INDUSTRIAL AUTOMATION & ROBOTICS TECHNOLOGY

Trajectory Planning

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Materials used

- Chapter 5, Introduction to Robotics, Saeed B. Niku

Chapter 5
Trajectory Planning

PATH VS. TRAJECTORY

- Path: A sequence of robot configurations in a particular order without regard to the timing of these configurations.
- Trajectory: It concerned about when each part of the path must be attained, thus specifying timing.

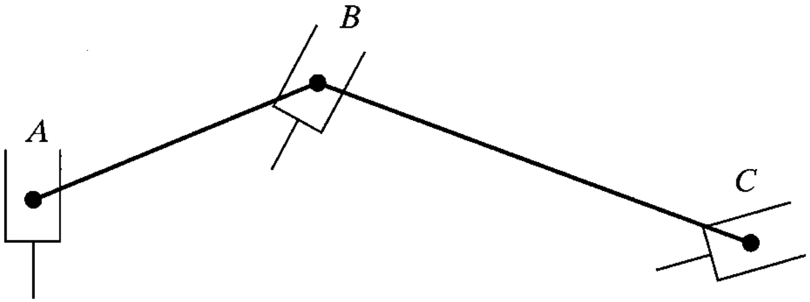

Fig. 5.1 Sequential robot movements in a path.

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JOINT-SPACE VS. CARTESIAN-SPACE DESCRIPTIONS

- Joint-space description:
  - The description of the motion to be made by the robot by its joint values.
  - The motion between the two points is unpredictable.
- Cartesian space description:
  - The motion between the two points is known at all times and controllable.
  - It is easy to visualize the trajectory, but is is difficult to ensure that singularity.

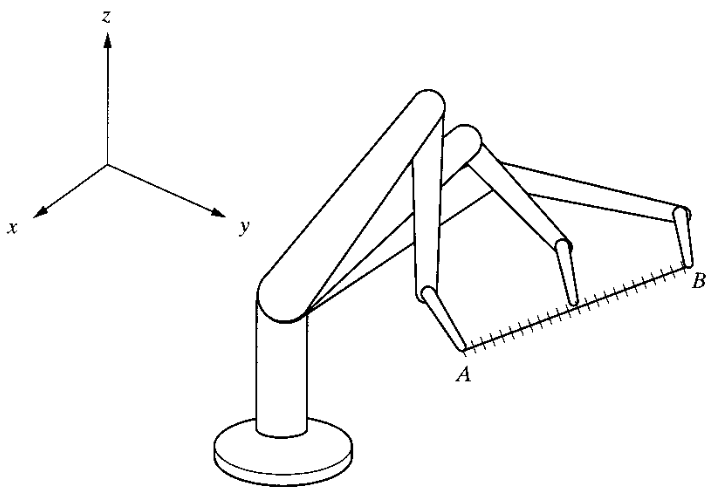

Fig. 5.2 Sequential motions of a robot to follow a straight line.

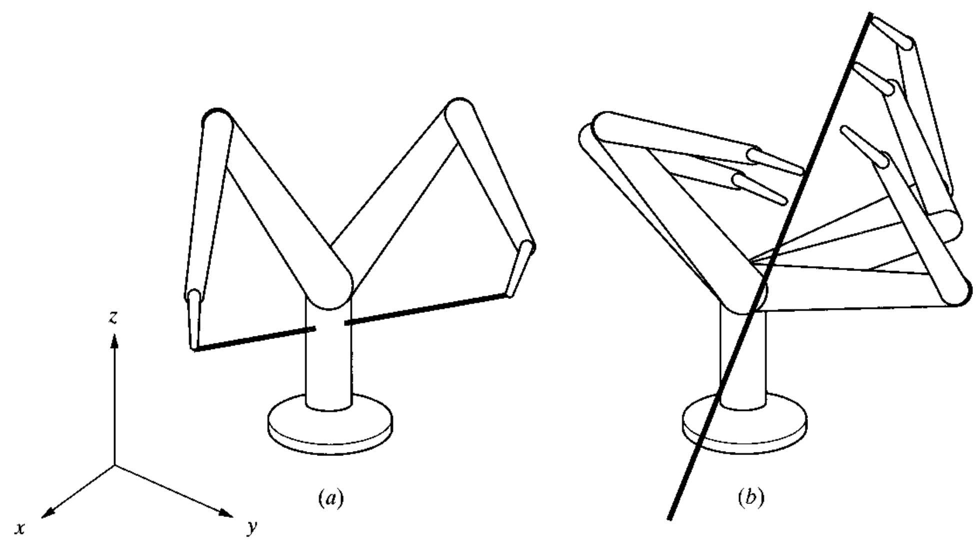

Fig. 5.3 Cartesian-space trajectory (a) The trajectory specified in Cartesian coordinates may force the robot to run into itself, and (b) the trajectory may requires a sudden change in the joint angles.

