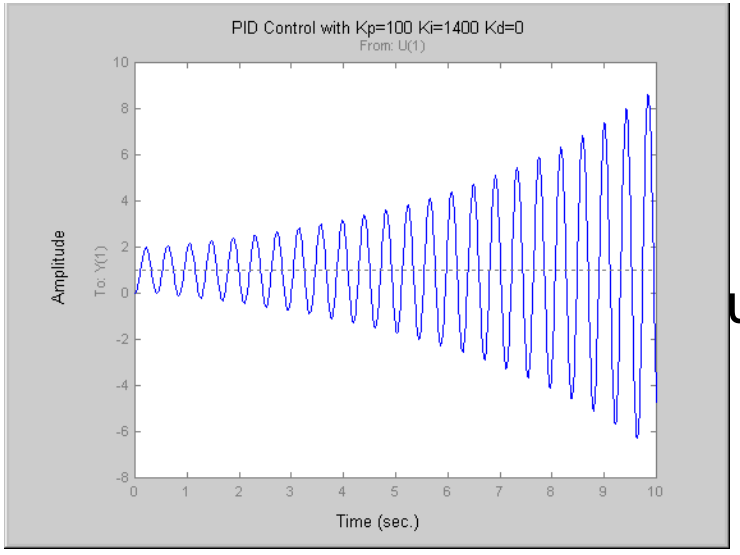
You've been integrated…





$K_p = 100$





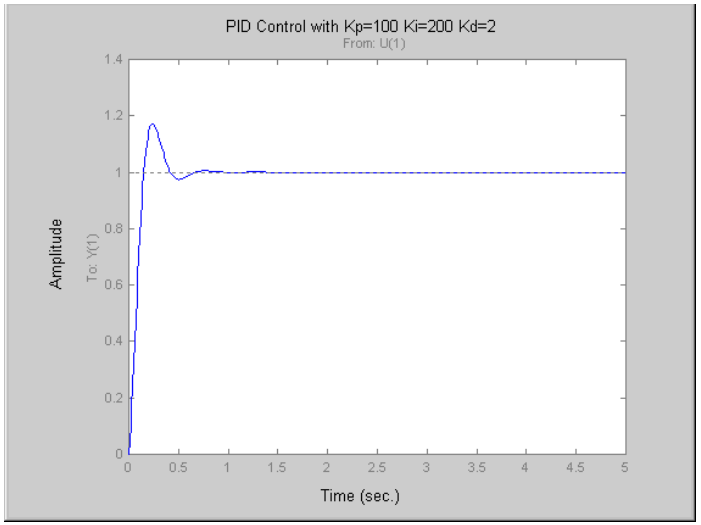
unstable & oscillation

Control Performance, PID-type

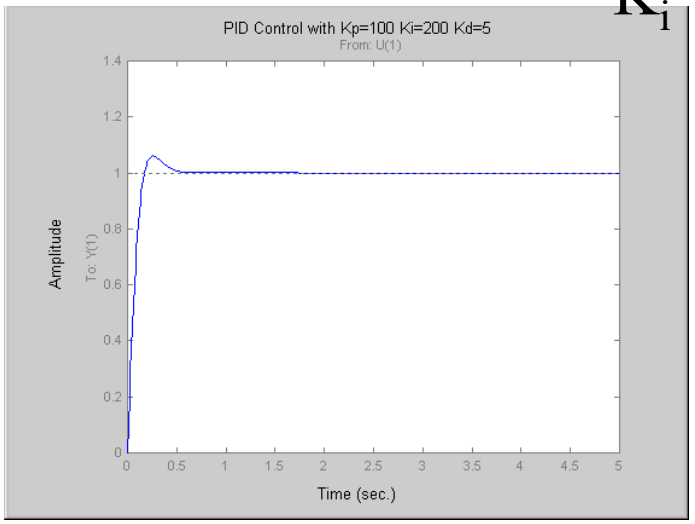
$K_p = 100$

$K_i = 200$

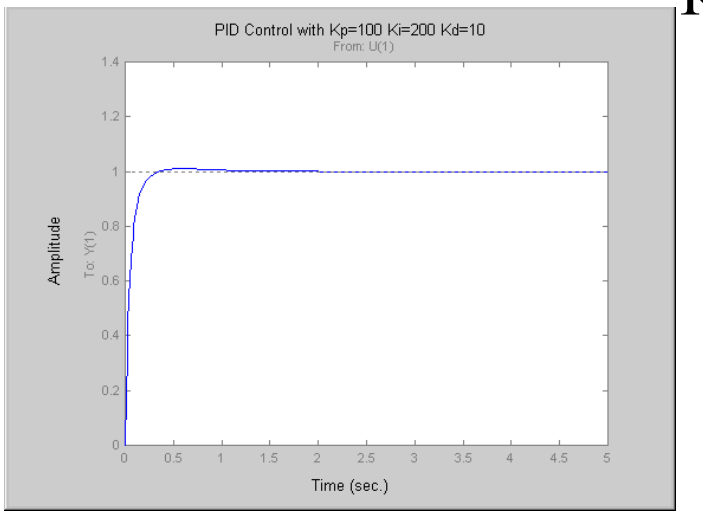
$K_d = 2$



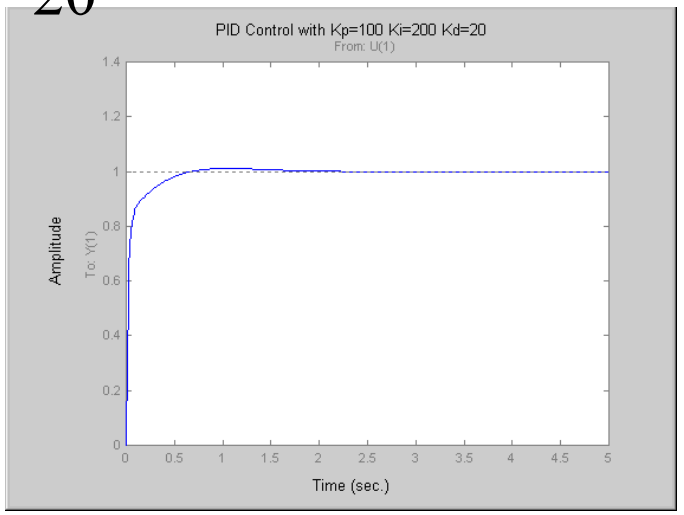
$K_d = 5$



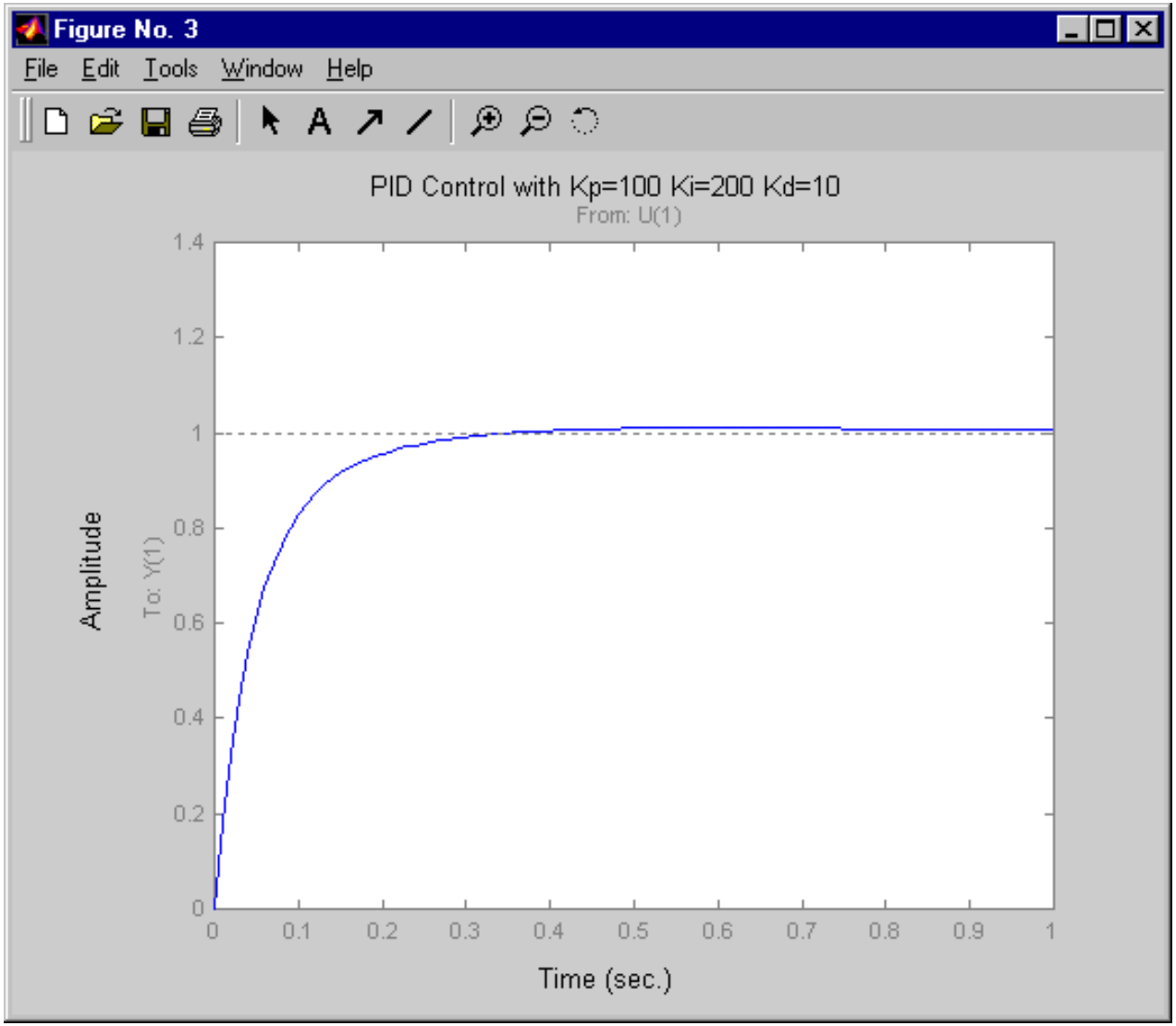
$K_d = 10$



$K_d = 20$



PID final control



Control Theory Review (II)

- Linear Control System
