Name: ____ Imza: ______________________________

Quiz – I

MECE 574 Industrial Automation and Robotics Technology

Fall 2025

Q1: For Euler angles with rotation sequence X-Z-Y, where

- Roll ϕ is about X-axis
- Yaw ψ is about Z-axis
- Pitch θ is about Y-axis

1. Build a generic rotation matrix for
   a. The extrinsic (axis fixed in space) rotations
   b. The intrinsic (rotations about the body's moving axis) rotations
2. Evaluate both composite rotation matrices for $(\phi, \psi, \theta) = \left(\frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{2}\right)$.
3. Compare the results for both extrinsic and intrinsic rotation matrices.

Solution:

1.a. Extrinsic rotation

As the rotations are around fixed axis, therefore pre-multiplication is performed

$$R_{XZY} = Rot(Y,\theta)Rot(Z,\psi)Rot(X,\phi)$$

$$R_{XZY} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}\begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$R_{XZY} = \begin{bmatrix} c\theta c\psi & s\theta s\phi - c\theta c\phi s\psi & c\phi s\theta + s\phi c\theta s\psi \\ s\psi & c\phi c\psi & -s\phi c\psi \\ -s\theta c\psi & c\theta s\phi + c\phi s\theta s\psi & c\phi c\theta - s\phi s\theta s\psi \end{bmatrix}$$

1.b. Intrinsic rotation

As the rotations are around moving axis, therefore post-multiplication is performed

$$R_{XZY} = Rot(X,\phi)Rot(Z,\psi)Rot(Y,\theta)$$

$$R_{XZY} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}\begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$R_{XZY} = \begin{bmatrix} c\theta c\psi & -s\psi & s\theta c\psi \\ s\phi s\theta + c\phi c\theta s\psi & c\phi c\psi & c\phi s\theta s\psi - s\phi c\theta \\ s\phi c\theta s\psi - c\phi s\theta & s\phi c\psi & c\phi c\theta + s\phi s\theta s\psi \end{bmatrix}$$

2. Evaluate

$$R_{ext} = \begin{bmatrix} 0 & 0.707 & 0.707 \\ 1 & 0 & 0 \\ 0 & 0.707 & -0.707 \end{bmatrix}$$

$$R_{int} = \begin{bmatrix} 0 & -1 & 0 \\ 0.707 & 0 & 0.707 \\ -0.707 & 0 & 0.707 \end{bmatrix}$$

3. The results for both operations are not equal due to the order of the rotation.

Moreover, once you form an expression, then mathematically the result operation from L-R or from R-L must both be same.

Q2: Calculate the forward kinematics of the 3-link robot manipulator. Draw link-coordinate diagram. Determine all kinematics parameters of the DH Table. Determine arm matrix from base-tool.

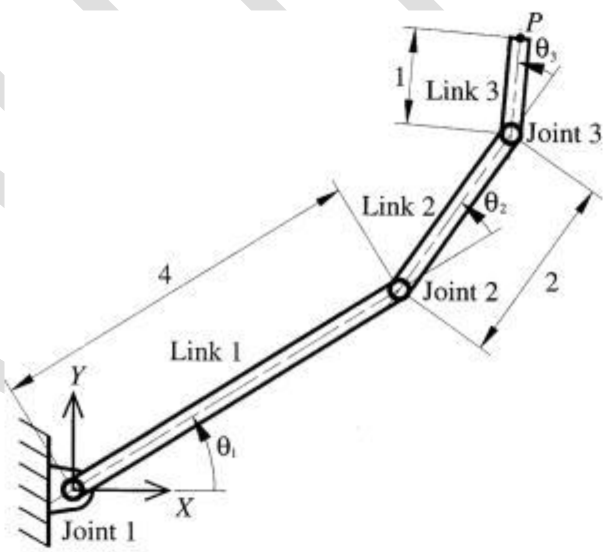

Figure 1: 3-link robot manipulator

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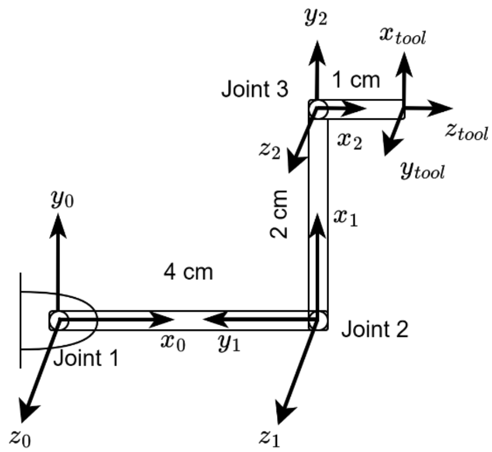

