INDUSTRIAL AUTOMATION & ROBOTICS TECHNOLOGY

Forward Kinematics

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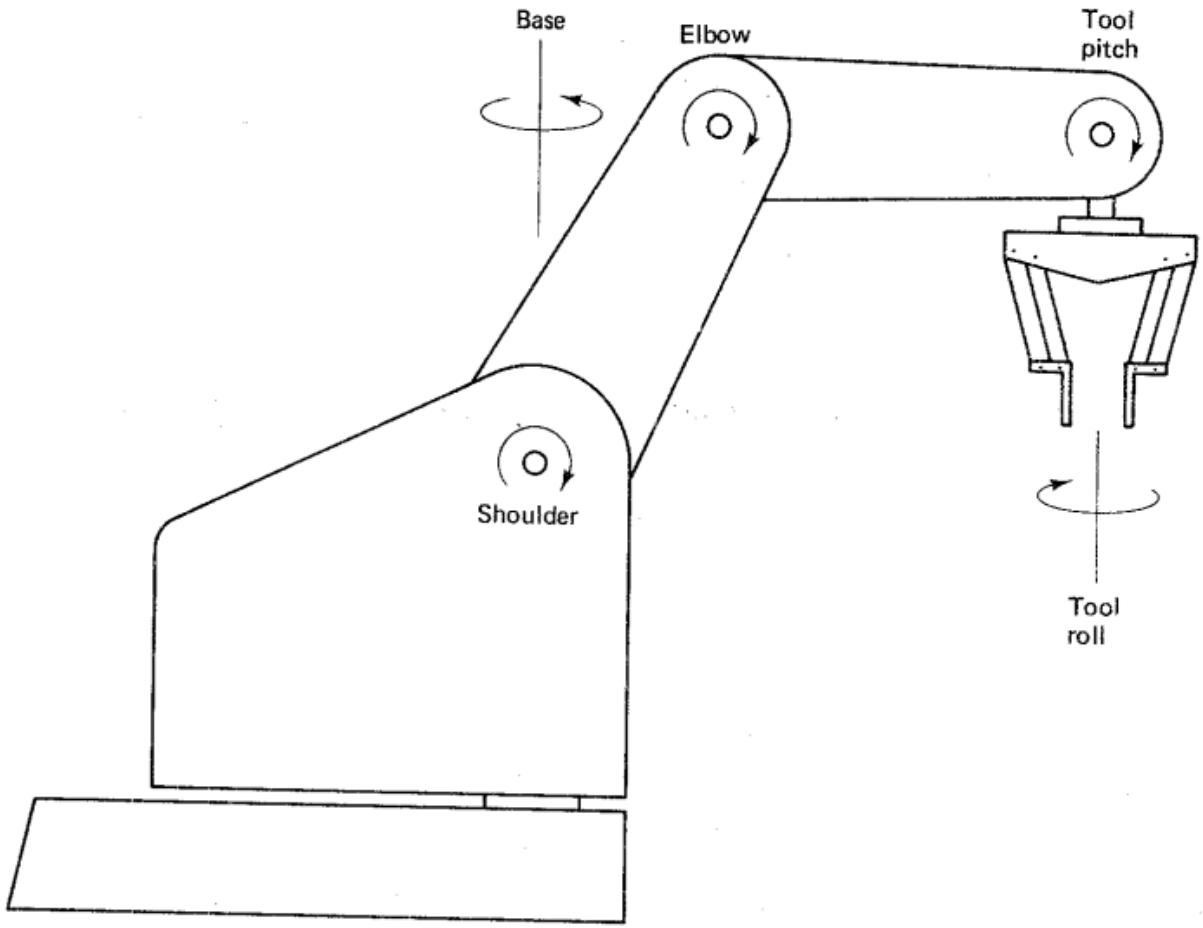
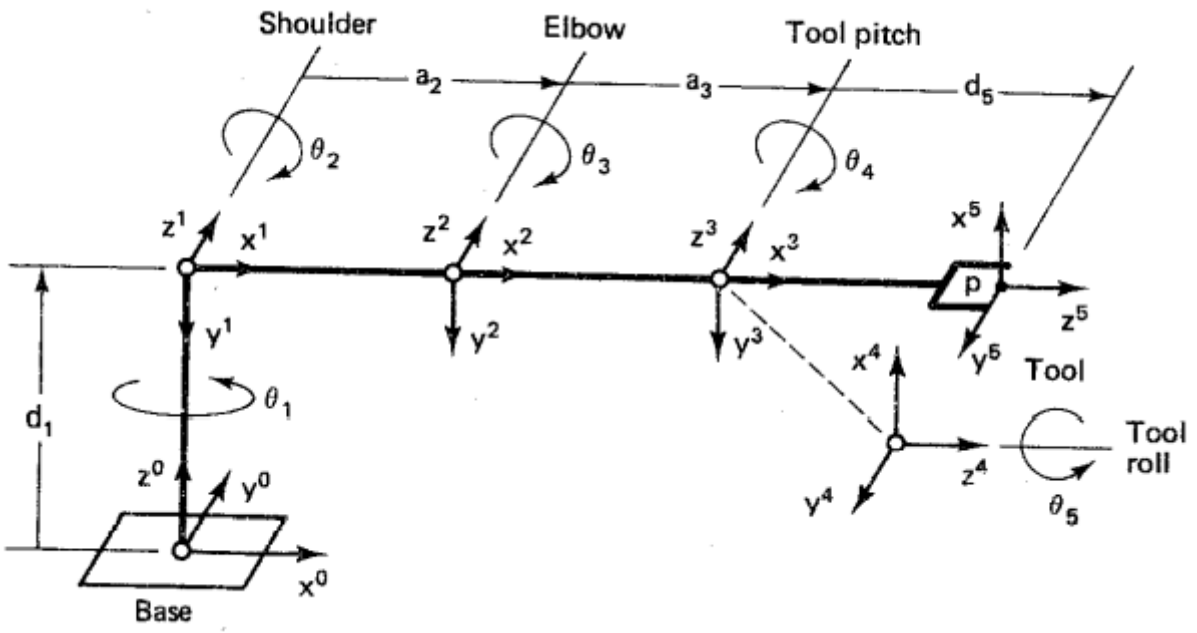
Materials used