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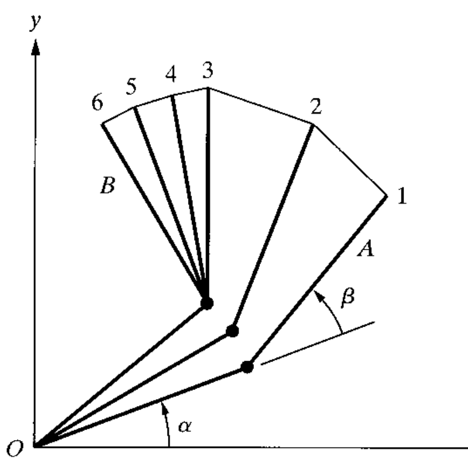
Trajectory Planning

BASICS OF TRAJECTORY PLANNING

- Let's consider a simple 2 degree of freedom robot.
- We desire to move the robot from Point A to Point B.
- Let's assume that both joints of the robot can move at the maximum rate of 10 degree/sec.



α	β
20	30
30	40
40	50
40	60
40	70
40	80

Fig. 5.4 Joint-space nonnormalized movements of a robot with two degrees of freedom.

- **Move the robot from A to B, to run both joints at their maximum angular velocities.**
- **After 2 [sec], the lower link will have finished its motion, while the upper link continues for another 3 [sec].**
- **The path is irregular and the distances traveled by the robot's end are not uniform.**

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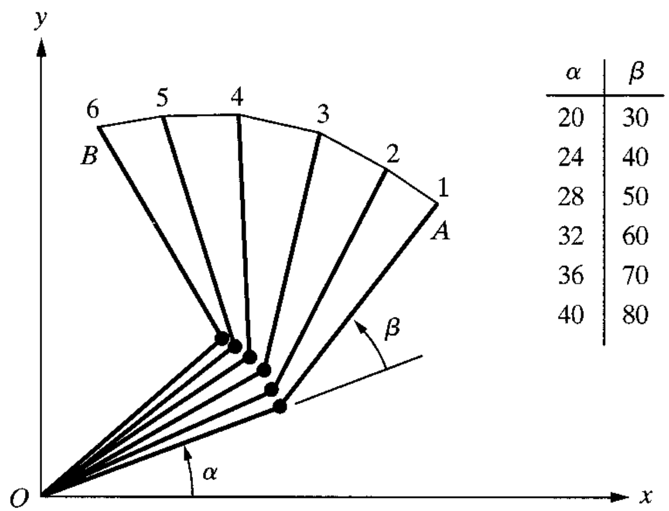
BASICS OF TRAJECTORY PLANNING

- Let's assume that the motions of both joints are normalized by a common factor such that the joint with smaller motion will move proportionally slower and the both joints will start and stop their motion simultaneously.



α	β
20	30
24	40
28	50
32	60
36	70
40	80

Fig. 5.5 Joint-space, normalized movements of a robot with two degrees of freedom.

- **Both joints move at different speeds, but move continuously together.**
- **The resulting trajectory will be different.**

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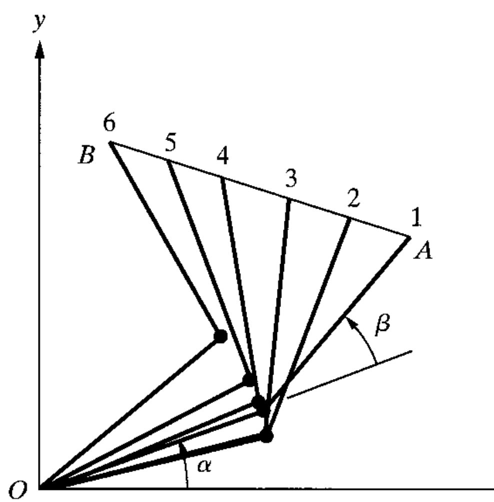
Trajectory Planning

BASICS OF TRAJECTORY PLANNING

- Let's assume that the robot's hand follow a known path between point A to B with straight line.
- The simplest solution would be to draw a line between points A and B, so called interpolation.