DH Table

	α	a	d	θ
1	0	4	0	θ_1
2	0	2	0	θ_2
3	90	1	0	θ_3

General Homogenous transformation matrix for DH transformation

$$T = \begin{bmatrix} cos\theta & -cos\alpha\ sin\theta & sin\alpha\ sin\theta & acos\theta \\ sin\theta & cos\alpha\ cos\theta & -sin\alpha\ cos\theta & asin\theta \\ 0 & sin\alpha & cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 4cos\theta_1 \\ sin\theta_1 & cos\theta_1 & 0 & 4sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & 2cos\theta_2 \\ sin\theta_2 & cos\theta_2 & 0 & 2sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} cos\theta_3 & 0 & -sin\theta_3 & cos\theta_3 \\ sin\theta_3 & 0 & cos\theta_3 & sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = T_0^1 T_1^2 T_2^3 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$T_0^3 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 4cos\theta_1 \\ sin\theta_1 & cos\theta_1 & 0 & 4sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & 2cos\theta_2 \\ sin\theta_2 & cos\theta_2 & 0 & 2sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_3 & 0 & -sin\theta_3 & cos\theta_3 \\ sin\theta_3 & 0 & cos\theta_3 & sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} c_{123} & 0 & s_{123} & 4c_1 + 2c_{12} + c_{123} \\ s_{123} & 0 & -c_{123} & 4s_1 + 2s_{12} + s_{123} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Q3: Determine joint angles for the 3-link robot manipulator shown in Figure 1 using Inverse Kinematics. Utilize both analytical and geometric approaches to perform IK operation.

Solution:

Analytical Approach

From Eq. (1),

$$\frac{s_{123}}{c_{123}} = \frac{t_{21}}{t_{11}}$$

$$\theta_{123} = tan^{-1}\left(\frac{t_{21}}{t_{11}}\right)$$

Also remember that $\theta_{123} = \theta_1 + \theta_2 + \theta_3$ and $\theta_{12} = \theta_1 + \theta_2$.

$$(T_0^1)^{-1} T_0^3 = T_1^2 T_2^3$$

$$(T_0^1)^{-1} = \begin{bmatrix} c\theta_1 & s\theta_1 & 0 & -4 \\ -s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c\theta_1 & s\theta_1 & 0 & -4 \\ -s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{23} & 0 & s_{23} & c_{23} + 2c_2 \\ s_{23} & 0 & -c_{23} & s_{23} + 2s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n_x c_1 + n_y s_1 & o_x c_1 + o_y s_1 & a_x c_1 + a_y s_1 & p_x c_1 + p_y s_1 - 4 \\ -n_x s_1 + n_y c_1 & -o_x s_1 + o_y c_1 & -a_x s_1 + a_y c_1 & -p_x s_1 + p_y c_1 \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{23} & 0 & s_{23} & c_{23} + 2c_2 \\ s_{23} & 0 & -c_{23} & s_{23} + 2s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

By comparing (2,2) on both sides from Eq. (2)

$$-o_x s_1 + o_y c_1 = 0$$

$$\theta_1 = tan^{-1}\left(\frac{o_y}{o_x}\right)$$

By comparing (1,4) and (2,4) on both sides from Eq. (2)

$$p_x c_1 + p_y s_1 - 4 = c_{23} + 2c_2$$

$$-p_x s_1 + p_y c_1 = s_{23} + 2s_2$$

By applying square on both sides of the equations above

$$\left(p_x c_1 + p_y s_1 - 4\right)^2 = (c_{23} + 2c_2)^2$$

$$\left(p_x c_1 + p_y s_1 - 4\right)^2 = {c_{23}}^2 + 4c_2^2 + 4c_{23}c_2 \tag{3}$$

$$\left(-p_x s_1 + p_y c_1\right)^2 = (s_{23} + 2s_2)^2$$

$$\left(-p_x s_1 + p_y c_1\right)^2 = {s_{23}}^2 + 4s_2^2 + 4s_{23}s_2 \tag{4}$$

By adding Eqs. (3) and (4)

$$\left(p_x c_1 + p_y s_1 - 4\right)^2 + \left(-p_x s_1 + p_y c_1\right)^2 = 5 + 4(c_{23}c_2 + s_{23}s_2)$$