- Chapter 2,3 Introduction to Robotics, John J. Craig
- Chapter 2, Introduction to Robotics, Saeed B. Niku

Alpha-II Robotic Arm



Alpha-II: Link coordinate frame



Alpha-II: Kinematic parameters

Axis	θ	d	a	α	Home
1	θ_1	215.0 mm	0	$-\pi/2$	0
2	θ_2	0	177.8 mm	0	0
3	θ_3	0	177.8 mm	0	0
4	θ_4	0	0	$-\pi/2$	$-\pi/2$
5	θ_5	129.5 mm	0	0	0

Link coordinate transformation and its inverse

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{k}^{k-1} = \begin{bmatrix} C\theta_k & S\theta_k & 0 & -a_k \\ -C\alpha_k S\theta_k & C\alpha_k C\theta_k & S\alpha_k & -d_k S\alpha_k \\ S\alpha_k S\theta_k & -S\alpha_k C\theta_k & C\alpha_k & -d_k C\alpha_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation between two frames

$$T_{\text{base}}^{\text{tool}}(q) = T_0^1(q_1)T_1^2(q_2)\cdots T_{n-1}^n(q_n) = T_0^n(q)$$

$$T_{\text{base}}^{\text{wrist}}(q) = T_0^1(q_1)T_1^2(q_2)T_2^3(q_3) = T_0^3(q)$$

$$T_{\text{wrist}}^{\text{tool}}(q) = T_3^4(q_4)T_4^5(q_5)\cdots T_{n-1}^n(q_n) = T_3^n(q)$$

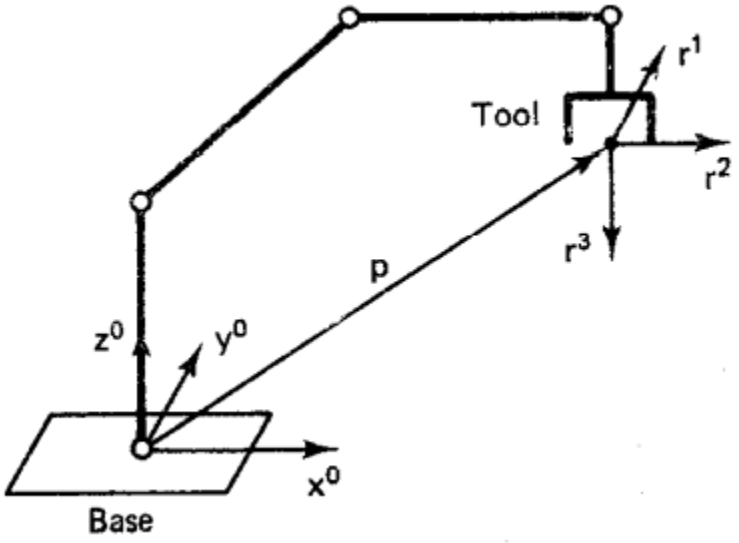
$$T_{\text{base}}^{\text{tool}}(q) = T_{\text{base}}^{\text{wrist}}(q_1, q_2, q_3)T_{\text{wrist}}^{\text{tool}}(q_4, q_5, \ldots, q_n)$$

$$T_{\text{tool}}^{\text{base}}(q) = [T_{\text{base}}^{\text{tool}}(q)]^{-1}$$

Arm Equation / Arm Matrix

Position and orientation of tool in base coordinate frame

$$T^{\text{tool}}_{\text{base}}(q) = \left[\begin{array}{c:c} R(q) & p(q) \\ \hdashline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$