α	β
20	30
14	55
16	69
21	77
29	81
40	80

Fig. 5.6 Cartesian-space movements of a two-degree-of-freedom robot.

- **Divide the line into five segments and solve for necessary angles α and β at each point.**
- **The joint angles are not uniformly changing.**

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BASICS OF TRAJECTORY PLANNING

- Let's assume that the robot's hand follow a known path between point A to B with straight line.

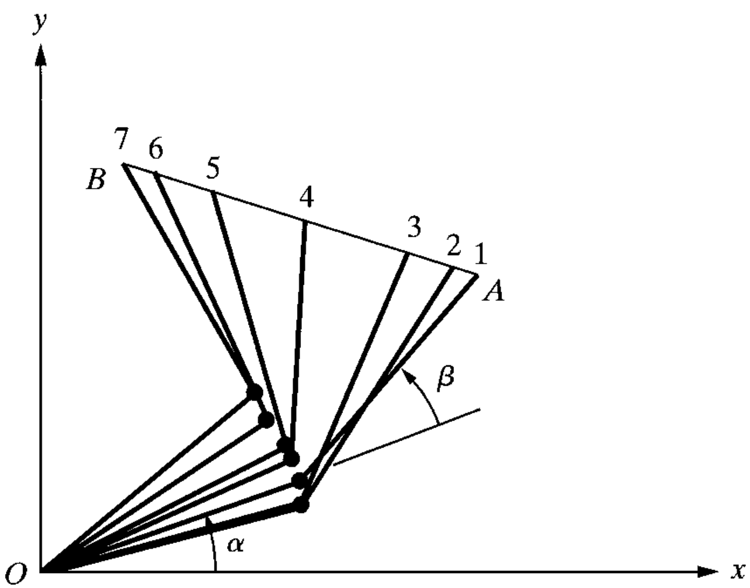

Fig. 5.7 Trajectory planning with an acceleration-deceleration regiment.

- The simplest solution would be to draw a line between points A and B, so called interpolation.

- **It is assumed that the robot's actuators are strong enough to provide large forces necessary to accelerate and decelerate the joints as needed.**

- **Divide the segments differently.**
  - **The arm move at smaller segments as we speed up at the beginning.**
  - Go at a constant cruising rate.
  - Decelerate with smaller segments as approaching point B.

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BASICS OF TRAJECTORY PLANNING

- Next level of trajectory planning is between multiple points for continuous movements.
- Stop-and-go motion create jerky motions with unnecessary stops.
- **Blend the two portions of the motion at point B.**
- **Specify two via point D and E before and after point B**

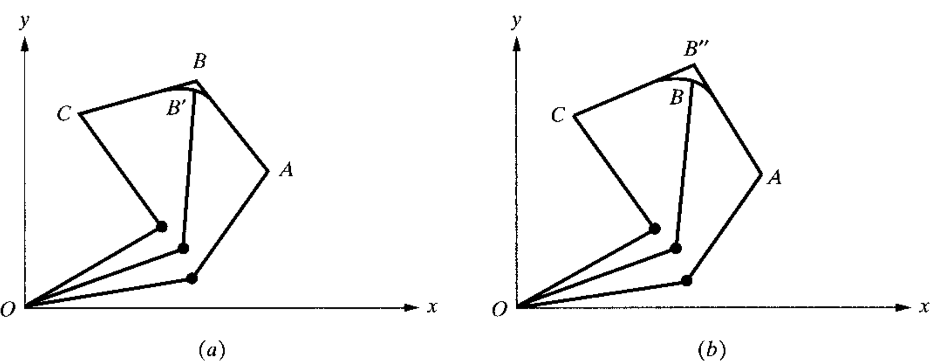

Fig. 5.8 Blending of different motion segments in a path.