Expanding $c_{23}c_2 + s_{23}s_2 = c_3$. Therefore

$$c_3 = \frac{\left(p_x c_1 + p_y s_1 - 4\right)^2 + \left(-p_x s_1 + p_y c_1\right)^2 - 5}{4}$$

Using identity $s_3 = \pm\sqrt{1 - c_3^2}$.

$$\theta_3 = tan^{-1}\left(\frac{s_3}{c_3}\right)$$

We can use the combined angle $\theta_{123} = \theta_1 + \theta_2 + \theta_3$ to determine our last unknown angle, θ_2.

$$\theta_{123} - \theta_1 - \theta_3 = \theta_2$$

This concludes our analytical analysis.

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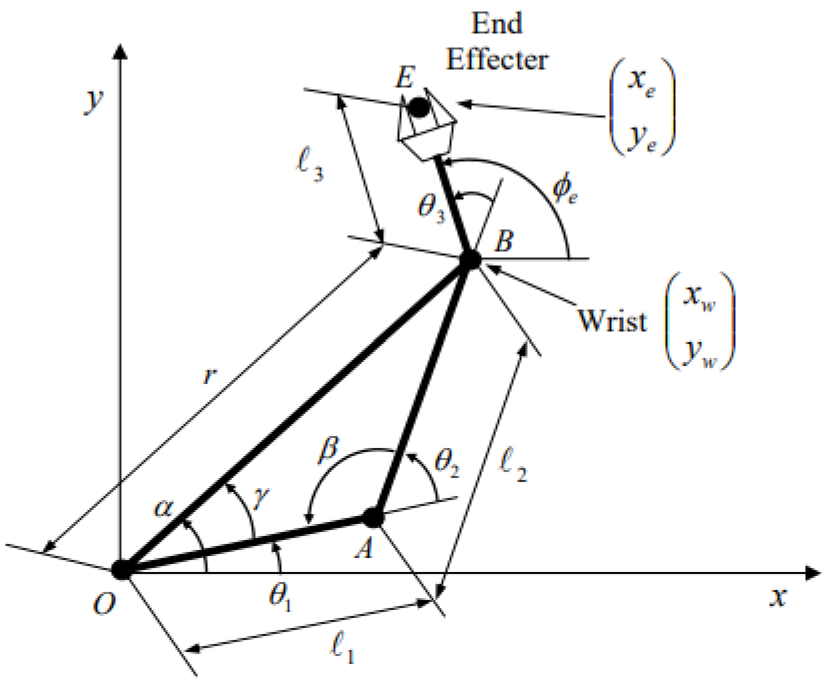
Geometric approach



The wrist coordinates are given by

$$x_w = x_e - l_3 cos\emptyset_e$$

$$y_w = y_e - l_3 sin\emptyset_e$$

Determine angle α as

$$\alpha = tan^{-1}\frac{y_w}{x_w}$$

Use law of cosine

$$l_1^2 + l_2^2 - 2l_1 l_2 cos\beta = r^2$$

Where $r^2 = x_w^2 + y_w^2$.

Determine angle θ_2.

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$$\theta_2 = \pi - \beta = \pi - \frac{l_1^2 + l_2^2 - x_w^2 - y_w^2}{2l_1 l_2}$$

Use law of cosine

$$r^2 + l_1^2 - 2rl_1 cos\gamma = l_2^2$$

Determine angle θ_1

$$\theta_1 = \alpha - \gamma = tan^{-1}\frac{y_w}{x_w} - cos^{-1}\frac{r^2 + l_1^2 - l_2^2}{2rl_1}$$

Determine angle θ_3

$$\theta_3 = \emptyset_e - \theta_1 - \theta_2$$