Alpha-II: Wrist w.r.t Base

$$
\begin{aligned}
T_{\text{base}}^{\text{wrist}} = T_0^1 T_1^2 T_2^3 &= \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & a_2C_1C_2 \\ S_1C_2 & -S_1S_2 & C_1 & a_2S_1C_2 \\ -S_2 & -C_2 & 0 & d_1 - a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_1C_{23} & -C_1S_{23} & -S_1 & C_1(a_2C_2 + a_3C_{23}) \\ S_1C_{23} & -S_1S_{23} & C_1 & S_1(a_2C_2 + a_3C_{23}) \\ -S_{23} & -C_{23} & 0 & d_1 - a_2S_2 - a_3S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Alpha-II: Tool w.r.t Wrist

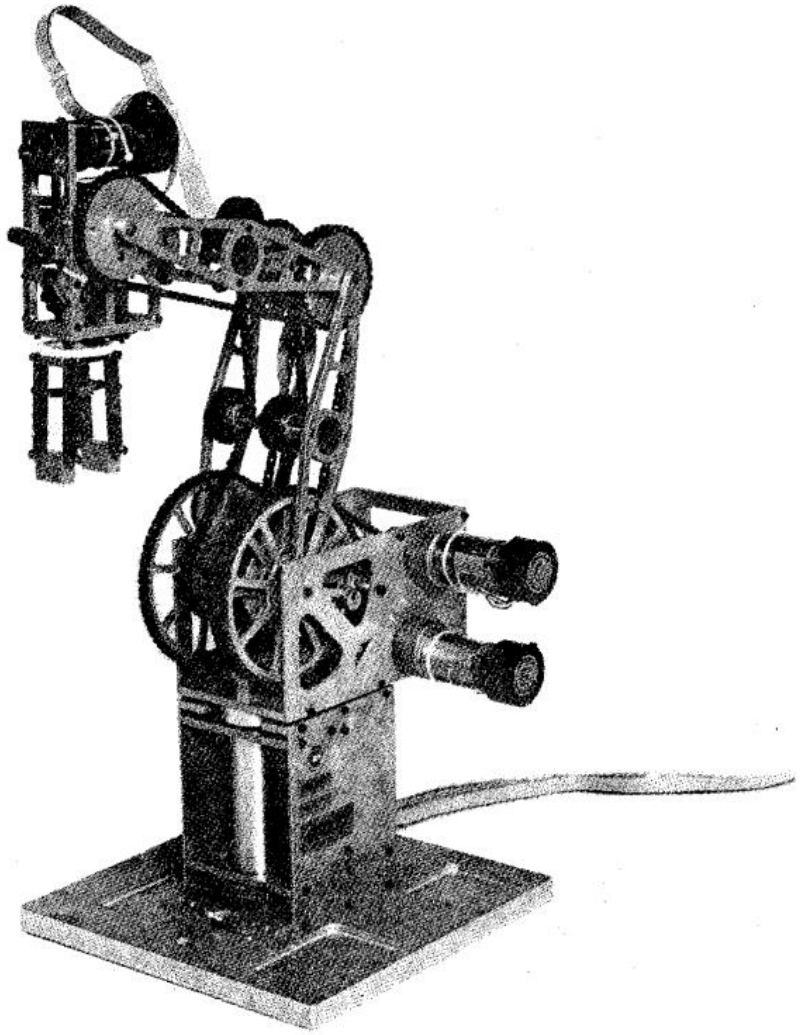
$$
\begin{aligned}
T^{\text{tool}}_{\text{wrist}} &= T^4_3 T^5_4 \\
&= \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_4C_5 & -C_4S_5 & -S_4 & -d_5S_4 \\ S_4C_5 & -S_4S_5 & C_4 & d_5C_4 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Alpha-II: Complete Arm matrix