  - State space equation of a system

$$\dot{x} = Ax + Bu$$

  - Example: a system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

  - Eigenvalue of A are the root of characteristic equation

$$|\lambda I - A| = 0 \quad \Leftrightarrow \quad |\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} = \lambda^2 = 0$$

  - Asymptotically stable all eigenvalues of A have negative real part

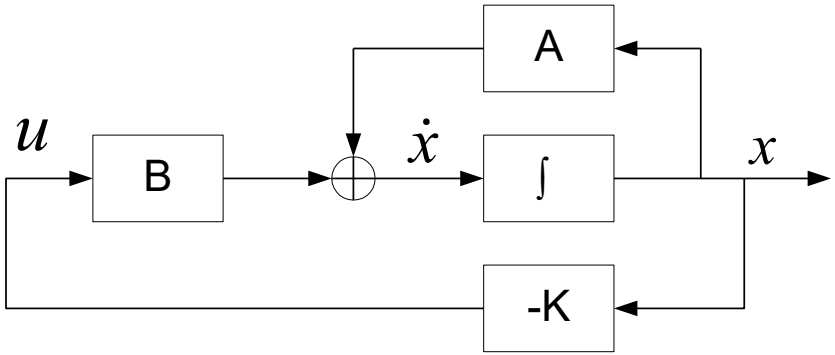
Control Theory Review (II)