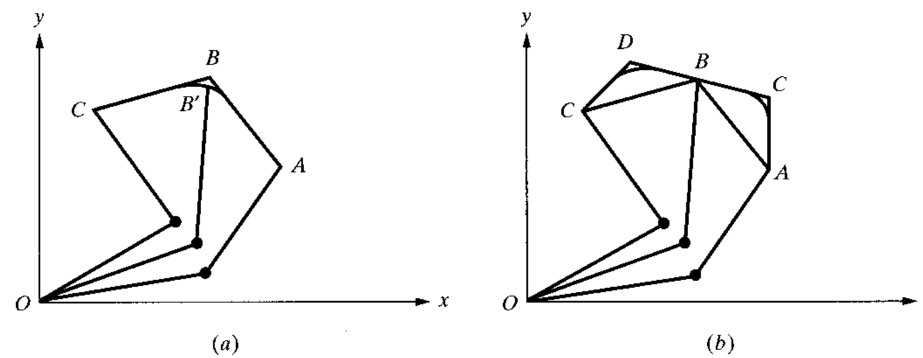

Fig. 5.9 An alternative scheme for ensuring that the robot will go through a specified point during blending of motion segments. Two via points D and E are picked such that point B will fall on the straight-line section of the segment ensuring that the robot will pass through point B.

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Trajectory Planning

JOINT-SPACE TRAJECTORY PLANNING

Third-Order Polynomial Trajectory Planning

- How the motions of a robot can be planned in joint-space with controlled characteristics.
- Polynomials of different orders
- Linear functions with parabolic blends

- **The initial location and orientation of the robot is known, and using the inverse kinematic equations, we find the final joint angles for the desired position and orientation.**

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\theta(t_i) = \theta_i$$

$$\theta(t_f) = \theta_f$$

$$\dot{\theta}(t_i) = 0$$

$$\dot{\theta}(t_f) = 0$$

- **Initial Condition**

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

- **First derivative of the polynomial of equation**

$$\theta(t_i) = c_0 + c_1 t_i + c_2 t_i^2 + c_3 t_i^3 = \theta_i$$

$$\theta(t_f) = c_0 + c_1 t_f + c_2 t_f^2 + c_3 t_f^3 = \theta_f$$

$$\dot{\theta}(t_i) = c_1 + 2c_2 t_i + 3c_3 t_i^2 = 0$$

$$\dot{\theta}(t_f) = c_1 + 2c_2 t_f + 3c_3 t_f^2 = 0$$

- **Substituting the initial and final conditions**

Example 5.1

- It is desired to have the first joint of a six-axis robot go from initial angle of 30° to a final angle of 75° in 5 seconds. Using a third-order polynomial, calculate the joint angle at 1, 2 3, and 4 seconds.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\theta(0) = c_0 = 30 \qquad \dot{\theta}(0) = c_1 = 0$$

Example 5.2

- Suppose the robot arm of Example 5.1 is to continue to the next point, where the joint is to reach 105 deg in another 3 seconds.
- Draw the position, velocity, and acceleration curves for the motion.

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JOINT-SPACE TRAJECTORY PLANNING

Fifth-Order Polynomial Trajectory Planning

- Specify the initial and ending accelerations for a segment.
- To use a fifth-order polynomial for planning a trajectory, the total number of boundary conditions is 6.

- **Calculation of the coefficients of a fifth-order polynomial with position, velocity and a acceleration boundary conditions can be possible with below equations.**

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3$$

Example 5.3

- Repeat example 5.1, but assume the initial acceleration and final deceleration will be 5deg/sec^2.

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JOINT-SPACE TRAJECTORY PLANNING
Linear Segments with Parabolic Blends

- Linear segment can be blended with parabolic sections at the beginning and the end of the motion segment, creating continuous position and velocity.
- Acceleration is constant for the parabolic sections, yielding a continuous velocity at the common points A and B.

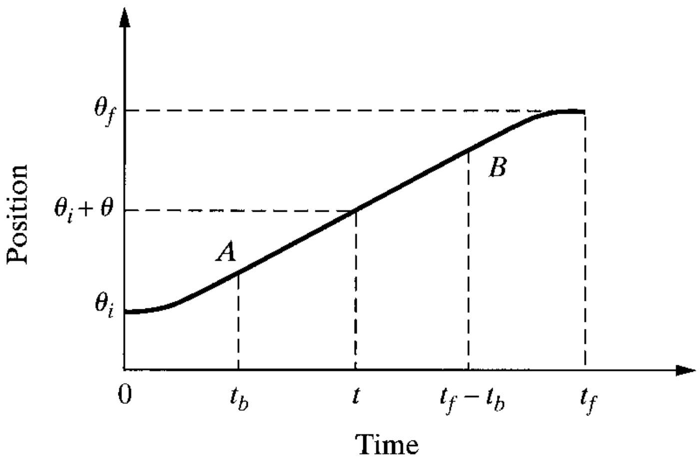

Fig. 5.13 Scheme for linear segments with parabolic blends.