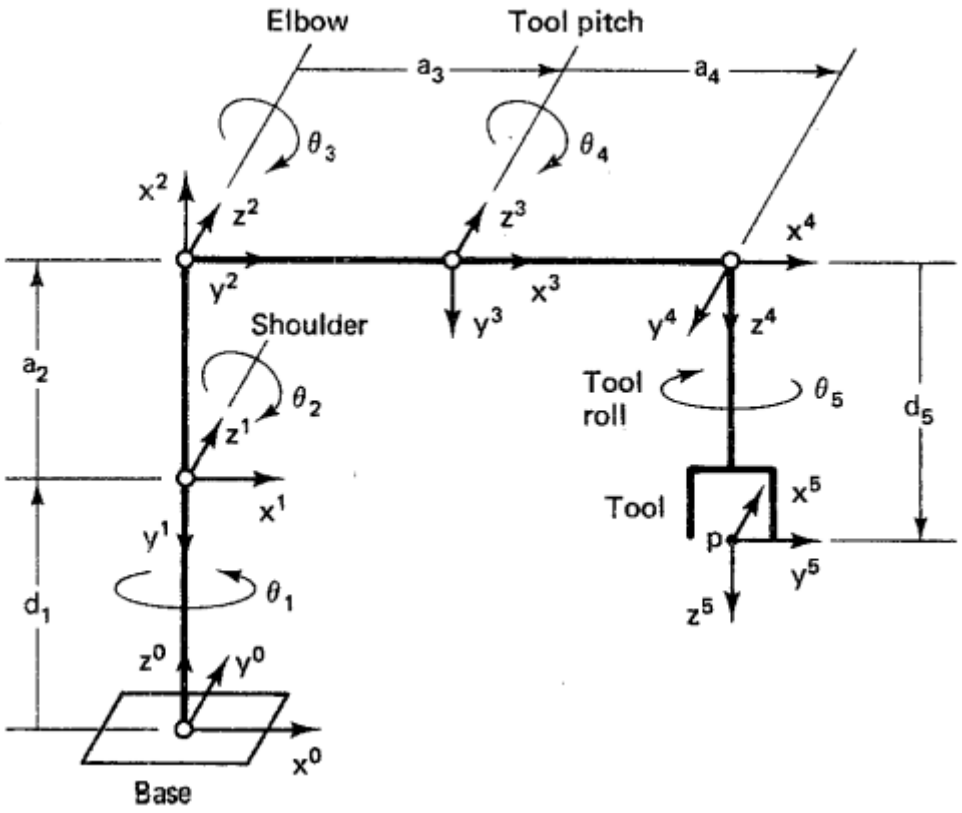
$C_{ijk} = cos(q_i + q_j + q_k)$ and $S_{ijk} = sin(q_i + q_j + q_k)$

$$\left[\begin{array}{ccc|c} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & -C_1S_{234} & C_1(117.8C_2 + 177.8C_{23} - 96.5S_{234}) \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & -S_1S_{234} & S_1(177.8C_2 + 117.8C_{23} - 96.5S_{234}) \\ -S_{234}C_5 & S_{234}S_5 & -C_{234} & 215.9 - 177.8S_2 - 177.8S_{23} - 96.5C_{234} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Five-axis articulated Robot: Rhino XR-3



Rhino XR-3: Link coordinated diagram



Rhino XR-3: Kinematic parameters

Axis	θ	d	a	α	Home
1	q_1	d_1	0	$-\pi/2$	0
2	q_2	0	a_2	0	$-\pi/2$
3	q_3	0	a_3	0	$\pi/2$
4	q_4	0	a_4	$-\pi/2$	0
5	q_5	d_5	0	0	$-\pi/2$

Rhino XR-3: Transformation matrix

$$
\begin{aligned}
T_{\text{base}}^{\text{wrist}} &= T_0^1 T_1^2 T_2^3 \\
&= \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_1 C_2 & -C_1 S_2 & -S_1 & a_2 C_1 C_2 \\ S_1 C_2 & -S_1 S_2 & C_1 & a_2 S_1 C_2 \\ -S_2 & -C_2 & 0 & d_1 - a_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & -S_1 & C_1(a_2 C_2 + a_3 C_{23}) \\ S_1 C_{23} & -S_1 S_{23} & C_1 & S_1(a_2 C_2 + a_3 C_{23}) \\ -S_{23} & -C_{23} & 0 & d_1 - a_2 S_2 - a_3 S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Rhino XR-3: Transformation matrix at home