- Find a state feedback control $u = -K \cdot x$
- such that the closed loop system is asymptotically stable

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- Closed loop system becomes

$$\dot{x} = (A - BK)x$$



- Chose K, such that all eigenvalues of A'=(A-BK) have negative real parts

$$\left|\lambda I - A'\right| = \begin{vmatrix} \lambda & -1 \\ k_1 & \lambda + k_2 \end{vmatrix} = \lambda^2 + k_2\lambda + k_1 = 0$$

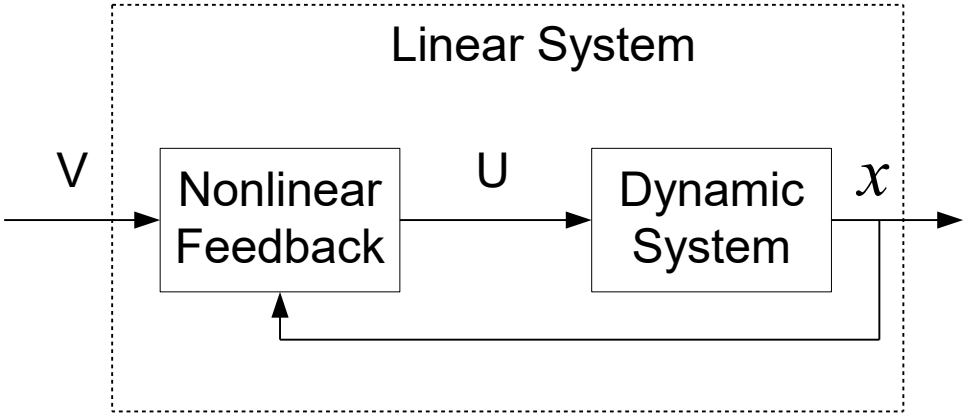
Control Theory Review (III)