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$$\theta(t) = c_0 + c_1 t + \frac{1}{2} c_2 t^2$$

$$\dot{\theta}(t) = c_1 + c_2 t$$

$$\ddot{\theta}(t) = c_2$$

$$\begin{cases} \theta(t=0) = \theta_i = c_0 \\ \dot{\theta}(t=0) = 0 = c_1 \\ \ddot{\theta}(t) = c_2 \end{cases} \rightarrow \begin{cases} c_0 = \theta_i \\ c_1 = 0 \\ c_2 = \ddot{\theta} \end{cases}$$

$$\theta(t) = \theta_i + \frac{1}{2} c_2 t^2$$

$$\dot{\theta}(t) = c_2 t$$

$$\ddot{\theta}(t) = c_2$$

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- Clearly, for the linear segment, the velocity will be constant and can be chosen based on the physical capabilities of the actuators. Substituting zero initial velocity, a constant known joint velocity v in the linear portion and zero final velocity in Equation (5.11), we find the joint positions and velocities for points A, B and the final point as follows:

$$\theta_A = \theta_i + \frac{1}{2}c_2 t_b^2$$

$$\dot{\theta}_A = c_2 t_b = \omega$$

$$\theta_B = \theta_A + \omega((t_f - t_b) - t_b) = \theta_A + \omega(t_f - 2t_b)$$

$$\dot{\theta}_B = \dot{\theta}_A = \omega$$

$$\theta_f = \theta_B + (\theta_A - \theta_i)$$

$$\dot{\theta}_f = 0$$

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The necessary blending time t_b can be found from

$$\begin{cases} c_2 = \dfrac{\omega}{t_b} \\ \theta_f = \theta_i + c_2 t_b^2 + \omega(t_f - 2t_b) \end{cases} \rightarrow \theta_f = \theta_i + \left(\frac{\omega}{t_b}\right) t_b^2 + \omega(t_f - 2t_b)$$

we calculate the blending time as:

$$t_b = \frac{\theta_i - \theta_f + \omega t_f}{\omega}$$

Obviously, t_b cannot be bigger than half of the total time t_f, which results in a parabolic speed-up and a parabolic slowdown, with no linear segment. A corresponding maximum velocity of $\omega_{max} = 2(\theta_f - \theta_i)/t_f$ can be found

The final parabolic segment is symmetrical with the initial parabola, but with a negative acceleration; therefore, it can be expressed as follows:

$$\theta(t) = \theta_f - \frac{1}{2}c_2(t_f - t)^2 \text{ where } c_2 = \frac{\omega}{t_b} \rightarrow \begin{cases} \theta(t) = \theta_f - \dfrac{\omega}{2t_b}(t_f - t)^2 \\ \dot{\theta}(t) = \dfrac{\omega}{t_b}(t_f - t) \\ \ddot{\theta}(t) = -\dfrac{\omega}{t_b} \end{cases}$$

Example 5.4

- Joint 1 of the 6-axis robot of Example 5.1 is to go from initial angle of $\theta_i = 30^\circ$ to the final angle of $\theta_f = 70^\circ$ in 5 seconds with a cruising velocity of $w_i = 10\ deg/sec$.
- Find the necessary time for blending and plot the joint positions, velocities, and accelerations.

JOINT-SPACE TRAJECTORY PLANNING

Higher Order Trajectories

- Incorporating the initial and final boundary conditions together with this information enables us to use higher order polynomials in the below form, so that the trajectory will pass through all specified points.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + \cdots\cdots + c_{n-1} t^{n-1} + c_n t^n$$

- It requires extensive calculation for each joint and higher order polynomials.

- Combinations of lower order polynomials for different segments of the trajectory and blending together to satisfy all required boundary conditions is required.

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CARTESIAN-SPACE TRAJECTORIES

- Cartesian-space trajectories relate to the motions of a robot relative to the Cartesian reference frame.
- In Cartesian-space, the joint values must be repeatedly calculated through the inverse kinematic equations of the robot.

- Computer Loop Algorithm

(1) Increment the time by t=t+Δt.

(2) Calculate the position and orientation of the hand based on the selected function for the trajectory.

(3) Calculate the joint values for the position and orientation through the inverse kinematic equations of the robot.

(4) Send the joint information to the controller.

(5) Go to the beginning of the loop

Thank you!