- $q = \left[0, -\frac{\pi}{2}, \frac{\pi}{2}, 0, -\pi/2\right]^T$

$$T_{\text{base}}^{\text{wrist}}(\text{home}) = \left[\begin{array}{ccc:c} 1 & 0 & 0 & a_3 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 + a_2 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Rhino XR-3: Transformation matrix

$$
\begin{aligned}
T^{\text{tool}}_{\text{wrist}} &= T^4_3 T^5_4 \\
&= \begin{bmatrix} C_4 & 0 & -S_4 & a_4 C_4 \\ S_4 & 0 & C_4 & a_4 S_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} C_4 C_5 & -C_4 S_5 & -S_4 & a_4 C_4 - d_5 S_4 \\ S_4 C_5 & -S_4 S_5 & C_4 & a_4 S_4 + d_5 C_4 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Rhino XR-3: Arm matrix

$$\left[\begin{array}{ccc|c} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & -C_1S_{234} & C_1(a_2C_2 + a_3C_{23} + a_4C_{234} - d_5S_{234}) \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & -S_1S_{234} & S_1(a_2C_2 + a_3C_{23} + a_4C_{234} - d_5S_{234}) \\ -S_{234}C_5 & S_{234}S_5 & -C_{234} & d_1 - a_2S_2 - a_3S_{23} - a_4S_{234} - d_5C_{234} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$q = \left[0, -\frac{\pi}{2}, \frac{\pi}{2}, 0, -\pi/2\right]^T$$

$$T_{\text{base}}^{\text{tool}}(\text{home}) = \left[\begin{array}{ccc|c} 0 & 1 & 0 & a_3 + a_4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & d_1 + a_2 - d_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Four-axis Scara robot: Adept one

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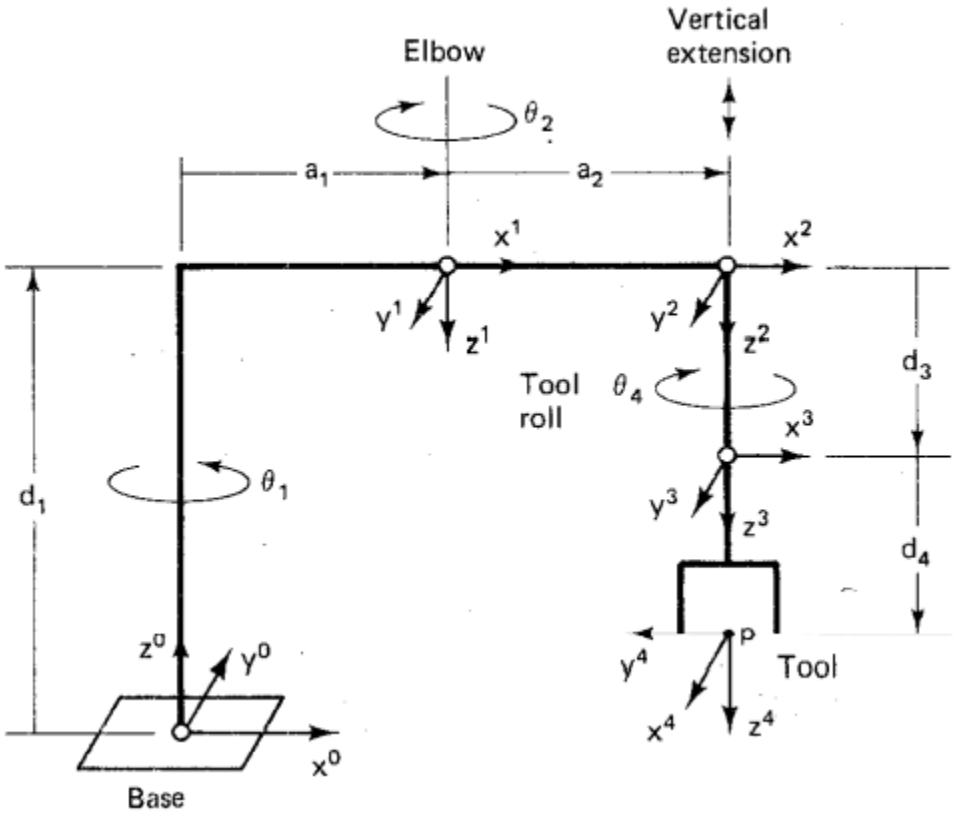

Four-axis Scara robot: Adept one

Kinematic parameters

Axis	θ	d	a	α	Home
1	q_1	d_1	a_1	π	0
2	q_2	0	a_2	0	0
3	0	q_3	0	0	100
4	q_4	d_4	0	0	$\pi/2$

Four-axis Scara robot: Adept one

Arm matrix

$$T_{\text{base}}^{\text{tool}} = T_0^1 T_1^2 T_2^3 T_3^4$$

$$= \begin{bmatrix} C_1 & S_1 & 0 & a_1 C_1 \\ S_1 & -C_1 & 0 & a_1 S_1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thank you!