- Feedback linearization
  - Nonlinear system $\dot{X} = f(x) + G(x)U$

$$U = [-G^{-1}(x) f(x) + G^{-1}(x) V]$$

$$\dot{X} = V$$

  - Example:



Original system:

$$\ddot{x} + \cos x = U$$

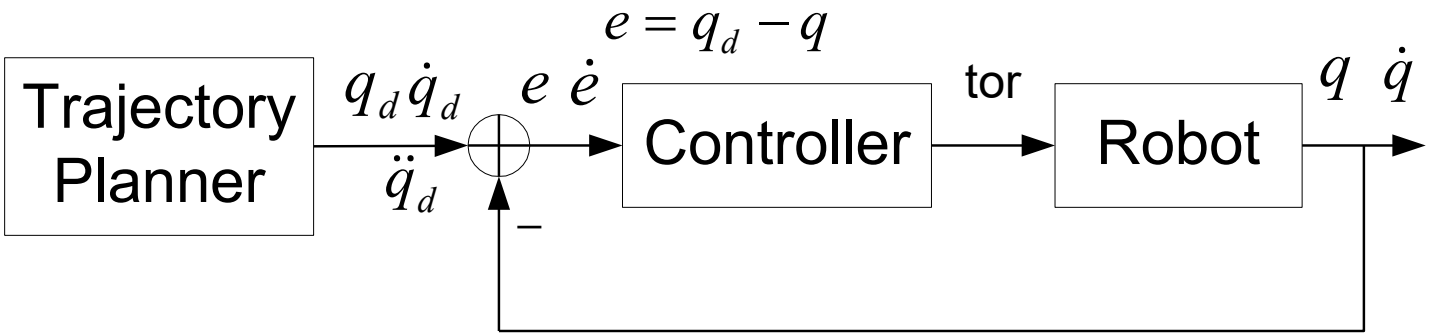
Nonlinear feedback:

$$U = \cos x + V$$

Linear system: $\ddot{x} = V$

Robot Motion Control (I)

- Joint level PID control
  - each joint is a servo-mechanism
  - adopted widely in industrial robot
  - neglect dynamic behavior of whole arm
  - degraded control performance especially in high speed
  - performance depends on configuration



Robot Motion Control (II)

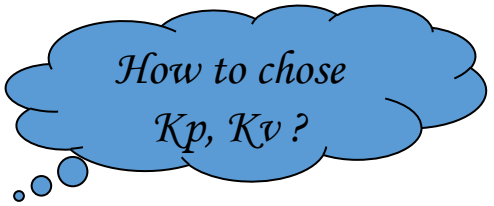
- Computed torque method
  - Robot system:

$$\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$$

  - Controller:

$$tor = D(q)[\ddot{q}^d + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q)] + H(q,\dot{q}) + C(q)$$

$$(\ddot{q}^d - \ddot{q}) + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q) = 0$$



Error dynamics $\quad \ddot{e} + k_v\dot{e} + k_p e = 0$

Advantage: compensated for the dynamic effects

Condition: robot dynamic model is known

Robot Motion Control (II)

How to chose Kp, Kv to make the system stable?

Error dynamics $\ddot{e} + k_v \dot{e} + k_p e = 0$

Define states:

$$x_1 = e \qquad \dot{x}_1 = x_2$$
$$x_2 = \dot{e} \qquad \dot{x}_2 = -k_v x_2 - k_p x_1$$

In matrix form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = AX$$

Characteristic equation:

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ k_p & \lambda + k_v \end{vmatrix} = \lambda^2 + k_v \lambda + k_p = 0$$

The eigenvalue of A matrix is:

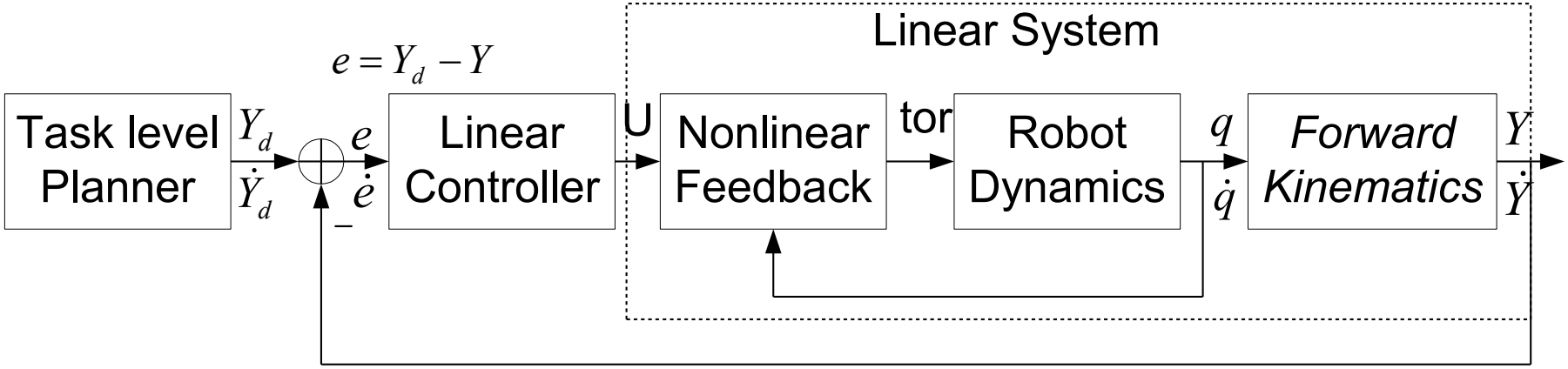
$$\lambda_{1,2} = \frac{-k_v \pm \sqrt{k_v^{\,2} - 4k_p}}{2}$$

Condition: λ have negative real part

One of a selections: $k_v > 0$, $k_p > 0$

Robot Motion Control (III)

- Non-linear Feedback Control



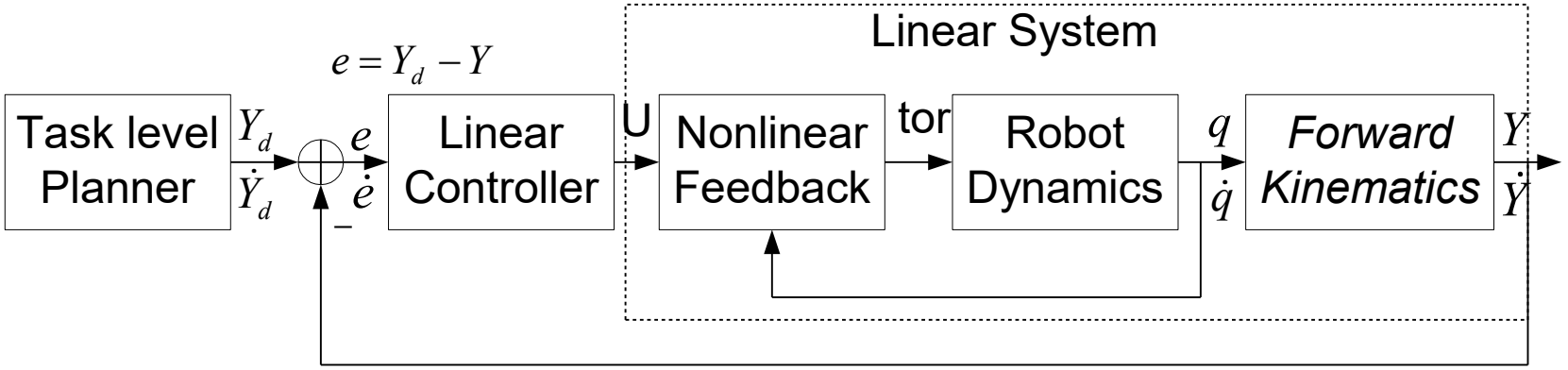
Robot System: $\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$

Jocobian: $\dot{Y} = \frac{d}{dq}[h(q)] \cdot \dot{q} = J\dot{q} \Rightarrow \ddot{Y} = \dot{J}\dot{q} + J\ddot{q} \Rightarrow \ddot{q} = J^{-1}(\ddot{Y} - \dot{J}\dot{q})$

$$D(q)J^{-1}(\ddot{Y} - \dot{J}\dot{q}) + H(q,\dot{q}) + C(q) = \tau$$

Robot Motion Control (III)

- Non-linear Feedback Control



Design the nonlinear feedback controller as:

$$tor = D(q)J^{-1}(U - \dot{J}\dot{q}) + H(q,\dot{q}) + C(q)$$

Then the linearized dynamic model:

$$D(q)J^{-1}\ddot{Y} = D(q)J^{-1}U \quad \Rightarrow \quad \ddot{Y} = U$$

Design the linear controller: $U = \ddot{Y}_d + k_v(\dot{Y}_d - \dot{Y}) + k_p(Y_d - Y)$

Error dynamic equation: $\ddot{e} + k_v\dot{e} + k_p e = 0$

Thank you!